Past-paper walk-through

Conservation of Electric Energy ▪ 9.3

eXercise

Questions in this set

- 2024 Set 2 Q1
- 2019 Set 2 Q1
- 2019 Set 2 Q2
- 2019 Set 2 Q3
- 2016 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\text{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost

Question 1

equipotential loops on the lower left, with A below B. A horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

Question 1

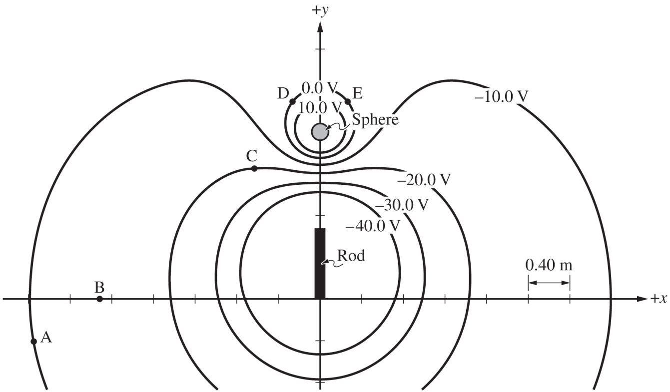


Figure 1

Question 1

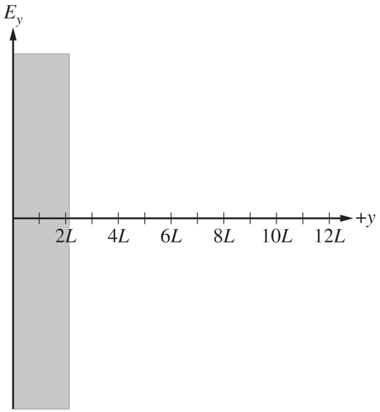


Figure 4

Question 1

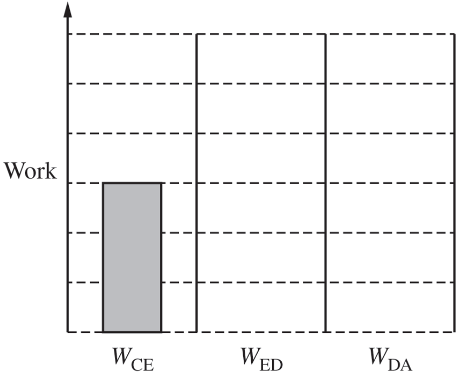


Figure 2

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the y-axis, in the xy-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to move the test charge to different positions in the order of C$\rightarrow$E$\rightarrow$D$\rightarrow$A. The test charge is held momentarily at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

Draw a bar to represent the relative absolute value of the work W_{ED}, done by the external force on the test charge to move the test charge from Position E to Position D.

Draw a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.

Question 1

The height of each bar should be proportional to the value of W_{CE}. If $W_{\mathrm{ED}} = 0$ and/or $W_{\mathrm{DA}} = 0$, write a "0" in the corresponding columns, as appropriate.
[Figure 2: A bar chart with vertical axis labeled "Work" (unlabeled scale, dashed horizontal gridlines) and three columns on the horizontal axis labeled W_{CE}, W_{ED}, W_{DA}. A single shaded bar of medium height is already drawn above the W_{CE} column; the W_{ED} and W_{DA} columns are empty for the student to complete.]
(b)(ii) Calculate the approximate magnitude of the *x*-component of the electric field at Position B.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(c) The positive test charge is placed at Position C. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

______ $+x$ ______ $+y$ ______ Directly away from the sphere

______ $-x$ ______ $-y$ ______ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(d)(i) [Figure 3: A vertical y-axis with tick marks labeled from bottom to top: $-\lambda$ at 0, then $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. A thick vertical bar labeled "Rod" spans from 0 to $2L$ on the axis, adjacent to the label $-\lambda$.]

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.
[Figure 4: A blank graph with vertical axis labeled E_y (no numeric scale shown) and horizontal axis labeled y with tick marks at $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. Axes are otherwise empty for the student to sketch a curve.]

Solution 1

(a)

Mark scheme

- Use $\Phi_E = Q/\varepsilon_0$ (from Gauss's law, $\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$) with $Q = 1.0 \times 10^{-9}$ C enclosed by the 0.0 V equipotential surface:
  $\Phi_E = \dfrac{1.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12}\ \mathrm{C}^2/(\mathrm{N} \cdot \mathrm{m}^2)} = 113\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}$. 1 point for using the correct flux equation, 1 point for the correct numerical answer (units not required for this point).

Solution 1

(b)(i)

Mark scheme

- Positions D and E lie on the same equipotential line, so no work is required to move the test charge between them: $W_{ED} = 0$ (bar of height 0). The bar representing W_{DA} should have a height of approximately one and a half units (about half the height of the given W_{CE} bar). 1 point for indicating $W_{ED} = 0$, 1 point for drawing the W_{DA} bar at roughly 1.5 units.

Solution 1

(b)(ii) Mark scheme

- Relate electric field to the potential gradient between adjacent equipotential lines: $E_y = -dV/dy$, so $|E_y| = |\Delta V/\Delta y|$. Substituting the potential values and spacing of the equipotential lines bracketing Position B: $|E_y| = \left|\dfrac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right| = 7.1\text{ V/m}$. 1 point for using a correct field-potential relationship (may start with numbers already substituted), 1 point for correctly substituting the potential values and equipotential-line spacing used to get the field magnitude at Position B.

Solution 1

(c) Mark scheme

- Direction: $-y$ (only). Justification: the electric field vector is perpendicular to the equipotential line (tangent) at Position C. Because the test charge is positive, it moves from higher to lower electric potential when a force acts on it; at Position C the field/force direction that is perpendicular to the equipotential line and points toward decreasing potential is $-y$ (directly toward the sphere). 1 point for selecting only $-y$ with an attempted justification, 1 point for stating the field is perpendicular to the equipotential line at C, 1 point for one of: the charge moves from higher to lower potential / the test charge and rod have opposite-sign charge / the test charge and sphere have same-sign charge.

Solution 1

(d)(i) Mark scheme

- Set up the potential from a continuous line charge: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$, i.e. $V_P = k\int \left(\frac{1}{y_P - y}\right) dq$, where $y_P - y$ is the distance from a charge element at height y on the rod to point P. Substitute the charge element of the negatively-charged rod, $dq = -\lambda\, dy$: $V_P = -k\lambda \int \left(\frac{1}{y_P - y}\right) dy$. Integrate over the rod's extent with limits $y = 0$ to $y = 2L$: $V_P = -k\lambda \int_0^{2L} \left(\frac{1}{y_P - y}\right) dy = k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$. 1 point for an appropriate potential-from-line-charge equation, 1 point for correctly determining $r = y_P - y$,

Solution 1

1 point for the correct integral with $\lambda\, dy$ substituted for dq, 1 point for the correct limits of integration (0 to 2L).

Solution 1

(d)(ii)

Mark scheme

- For $2L < y < 12L$, sketch E_y as a curve that is always negative (field points toward the negatively-charged rod, i.e. in $-y$) and continually approaches (but never reaches) the horizontal axis as y increases — i.e. a concave-down curve that asymptotically rises toward zero from below, steepest just above $y = 2L$ and flattening out toward $y = 12L$. 1 point for a curve/line that continually approaches the horizontal axis as position increases, 1 point for a concave-down curve that stays always negative.

Question 1

2019 Set 2 ▪ Q1

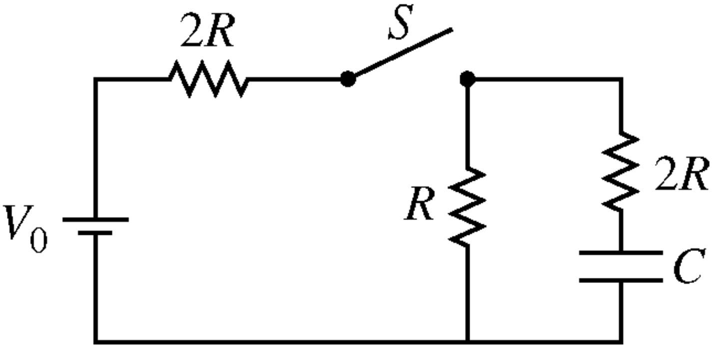
[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) Derive an expression for the steady-state current supplied by the battery.

Question 1

Question 1



Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(b) Derive an expression for the charge on the capacitor.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(c) Derive an expression for the energy stored in the capacitor.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(d) Now the switch is opened at time $t = 0$.

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(e)(i) Calculate the current in resistor R immediately after the switch is opened.

(e)(ii) On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

Question 1

[Graph with vertical axis labeled "Current" and horizontal axis labeled "Time"; origin labeled O; axes are blank/unscaled for the student to sketch on.]

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Solution 1

(a) Mark scheme

- At steady state the capacitor is fully charged and draws no current, so no current flows through the branch containing the second $2R$ resistor and C. The circuit reduces to the top $2R$ in series with R (effective resistance $3R$). By Ohm's law: $I = \dfrac{V_0}{R_{eff}} = \dfrac{V_0}{2R + R}$, so $I = \dfrac{V_0}{3R}$. (1 pt: using Ohm's law with the correct effective resistance; 1 pt: correct substitution leading to the correct answer.)

Solution 1

(b) Mark scheme

- The stored charge is $q = CV_C$. At steady state no current flows through the $2R$-C branch, so $V_C = V_R$ (the parallel branch's voltage). $V_R = IR = \left(\frac{V_0}{3R}\right) R = \frac{1}{3} V_0$. Therefore $q = CV_R = \frac{1}{3} CV_0$. (1 pt: using the equation relating stored charge to capacitance; 1 pt: correctly determining V_R and substituting.)

Solution 1

(c) Mark scheme

- Energy stored in a capacitor: $U = \dfrac{q^2}{2C}$. Substituting $q = \dfrac{1}{3}CV_0$ from part (b): $U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$. (1 pt: any correct equation for energy stored in a capacitor; 1 pt: correct substitution of charge and/or voltage from part (b).)

Solution 1

(d) Mark scheme

- After the switch opens, current can only flow around the loop containing R, the second $2R$, and C. Voltage loop equation: $V_C - V_R - V_{2R} = 0 \Rightarrow \dfrac{q(t)}{C} = I(R + 2R)$. Since the capacitor is discharging, $I = -\dfrac{dq}{dt}$, giving the differential equation $\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$ (do not solve it). (1 pt: any correct voltage loop equation for when the switch is open; 1 pt: substituting $-dq/dt$ or dq/dt for the current, consistent with the loop equation.)

Solution 1

(e)(i) Mark scheme

- The capacitor voltage cannot change instantaneously, so immediately after the switch opens $V_C(0^+) = V_R(0^+) = \dfrac{V_0}{3}$ (from part (b)). Current now flows through the total resistance $R + 2R = 3R$: $I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$. (1 pt: using a voltage consistent with part (b) in $I = V/R$; 1 pt: substituting the correct total resistance $3R$ into Ohm's law.)

Solution 1

(e)(ii) Mark scheme

- The current starts at its maximum value $I_0 = \dfrac{V_0}{9R}$ (from part (e)(i)) at $t = 0$ and decays exponentially toward zero as the capacitor discharges: $I(t) = \dfrac{V_0}{9R}e^{-t/3RC}$. The sketch should show a curve that is concave up throughout, decreasing monotonically toward the horizontal (time) axis as an asymptote, with the initial value explicitly labeled $V_0/9R$ on the vertical axis at $t = 0$. (1 pt: curve concave up throughout the graph; 1 pt: horizontal axis as an asymptote; 1 pt: labeling the maximum current, consistent with part (e)(i).)

Solution 1

(f) Mark scheme

- Equal to. Justification: after the switch opens, the capacitor discharges all of its stored energy and charge; assuming no energy is lost in the connecting wires, the only circuit elements that dissipate this energy are the two resistors (R and 2R) in series with the capacitor, so the total energy dissipated in the resistors equals the energy that was stored in the capacitor (conservation of energy). (1 pt: selecting \"Equal to\"; 1 pt: a correct justification invoking conservation of energy.) [An alternate full-credit path selects \"Less than\" with justification that some energy is additionally lost due to resistance in the connecting wires and/or losses in the capacitor.]

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

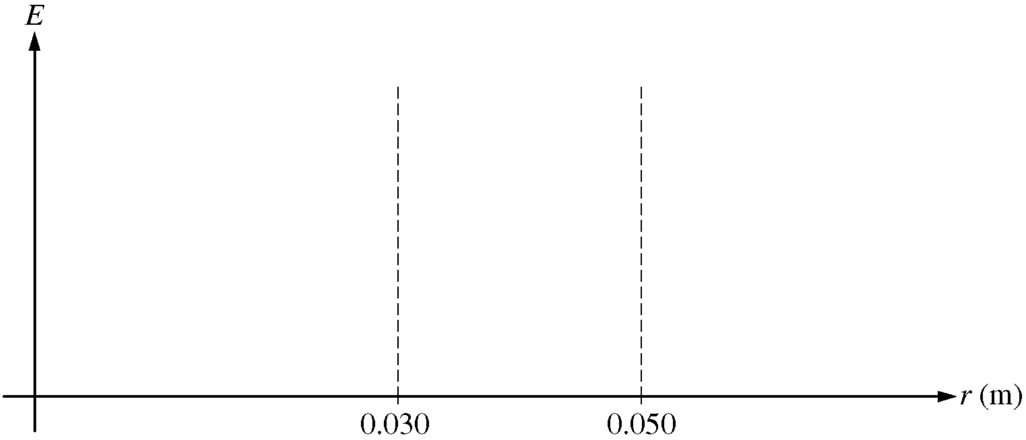
Question 2

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

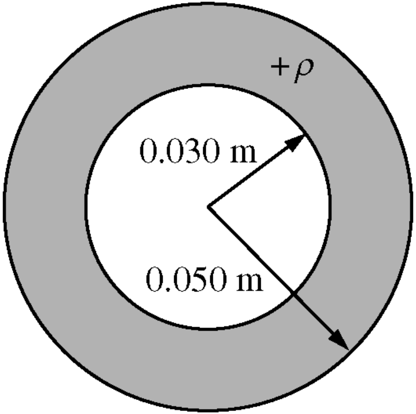
$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

Question 2



Question 2



Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030\ \text{m} : \quad \rho = 0$$

$$0.030\ \text{m} < r < 0.050\ \text{m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6}\ \text{C/m}^2$$

Question 2

$$r > 0.050\ \text{m} : \quad \rho = 0$$

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.
[Graph with vertical axis labeled E and horizontal axis labeled r (m); two vertical dashed reference lines are marked at $r = 0.030$ and $r = 0.050$; the axes are otherwise blank for the student to sketch on.]

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050\ \text{m}: \quad \rho = 0$$

(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(e)(i) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s. Calculate the magnitude of the initial acceleration of the proton.

(e)(ii) Calculate the speed of the proton after a long time.

Solution 2

(a) Mark scheme

- $Q = \int \rho \, dV = \int_{0.03}^{0.05} \frac{b}{r}(4\pi r^2)\, dr = 4\pi b \int_{0.03}^{0.05} r \, dr = 4\pi b \left[\frac{r^2}{2}\right]_{0.03}^{0.05} = 2\pi b \left(0.05^2 - 0.03^2\right)$. With $b = 1.6 \times 10^{-6}$ C/m^2: $Q = 2\pi(1.6 \times 10^{-6})(0.0016) = 1.61 \times 10^{-8}$ C. (1 pt: recognizing the need to integrate the charge density expression, $Q = \int \rho \, dV$, with the spherical shell volume element $4\pi r^2 \, dr$; 1 pt: correct substitution of $\rho = b/r$ into the integral; 1 pt: using the correct limits of integration and obtaining the correct numeric answer.)

Solution 2

(b) Mark scheme

- By symmetry, Gauss's law gives $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$. At the outer surface ($r = 0.05$ m) all the charge is enclosed, so $Q_{enc} = Q = 1.61 \times 10^{-8}$ C (from part (a)). $E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{1.61 \times 10^{-8}}{4\pi(8.85 \times 10^{-12})(0.05)^2} = 5.79 \times 10^4$ N/C. (1 pt: correctly evaluating the surface integral in Gauss's law; 1 pt: substituting the answer from part (a) and the correct radius into the equation; 1 pt: correct final answer with correct units, consistent with part (a).)

Solution 2

(c) Mark scheme

- $E = 0$ for $r < 0.030$ m (no enclosed charge). For 0.030 m $< r < 0.050$ m, E rises continuously from zero, concave down, increasing up to the value found in part (b) at $r = 0.050$ m (as Q_{enc} grows with r inside the shell). For $r > 0.050$ m, all the charge is enclosed and E falls off as $1/r^2$, decreasing asymptotically toward the horizontal (r) axis. (1 pt: clearly showing $E = 0$ for $r < 0.030$ m; 1 pt: a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$; 1 pt: a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m.)

Solution 2

(d) Mark scheme

- With $V = 0$ at infinity, and since at $r = 0.05$ m all the charge is enclosed (the sphere behaves as a point charge from that radius outward): $V_R = \frac{Q_{tot}}{4\pi\varepsilon_0 r} = \frac{(9\times10^9)(1.61\times10^{-8}\ \text{C})}{0.05\ \text{m}} \approx 2900$ V. (Equivalent alternate method: integrate $\Delta V_R = -\int_\infty^r E\,dr = -\int_\infty^{0.05} \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}\,dr = 2900$ V.) (1 pt: substituting the total charge from part (a) into a correct expression for electric potential; 1 pt: substituting $r = 0.05$ m (or the correct limits, if integrating) into that expression.)

Solution 2

(e)(i)

Mark scheme

- Newton's second law with the electric force: $F = ma \Rightarrow qE = ma \Rightarrow a = \dfrac{qE}{m}$. Using the proton's charge $q = 1.6 \times 10^{-19}$ C, mass $m = 1.67 \times 10^{-27}$ kg, and $E = 5.79 \times 10^{4}$ N/C from part (b): $a = \dfrac{(1.6 \times 10^{-19})(5.79 \times 10^{4})}{1.67 \times 10^{-27}} = 5.55 \times 10^{12}$ m/s^2. (1 pt: using a correct expression of Newton's second law in terms of the electric field; 1 pt: correctly substituting values into that equation.)

Solution 2

(e)(ii) Mark scheme

- Using the work-energy theorem with electric potential: $-q\Delta V = \frac{1}{2}mv^2$, with $\Delta V = V_\infty - V_R = 0 - 2900$ V (the proton is repelled outward, converting electric potential energy into kinetic energy as it moves to infinity).
  $v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19})(0 - 2900)}{1.67 \times 10^{-27}}} = 7.45 \times 10^5$ m/s. (1 pt: a correct expression of kinetic energy in terms of the electric potential difference; 1 pt: correctly substituting into the equation and obtaining the correct final answer.)

Question 3

2019 Set 2 ▪ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled ΔV. To the left of and between the plates is a small filled circle labeled "mass $= m$, charge $= e^-$" with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of $\times$ symbols labeled B, representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

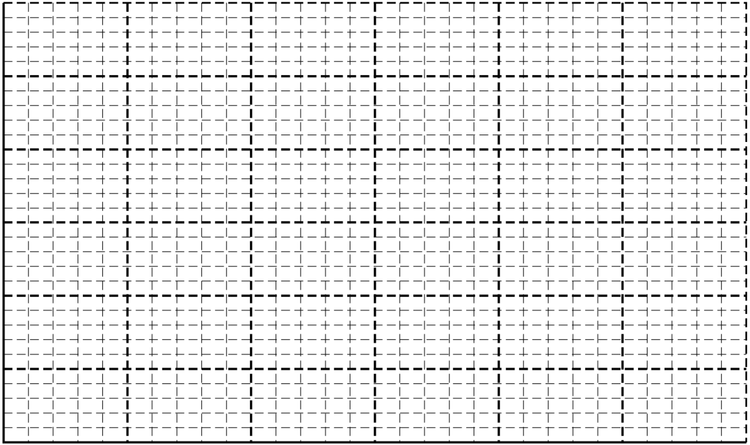
Question 3

(a)(i) What is the initial direction of the force on the sphere as it enters the magnetic field?

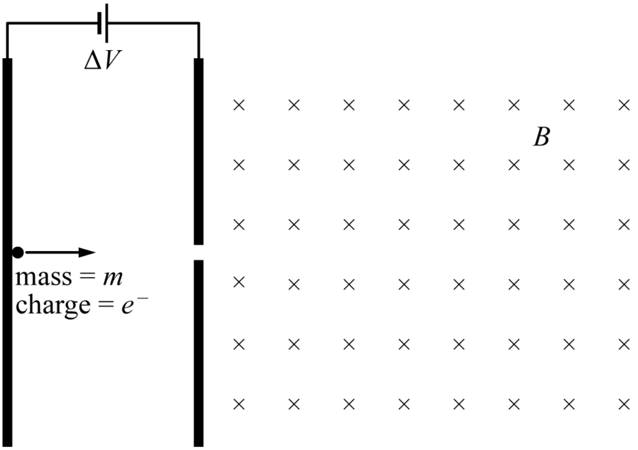
_____ Into the page _____ Out of the page _____ Toward the top of the page _____ Toward the bottom of the page

(a)(ii) Describe the path taken by the sphere after it enters the magnetic field.

Question 3



Question 3



Question 3

Potential difference (V)	60	70	100	110	120	140
Magnetic field $\left(\mathrm{T} \times 10^{-3}\right)$	2.62	2.78	3.39	3.54	3.78	3.99

Question 3

2019 Set 2 ▪ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled ΔV. To the left of and between the plates is a small filled circle labeled "mass $= m$, charge $= e^{-}$" with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of $\times$ symbols labeled B, representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(b) Derive an expression for the speed of the sphere as it passes through the small opening.

Question 3

2019 Set 2 ▪ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled ΔV. To the left of and between the plates is a small filled circle labeled "mass $= m$, charge $= e^-$" with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of $\times$ symbols labeled B, representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(c) Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

Question 3

2019 Set 2 ▪ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled ΔV. To the left of and between the plates is a small filled circle labeled "mass $= m$, charge $= e^-$" with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of $\times$ symbols labeled B, representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(d) An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through

Question 3

the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field (T $\times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:

Horizontal axis:

Question 3

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

Question 3

2019 Set 2 ▪ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled ΔV. To the left of and between the plates is a small filled circle labeled "mass $= m$, charge $= e^-$" with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of $\times$ symbols labeled B, representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

Question 3

(e) On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.
[Blank grid graph for plotting data.]

Question 3

2019 Set 2 ▪ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled ΔV. To the left of and between the plates is a small filled circle labeled "mass $= m$, charge $= e^-$" with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of $\times$ symbols labeled B, representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(f) Using the straight line from part (e), determine the mass-to-charge ratio of an electron.