Past-paper walk-through

Electric Potential Energy ▪ 9.1

eXercise

Questions in this set

- 2024 Set 1 Q1
- 2024 Set 2 Q1
- 2019 Set 2 Q2
- 2016 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the *x*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and the *y*-axis are 0.40 m apart.
[Figure 1: An *x*-*y* coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the *x*-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the *x*-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and

Question 1

between the rod and sphere. A horizontal double-headed arrow near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]
(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

Question 1

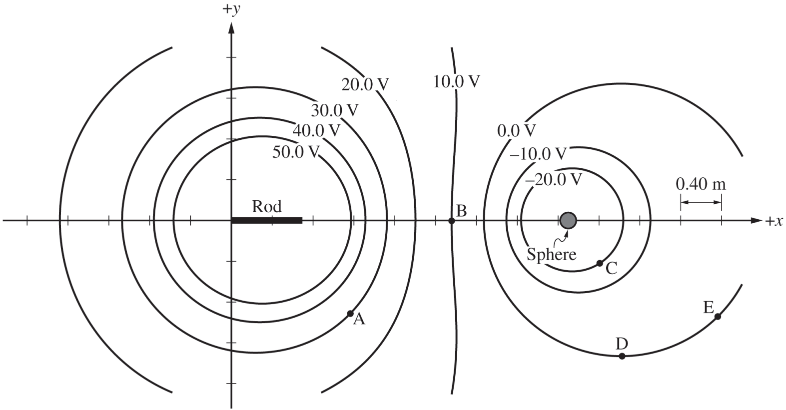

Figure 1

Question 1

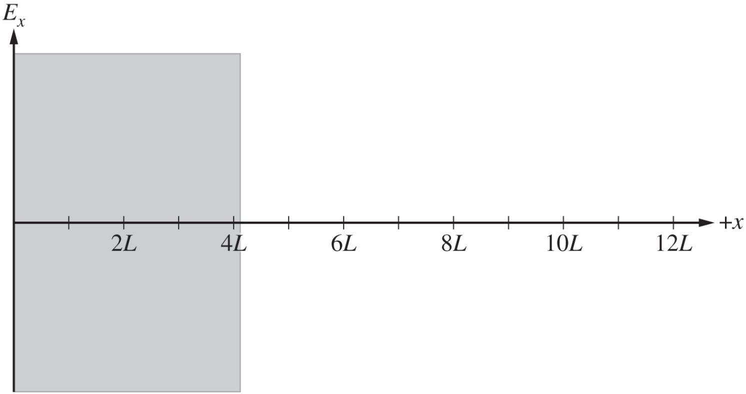

Figure 4

Question 1

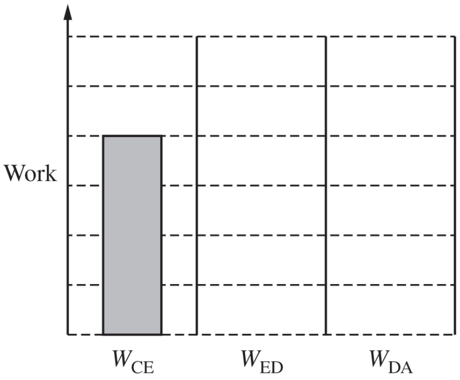

Figure 2

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the *x*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and the *y*-axis are 0.40 m apart.

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Question 1

near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E. **Complete** the following bar graph in Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D. - **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to

Question 1

Position A. - The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.
[Figure 2: A bar-chart grid with a vertical axis labeled "Work" (unmarked scale, gridlines shown) and three labeled columns along the horizontal axis: W_{CE}, W_{ED}, W_{DA}. The bar for W_{CE} is already drawn at a fixed reference height (shaded); the bars for W_{ED} and W_{DA} are to be drawn by the student.]
(b)(ii) Calculate the approximate magnitude of the x-component of the electric field at Position B.

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2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the *x*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and the *y*-axis are 0.40 m apart.

[Figure 1: An *x*-*y* coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the *x*-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the *x*-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and between the rod and sphere. A horizontal double-headed arrow

Question 1

near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(c) The positive test charge is placed at Position D. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the *x*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and the *y*-axis are 0.40 m apart.

[Figure 1: An *x*-*y* coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the *x*-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the *x*-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and between the rod and sphere. A horizontal double-headed arrow

Question 1

near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(d)(i) [Figure 3: A horizontal rod labeled "Rod" lies along the x-axis from $x = 0$ to $x = 4L$, with linear charge density $+\lambda$ indicated by a bracket labeled $+\lambda$ above the rod near $x = 0$. The x-axis is marked with tick labels $0, 2L, 4L, 6L, 8L, 10L, 12L$.]

The sphere and the test charge are removed. The rod remains. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

The electric potential V_P at x_P is $V_P = k\lambda \ln\left[\dfrac{x_P}{x_P - 4L}\right]$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) [Figure 4: A blank graph with vertical axis E_x (unlabeled scale) and horizontal axis x marked with tick labels $2L, 4L, 6L, 8L, 10L, 12L$; a shaded region spans from $x = 4L$ to $x = 12L$ indicating the region to be graphed.]
On Figure 4, **sketch** a graph of the x-component of the electric field from the rod as a function of x in the region $4L < x < 12L$.

Solution 1

(a)

Mark scheme

- Use Gauss's law: $\Phi_E = Q/\varepsilon_0$ (1 pt for the correct equation). With enclosed charge $Q = -2.0 \times 10^{-9}$ C (the charge inside the -20.0 V equipotential Gaussian surface) and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/(N $\cdot$ m^2):
$$\Phi_E = \frac{-2.0 \times 10^{-9}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = -226\ \text{N} \cdot \text{m}^2/\text{C}.$$ So $|\Phi_E| = 226$ N $\cdot$ m^2/C
(1 pt for the correct numerical answer; a negative sign or missing/incorrect units still earns this point).

Solution 1

(b)(i)

Mark scheme

- $W_{ED} = 0$ (1 pt), because Positions E and D lie on the same equipotential line so no work is done by the external force moving the test charge between them. On the bar graph, the bar representing W_{DA} should be drawn with a height of six units, using the same vertical scale as the given W_{CE} bar (1 pt).

Solution 1

(b)(ii) Mark scheme

- Start from the field-potential relation $E_x = -\dfrac{dV}{dx}$, so $|E_x| = \left|\dfrac{\Delta V}{\Delta x}\right|$ (1 pt for using a correct relating equation). Substitute the potential values and separation of the equipotential lines bracketing Position B: $|E_x| = \left|-\dfrac{20.0\ \text{V} - 0.0\ \text{V}}{0.65\ \text{m}}\right|$ (1 pt for correct substitution of values). This gives $|E_x| = 31\ \text{V/m}$.

Solution 1

(c) Mark scheme

- Select only $+y$, with an attempt at justification (1 pt). The justification must state that the electric field vector is perpendicular to the line tangent to the equipotential line at Position D (1 pt), and must also indicate one of: the positive test charge moves from a higher electric potential to a lower electric potential, or the test charge and the sphere have charges of opposite sign, or the test charge moves in the direction of the electric field, which is directed upward (1 pt). Model justification: the field is perpendicular to the equipotential line at D; since the test charge is positive it accelerates from higher to lower potential, so the net force (and field) at D points in $+y$, perpendicular to the equipotential line and in the direction of decreasing potential.

Solution 1

(d)(i) Mark scheme

- Derive V_P for the uniformly charged rod (length $4L$, density $+\lambda$, from $x = 0$ to $x = 4L$), 4 points total (1 pt each step): (1) start from an appropriate equation for potential from a charge distribution, $V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$ or $V_P = k\int\left(\frac{1}{x_P - x}\right) dq$; (2) correctly identify $r = x_P - x$, the distance from Point P (at x_P) to a point on the rod; (3) substitute $dq = \lambda\, dx$ to get $V_P = k\lambda\int\left(\frac{1}{x_P - x}\right) dx$; (4) use the correct limits of integration, 0 to $4L$: $V_P = k\lambda\int_0^{4L}\left(\frac{1}{x_P - x}\right) dx$. Evaluating

Solution 1

gives the final result $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$ (equivalently obtainable by integrating the field $E(x) = k\lambda\left(\frac{1}{x-4L} - \frac{1}{x}\right)$ from ∞ to x_P via $\Delta V = -\int \vec{E} \cdot d\vec{r}$).

Solution 1

(d)(ii)

Mark scheme

- Sketch the x-component of the rod's electric field for $4L < x < 12L$: the curve must continually approach (get closer to, without necessarily reaching) the horizontal axis as x increases (1 pt), and it must be a concave-up curve that stays always positive — falling steeply just past $x = 4L$ and flattening out toward (but not crossing) zero by $x = 12L$ (1 pt), consistent with the field weakening with distance from the charged rod.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.
[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost

Question 1

equipotential loops on the lower left, with A below B. A horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]
(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

Question 1

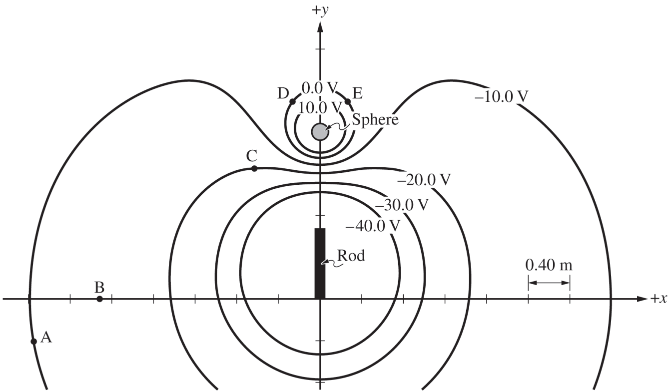

Figure 1

Question 1

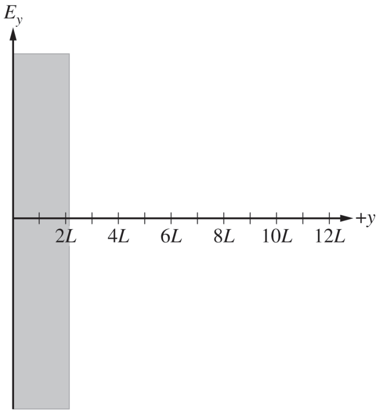

Figure 4

Question 1

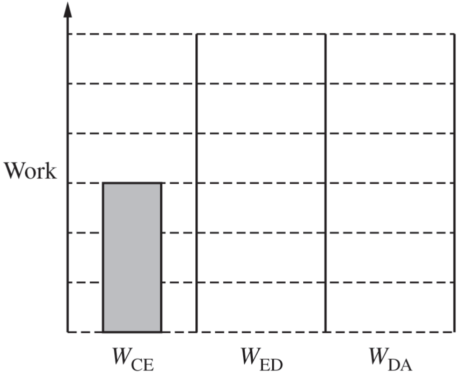

Figure 2

Question 1

2024 Set 2 ▪ Q1

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Question 1

horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to move the test charge to different positions in the order of C$\rightarrow$E$\rightarrow$D$\rightarrow$A. The test charge is held momentarily at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

Draw a bar to represent the relative absolute value of the work W_{ED}, done by the external force on the test charge to move the test charge from Position E to Position D.

Draw a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.

Question 1

The height of each bar should be proportional to the value of W_{CE}. If $W_{\mathrm{ED}} = 0$ and/or $W_{\mathrm{DA}} = 0$, write a "0" in the corresponding columns, as appropriate.
[Figure 2: A bar chart with vertical axis labeled "Work" (unlabeled scale, dashed horizontal gridlines) and three columns on the horizontal axis labeled W_{CE}, W_{ED}, W_{DA}. A single shaded bar of medium height is already drawn above the W_{CE} column; the W_{ED} and W_{DA} columns are empty for the student to complete.]
(b)(ii) Calculate the approximate magnitude of the x-component of the electric field at Position B.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(c) The positive test charge is placed at Position C. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_______ $+x$ _______ $+y$ _______ Directly away from the sphere

_______ $-x$ _______ $-y$ _______ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 2 ▪ Q1

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Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(d)(i) [Figure 3: A vertical y-axis with tick marks labeled from bottom to top: $-\lambda$ at 0, then $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. A thick vertical bar labeled "Rod" spans from 0 to $2L$ on the axis, adjacent to the label $-\lambda$.]

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.
[Figure 4: A blank graph with vertical axis labeled E_y (no numeric scale shown) and horizontal axis labeled y with tick marks at $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. Axes are otherwise empty for the student to sketch a curve.]

Solution 1

(a)

Mark scheme

- Use $\Phi_E = Q/\varepsilon_0$ (from Gauss's law, $\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$) with $Q = 1.0 \times 10^{-9}$ C enclosed by the 0.0 V equipotential surface:
  $\Phi_E = \dfrac{1.0 \times 10^{-9}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = 113\ \text{N} \cdot \text{m}^2/\text{C}$. 1 point for using the correct flux equation, 1 point for the correct numerical answer (units not required for this point).

Solution 1

(b)(i)

Mark scheme

- Positions D and E lie on the same equipotential line, so no work is required to move the test charge between them: $W_{ED} = 0$ (bar of height 0). The bar representing W_{DA} should have a height of approximately one and a half units (about half the height of the given W_{CE} bar). 1 point for indicating $W_{ED} = 0$, 1 point for drawing the W_{DA} bar at roughly 1.5 units.

Solution 1

(b)(ii) Mark scheme

- Relate electric field to the potential gradient between adjacent equipotential lines: $E_y = -dV/dy$, so $|E_y| = |\Delta V/\Delta y|$. Substituting the potential values and spacing of the equipotential lines bracketing Position B: $|E_y| = \left|\dfrac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right| = 7.1\text{ V/m}$. 1 point for using a correct field-potential relationship (may start with numbers already substituted), 1 point for correctly substituting the potential values and equipotential-line spacing used to get the field magnitude at Position B.

Solution 1

(c) Mark scheme

- Direction: $-y$ (only). Justification: the electric field vector is perpendicular to the equipotential line (tangent) at Position C. Because the test charge is positive, it moves from higher to lower electric potential when a force acts on it; at Position C the field/force direction that is perpendicular to the equipotential line and points toward decreasing potential is $-y$ (directly toward the sphere). 1 point for selecting only $-y$ with an attempted justification, 1 point for stating the field is perpendicular to the equipotential line at C, 1 point for one of: the charge moves from higher to lower potential / the test charge and rod have opposite-sign charge / the test charge and sphere have same-sign charge.

Solution 1

(d)(i) Mark scheme

- Set up the potential from a continuous line charge: $V = \dfrac{1}{4\pi\varepsilon_0}\int \dfrac{dq}{r}$, i.e. $V_P = k\int\left(\dfrac{1}{y_P - y}\right)dq$, where $y_P - y$ is the distance from a charge element at height y on the rod to point P. Substitute the charge element of the negatively-charged rod, $dq = -\lambda\,dy$: $V_P = -k\lambda\int\left(\dfrac{1}{y_P - y}\right)dy$. Integrate over the rod's extent with limits $y = 0$ to $y = 2L$: $V_P = -k\lambda\int_0^{2L}\left(\dfrac{1}{y_P - y}\right)dy = k\lambda\ln\left(\dfrac{y_P}{y_P - 2L}\right)$. 1 point for an appropriate potential-from-line-charge equation, 1 point for correctly determining $r = y_P - y$,

Solution 1

1 point for the correct integral with $\lambda\, dy$ substituted for dq, 1 point for the correct limits of integration (0 to 2L).

Solution 1

(d)(ii)

Mark scheme

- For $2L < y < 12L$, sketch E_y as a curve that is always negative (field points toward the negatively-charged rod, i.e. in $-y$) and continually approaches (but never reaches) the horizontal axis as y increases — i.e. a concave-down curve that asymptotically rises toward zero from below, steepest just above $y = 2L$ and flattening out toward $y = 12L$. 1 point for a curve/line that continually approaches the horizontal axis as position increases, 1 point for a concave-down curve that stays always negative.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]
A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

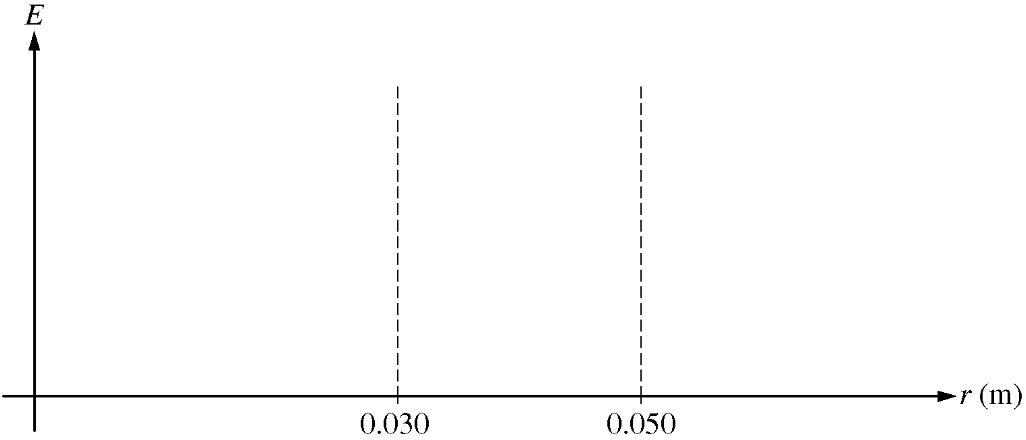
Question 2

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

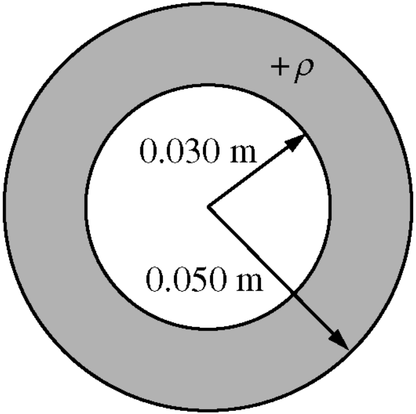
$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

Question 2



Question 2



Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m} : \quad \rho = 0$$

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

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Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.
[Graph with vertical axis labeled E and horizontal axis labeled r (m); two vertical dashed reference lines are marked at $r = 0.030$ and $r = 0.050$; the axes are otherwise blank for the student to sketch on.]

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

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Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

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Question 2

$$r > 0.050 \text{ m} : \quad \rho = 0$$

(e)(i) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s. Calculate the magnitude of the initial acceleration of the proton.
(e)(ii) Calculate the speed of the proton after a long time.

Solution 2

(a) Mark scheme

- $Q = \int \rho\, dV = \int_{0.03}^{0.05} \frac{b}{r}(4\pi r^2)\, dr = 4\pi b \int_{0.03}^{0.05} r\, dr = 4\pi b \left[\frac{r^2}{2}\right]_{0.03}^{0.05} =$ $2\pi b \left(0.05^2 - 0.03^2\right)$. With $b = 1.6 \times 10^{-6}$ C/m^2: $Q = 2\pi(1.6 \times 10^{-6})(0.0016) = 1.61 \times 10^{-8}$ C. (1 pt: recognizing the need to integrate the charge density expression, $Q = \int \rho\, dV$, with the spherical shell volume element $4\pi r^2\, dr$; 1 pt: correct substitution of $\rho = b/r$ into the integral; 1 pt: using the correct limits of integration and obtaining the correct numeric answer.)

Solution 2

(b) Mark scheme

- By symmetry, Gauss's law gives $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \dfrac{Q_{enc}}{\varepsilon_0}$. At the outer surface ($r = 0.05$ m) all the charge is enclosed, so $Q_{enc} = Q = 1.61 \times 10^{-8}$ C (from part (a)). $E = \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \dfrac{1.61 \times 10^{-8}}{4\pi(8.85 \times 10^{-12})(0.05)^2} = 5.79 \times 10^4$ N/C. (1 pt: correctly evaluating the surface integral in Gauss's law; 1 pt: substituting the answer from part (a) and the correct radius into the equation; 1 pt: correct final answer with correct units, consistent with part (a).)

Solution 2

(c) Mark scheme

- $E = 0$ for $r < 0.030$ m (no enclosed charge). For 0.030 m $< r < 0.050$ m, E rises continuously from zero, concave down, increasing up to the value found in part (b) at $r = 0.050$ m (as Q_{enc} grows with r inside the shell). For $r > 0.050$ m, all the charge is enclosed and E falls off as $1/r^2$, decreasing asymptotically toward the horizontal (r) axis. (1 pt: clearly showing $E = 0$ for $r < 0.030$ m; 1 pt: a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$; 1 pt: a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m.)

Solution 2

(d) Mark scheme

- With $V = 0$ at infinity, and since at $r = 0.05$ m all the charge is enclosed (the sphere behaves as a point charge from that radius outward): $V_R = \dfrac{Q_{tot}}{4\pi\varepsilon_0 r} = \dfrac{(9 \times 10^9)(1.61 \times 10^{-8}\ \text{C})}{0.05\ \text{m}} \approx 2900$ V. (Equivalent alternate method: integrate $\Delta V_R = -\int_{\infty}^{r} E\,dr = -\int_{\infty}^{0.05} \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2}\,dr = 2900$ V.) (1 pt: substituting the total charge from part (a) into a correct expression for electric potential; 1 pt: substituting $r = 0.05$ m (or the correct limits, if integrating) into that expression.)

Solution 2

(e)(i)

Mark scheme

- Newton's second law with the electric force: $F = ma \Rightarrow qE = ma \Rightarrow a = \dfrac{qE}{m}$.
  Using the proton's charge $q = 1.6 \times 10^{-19}$ C, mass $m = 1.67 \times 10^{-27}$ kg, and $E = 5.79 \times 10^{4}$ N/C from part (b):
  $a = \dfrac{(1.6 \times 10^{-19})(5.79 \times 10^{4})}{1.67 \times 10^{-27}} = 5.55 \times 10^{12}$ m/s^2. (1 pt: using a correct expression of Newton's second law in terms of the electric field; 1 pt: correctly substituting values into that equation.)

Solution 2

(e)(ii) Mark scheme

- Using the work-energy theorem with electric potential: $-q\Delta V = \frac{1}{2}mv^2$, with $\Delta V = V_\infty - V_R = 0 - 2900$ V (the proton is repelled outward, converting electric potential energy into kinetic energy as it moves to infinity).
$v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19})(0 - 2900)}{1.67 \times 10^{-27}}} = 7.45 \times 10^5$ m/s. (1 pt: a correct expression of kinetic energy in terms of the electric potential difference; 1 pt: correctly substituting into the equation and obtaining the correct final answer.)

Question 1

2016 ▪ Q1

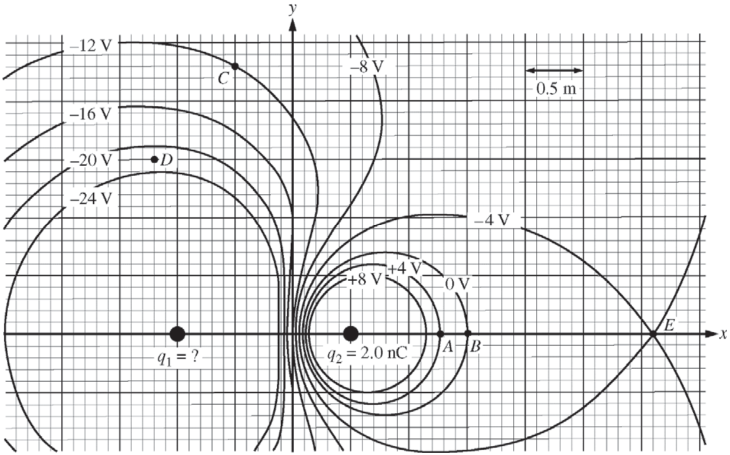
Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.
[Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside

Question 1

in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and (right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(a) Calculate the value of q_1.

Question 1



Question 1

2016 ▪ Q1

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Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

Question 1

2016 ▪ Q1

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Question 1

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(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

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(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

Question 1

2016 ▪ Q1

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Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(e)(i) A proton is placed at point A and then released from rest. Calculate the work done by the electric field on the proton as it moves from point A to point E.

(e)(ii) Calculate the speed of the proton when it reaches point E.

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

______ Up ______ Down ______ Left ______ Right ______ Into the page ______ Out of the page ______ The direction is undefined since the acceleration is zero.

Justify your answer.

Solution 1

(a) Mark scheme

- Total potential is the sum of the potentials from the individual point charges. At point B, $V_B = 0 = V_1 + V_2$, so $-V_1 = V_2$, i.e. $-\dfrac{kq_1}{r_1} = \dfrac{kq_2}{r_2}$. Substituting $r_1 = 5 \times 0.5$ m, $r_2 = 2 \times 0.5$ m, $q_2 = 2.0$ nC: $-\dfrac{q_1}{(5 \times 0.5\ \text{m})} = \dfrac{2.0\ \text{nC}}{(2 \times 0.5\ \text{m})}$. Solving gives $q_1 = -5.0$ nC. Points: 1 for indicating total potential = sum of individual potentials; 1 for correctly substituting into that equation (signs ignored at this step); 1 for the correct answer with correct sign and units, $q_1 = -5.0$ nC.

Solution 1

(b)

Mark scheme

- This is a drawing task: sketch, at point C, a vector for the electric field direction. The field must be drawn perpendicular to the equipotential line passing through C, and it must point in the direction of decreasing potential — i.e. from the -12 V line toward the -16 V line (field points from high to low potential). Points: 1 for drawing a vector perpendicular to the equipotential line for C; 1 for drawing the vector in the direction of the -16 V line.

Solution 1

(c) Mark scheme

- Use $E = -\dfrac{dV}{dx}$, so $|E| \approx \dfrac{\Delta V}{\Delta x}$. Reading values near point D from the equipotential diagram (lines -20 V and -24 V, separated by $\Delta x = 2 \times 0.1$ m):
  $E = \dfrac{(-20\text{ V}) - (-24\text{ V})}{2 \times 0.1\text{ m}} = 20\text{ N/C}$. Points: 1 for using the equation relating electric field to potential difference/gradient; 1 for correctly substituting the values from the figure to get $E = 20\text{ N/C}$.

Solution 1

(d)

Mark scheme

- By Gauss's law, $\Phi_E = \frac{q_{enc}}{\varepsilon_0}$, where $q_{enc} = 2.0$ nC is the charge enclosed by the 0 V equipotential surface. $\Phi_E = \frac{2.0 \text{ nC}}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 226 \text{ N} \cdot \text{m}^2/\text{C}$.
  Points: 1 for using the correct equation for electric flux (Gauss's law); 1 for the correct answer with units, $\Phi_E = 226 \text{ N} \cdot \text{m}^2/\text{C}$.

Solution 1

(e)(i)

Mark scheme

- Work equals the negative change in potential energy: $W = -q\Delta V$. With the proton's charge $q = 1.6 \times 10^{-19}$ C and $\Delta V = V_E - V_A = (-4 \text{ V}) - (4 \text{ V}) = -8$ V: $W = -(1.6 \times 10^{-19} \text{ C})(-8 \text{ V}) = 1.28 \times 10^{-18}$ J $= 8.0$ eV. Points: 1 for using an equation relating work done to the change in potential energy ($W = -q\Delta V$); 1 for a correct answer including sign and units, $W = 1.28 \times 10^{-18}$ J $= 8.0$ eV.

Solution 1

(e)(ii)

Mark scheme

- The work done on the proton equals its kinetic energy gain: $W = \Delta K = \frac{1}{2}mv^2$. Substituting the part (e)(i) result and the proton mass $m = 1.67 \times 10^{-27}$ kg: 1.28×10^{-18} J $= \frac{1}{2}(1.67 \times 10^{-27}$ kg$)v^2 \Rightarrow v = 3.92 \times 10^4$ m/s. Points: 1 for indicating that the proton's kinetic energy at E equals the work done; 1 for correctly substituting the part (e)(i) answer to get $v = 3.92 \times 10^4$ m/s.

Solution 1

(f)

Mark scheme

- Correct choice: Left. Justification: an electron (negative charge) accelerates in the direction perpendicular to the equipotential surfaces, toward higher potential (opposite the electric field direction). At point B this direction is toward the left. Points: 1 for correctly selecting 'Left'; 1 for a correct justification along these lines. No points are earned if the wrong direction is selected.