Past-paper walk-through

Gauss's Law ▪ 8.6

eXercise

Questions in this set

- 2025 Q1
- 2024 Set 2 Q1
- 2022 Set 1 Q1
- 2022 Set 2 Q1
- 2019 Set 1 Q1
- 2019 Set 2 Q2
- 2018 Q1
- 2017 Q1
- 2015 Q1
- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

2025 ▪ Q1

An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.
[Figure 1: Two diagrams. "Side View" shows a coaxial cylindrical capacitor of length L, with the inner shell labeled $+\sigma_1$ and the outer shell labeled $-\sigma_2$. "Cross-Sectional View" shows two concentric circles, the inner one with radius R_1 and the outer one with radius R_2. Note: Figures not drawn to scale.]
(a)(i) Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region

Question 1

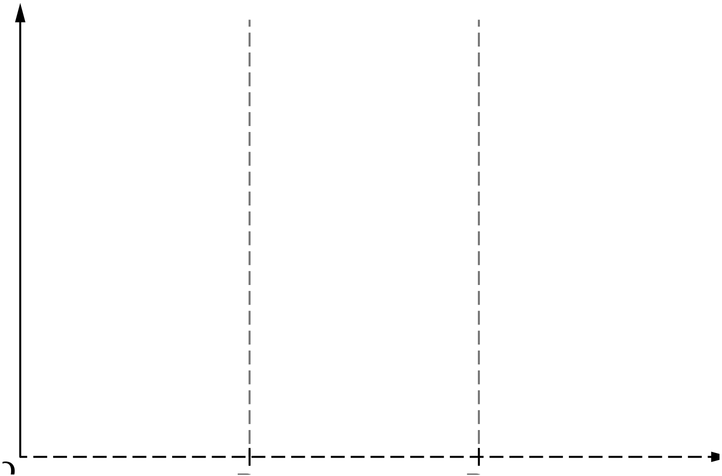
$R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

(a)(ii) **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

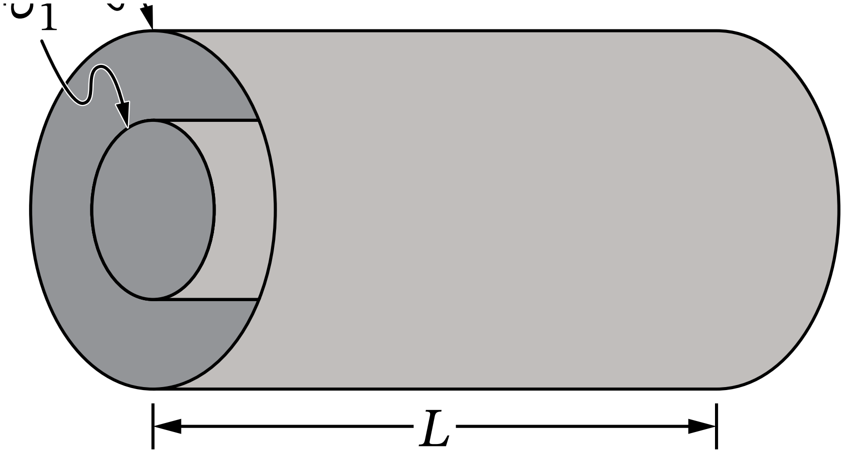
(a)(iii) On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

[Figure 2: A blank set of axes with vertical axis labeled E and horizontal axis labeled r, origin O. Two vertical dashed reference lines are marked on the r-axis at positions labeled R_1 and R_2.]

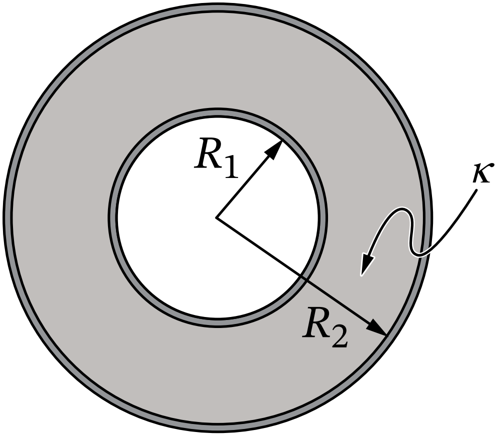
Question 1



Question 1



Question 1



Question 1

2025 ▪ Q1

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[Figure 1: Two diagrams. "Side View" shows a coaxial cylindrical capacitor of length L, with the inner shell labeled $+\sigma_1$ and the outer shell labeled $-\sigma_2$. "Cross-Sectional View" shows two concentric circles, the inner one with radius R_1 and the outer one with radius R_2. Note: Figures not drawn to scale.]

(b) A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.

Question 1

[Figure 3: "Cross-Sectional View" showing two concentric circles, the inner one with radius R_1 and the outer one with radius R_2; the annular region between them is shaded and labeled with κ, indicating the dielectric material fills that region.]

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Solution 1

(a)(i)

Mark scheme

- Apply Gauss's law: $\oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0. ChooseacylindricalGaussiansurfaceofradiusr(R_1 < r < R_2)andlength\ell, sothefluxtermisE(2\pi r\ell). Theenclosedchargeisonlyontheinnershell: q_{enc} = \sigma_1(2\pi R_1\ell).Substituting : E(2\pi r\ell) = \sigma_1(2\pi R_1\ell)/\varepsilon_0, givingE = \sigma_1 R_1/(\varepsilon_0 r).Points : (1)writingGauss'slaw, (2)correctGaussian - surfacearea2\pi r\ell forR_1 < r < R_2, (3)correctenclosedcharge\sigma_1(2\pi R_1\ell).$

Solution 1

(a)(ii) Mark scheme

- Substitute E from part (a)(i) into $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}. Then |\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r\varepsilon_0} dr = \frac{\sigma_1 R_1}{\varepsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr = \frac{\sigma_1 R_1}{\varepsilon_0} \ln(r)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right). Points: (1) substituting E into the potential-difference integral, (2) attempting/solving the integral with correct limits R_1 to R_2 (order of limits doesn$

Solution 1

(a)(iii)

Mark scheme

- Sketch E vs r: E = 0 for 0<r<$R_1 andforr > R_2(fieldiszeroinsidetheinnershellandoutsidetheoutershell, sinceaGaussiansurfacethereenclosesz$ $r < R_2, E = \sigma_1 R_1/(\varepsilon_0 r), acurvethatisdecreasingandconcaveup(doesnotneedtotouchthedashe$ $(1) zerofor 0 < r < R_1 andr > R_2, (2) decreasingandconcave - upcurvebetweenR_1 andR_2.$

Solution 1

(b) Mark scheme

- Start from C $= Q/\Delta V. With the dielectric filling R_1 < r < R_2, the capacitance is \kappa times the capacitance without the dielectric: C = \kappa Q/\Delta V. Using Q = \sigma_1(2\pi L R_1) and \Delta V = \frac{\sigma_1 R_1}{\varepsilon_0}\ln(R_2/R_1) from part (a): without the dielectric, C = \frac{\sigma_1(2\pi L R_1)}{\frac{\sigma_1 R_1}{\varepsilon_0}\ln(R_2/R_1)} = \frac{2\pi L\varepsilon_0}{\ln(R_2/R_1)}. With the dielectric, C = \kappa\frac{Q}{\Delta V} = \frac{2\pi L\kappa\varepsilon_0}{\ln(R_2/R_1)}. Points: (1) C = Q/\Delta V, (2) C_{with dielectric} = \kappa \times C_{without}, (3) correct substitution of Q and \Delta V consistent with part (a).$

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.
[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost

Question 1

equipotential loops on the lower left, with A below B. A horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]
(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

Question 1

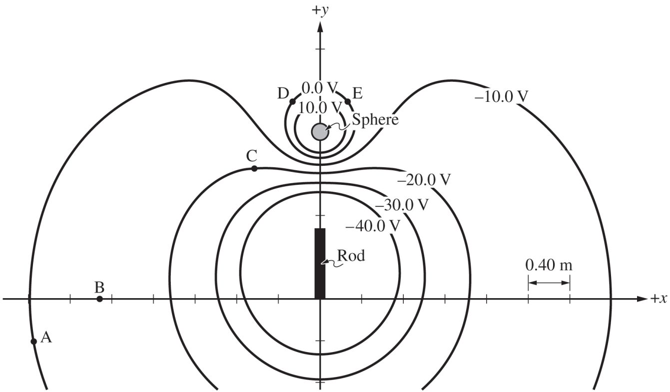

Figure 1

Question 1

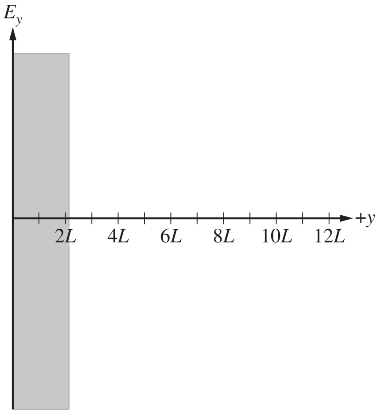

Figure 4

Question 1

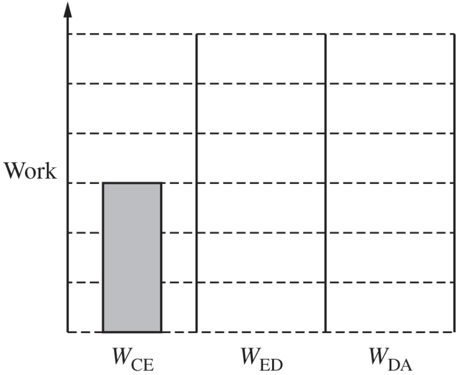

Figure 2

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to move the test charge to different positions in the order of C$\rightarrow$E$\rightarrow$D$\rightarrow$A. The test charge is held momentarily at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

Draw a bar to represent the relative absolute value of the work W_{ED}, done by the external force on the test charge to move the test charge from Position E to Position D.

Draw a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.

Question 1

The height of each bar should be proportional to the value of W_{CE}. If $W_{\mathrm{ED}} = 0$ and/or $W_{\mathrm{DA}} = 0$, write a "0" in the corresponding columns, as appropriate.
[Figure 2: A bar chart with vertical axis labeled "Work" (unlabeled scale, dashed horizontal gridlines) and three columns on the horizontal axis labeled W_{CE}, W_{ED}, W_{DA}. A single shaded bar of medium height is already drawn above the W_{CE} column; the W_{ED} and W_{DA} columns are empty for the student to complete.]
(b)(ii) Calculate the approximate magnitude of the x-component of the electric field at Position B.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(c) The positive test charge is placed at Position C. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

______ $+x$ ______ $+y$ ______ Directly away from the sphere

______ $-x$ ______ $-y$ ______ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 2 ▪ Q1

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Question 1

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(d)(i) [Figure 3: A vertical *y*-axis with tick marks labeled from bottom to top: $-\lambda$ at 0, then $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. A thick vertical bar labeled "Rod" spans from 0 to $2L$ on the axis, adjacent to the label $-\lambda$.]

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the *y*-axis in the orientation shown in Figure 3. Position P (not shown) is located on the *y*-axis a distance y_P from the origin, where $y_P > 2L$.

The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.
[Figure 4: A blank graph with vertical axis labeled E_y (no numeric scale shown) and horizontal axis labeled y with tick marks at $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. Axes are otherwise empty for the student to sketch a curve.]

Solution 1

(a)

Mark scheme

- Use $\Phi_E = Q/\varepsilon_0$ (from Gauss's law, $\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$) with $Q = 1.0 \times 10^{-9}$ C enclosed by the 0.0 V equipotential surface:
$\Phi_E = \dfrac{1.0 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 113 \text{ N} \cdot \text{m}^2/\text{C}$. 1 point for using the correct flux equation, 1 point for the correct numerical answer (units not required for this point).

Solution 1

(b)(i)

Mark scheme

- Positions D and E lie on the same equipotential line, so no work is required to move the test charge between them: $W_{ED} = 0$ (bar of height 0). The bar representing W_{DA} should have a height of approximately one and a half units (about half the height of the given W_{CE} bar). 1 point for indicating $W_{ED} = 0$, 1 point for drawing the W_{DA} bar at roughly 1.5 units.

Solution 1

(b)(ii) Mark scheme

- Relate electric field to the potential gradient between adjacent equipotential lines: $E_y = -dV/dy$, so $|E_y| = |\Delta V/\Delta y|$. Substituting the potential values and spacing of the equipotential lines bracketing Position B: $|E_y| = \left|\frac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right| = 7.1\text{ V/m}$. 1 point for using a correct field-potential relationship (may start with numbers already substituted), 1 point for correctly substituting the potential values and equipotential-line spacing used to get the field magnitude at Position B.

Solution 1

(c) Mark scheme

- Direction: $-y$ (only). Justification: the electric field vector is perpendicular to the equipotential line (tangent) at Position C. Because the test charge is positive, it moves from higher to lower electric potential when a force acts on it; at Position C the field/force direction that is perpendicular to the equipotential line and points toward decreasing potential is $-y$ (directly toward the sphere). 1 point for selecting only $-y$ with an attempted justification, 1 point for stating the field is perpendicular to the equipotential line at C, 1 point for one of: the charge moves from higher to lower potential / the test charge and rod have opposite-sign charge / the test charge and sphere have same-sign charge.

Solution 1

(d)(i) Mark scheme

- Set up the potential from a continuous line charge: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$, i.e. $V_P = k\int\left(\frac{1}{y_P - y}\right)dq$, where $y_P - y$ is the distance from a charge element at height y on the rod to point P. Substitute the charge element of the negatively-charged rod, $dq = -\lambda\, dy$: $V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$. Integrate over the rod's extent with limits $y = 0$ to $y = 2L$: $V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$. 1 point for an appropriate potential-from-line-charge equation, 1 point for correctly determining $r = y_P - y$,

Solution 1

1 point for the correct integral with $\lambda\, dy$ substituted for dq, 1 point for the correct limits of integration (0 to 2L).

Solution 1

(d)(ii)

Mark scheme

- For $2L < y < 12L$, sketch E_y as a curve that is always negative (field points toward the negatively-charged rod, i.e. in $-y$) and continually approaches (but never reaches) the horizontal axis as y increases — i.e. a concave-down curve that asymptotically rises toward zero from below, steepest just above $y = 2L$ and flattening out toward $y = 12L$. 1 point for a curve/line that continually approaches the horizontal axis as position increases, 1 point for a concave-down curve that stays always negative.

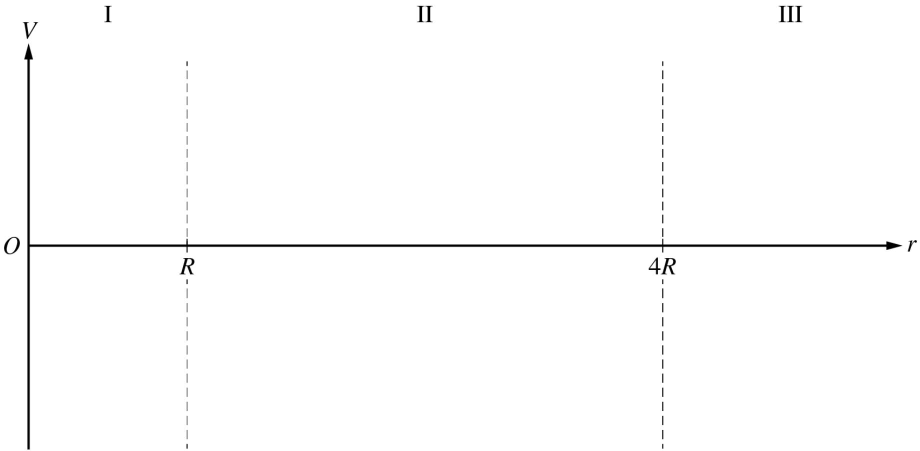
Question 1

2022 Set 1 ▪ Q1

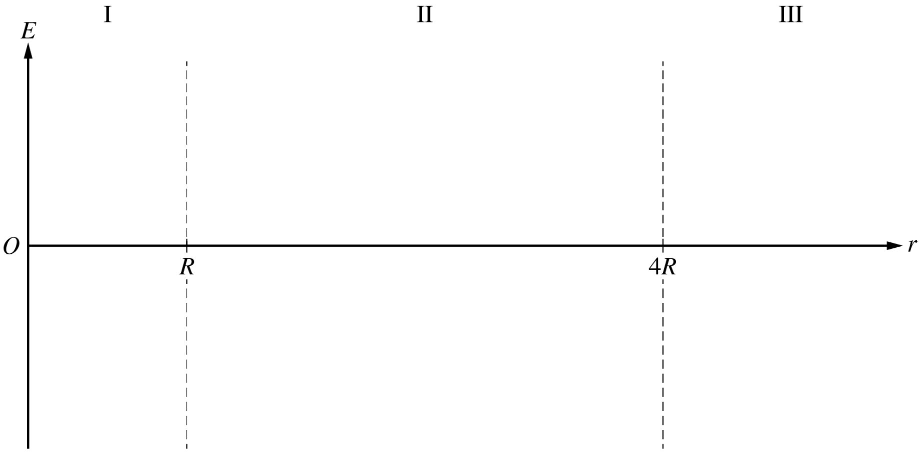
A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view. [Two figures side by side, captioned "Note: Figures not drawn to scale." Left figure, labeled "Cutout View": a large shaded circular shell with a wedge cut out to reveal a smaller shaded sphere at the center. Right figure, labeled "Cross-Section View": a large circle (the shell) with a smaller circle at the center labeled R, and an arrow from the center to the outer circle labeled $4R$.]

(a) Determine the charge on the outer surface of the shell.

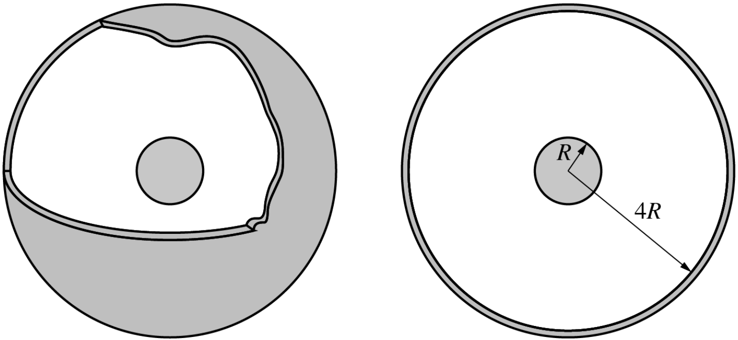
Question 1



Question 1



Question 1



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

Question 1

2022 Set 1 ▪ Q1

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(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

Question 1

2022 Set 1 ▪ Q1

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(c) The magnitude of the electric field at $r = R$ is 8 N/C. Calculate the value of the electric field at $r = 2R$.

Question 1

2022 Set 1 ▪ Q1

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(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

Question 1

2022 Set 1 ▪ Q1

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(e)(i) On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.

Question 1

[Graph axes: horizontal axis labeled r, with vertical dashed reference lines at $r = R$ and $r = 4R$ dividing the plot into three labeled regions I, II, III (left to right). Vertical axis labeled E, origin marked O. Axes are blank, to be sketched on by the student.]

(e)(ii) On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.

[Graph axes: horizontal axis labeled r, with vertical dashed reference lines at $r = R$ and $r = 4R$ dividing the plot into three labeled regions I, II, III (left to right). Vertical axis labeled V, origin marked O. Axes are blank, to be sketched on by the student.]

Solution 1

(a)

Mark scheme

- $q_{net} = q_{inner} + q_{outer}$, so $q_{outer} = q_{net} - q_{inner} = +3Q - (+Q) = +2Q$. The charge on the outer surface of the shell is $+2Q$. (A correct final answer earns the point even with no work shown.)

Solution 1

(b) Mark scheme

- Apply Gauss's law: $\dfrac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA = E(4\pi r^2)$ (1 pt: using Gauss's law, substituting for area or enclosed charge). The sphere has uniform charge density $\rho = \dfrac{-Q}{\frac{4}{3}\pi R^3}$ (net charge on the sphere is $-Q$), so $q_{enc} = \rho V = \left(\dfrac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\dfrac{4}{3}\pi r^3\right) = -Q\dfrac{r^3}{R^3}$ (1 pt: correct $q_{enc}(r)$, regardless of sign). Then $E = q_{enc}\dfrac{1}{\varepsilon_0(4\pi r^2)} = \left(-Q\dfrac{r^3}{R^3}\right)\dfrac{1}{\varepsilon_0(4\pi r^2)}$ (1 pt: using the correct area), giving $E(r) = -\dfrac{Qr}{4\pi\varepsilon_0 R^3}$ for $r < R$ (sign is not graded).

Solution 1

Solution 1

(c)

Mark scheme

- E varies as $E \propto \frac{1}{r^2}$ from the center of the nonconducting sphere (1 pt: recognizing the inverse-square dependence). Since $r = 2R$ is twice $r = R$, where $E = 8$ N/C: $E_{new} = \frac{E_{old}}{4} = \frac{8 \text{ N/C}}{4} = 2$ N/C (1 pt: correct magnitude, with units).

Solution 1

(d) Mark scheme

- $V_f - V_i = -\int_{s_i}^{s_f} E\,dr$, so $\Delta V = -\int_{r=R}^{r=4R} E\,dr$ (1 pt: relating ΔV to E with an attempt at integration limits or evaluating the integral). Substituting $E = -\frac{Q}{4\pi\varepsilon_0 r^2}$ (consistent with part (c)):
$\Delta V = -\int_R^{4R} \left(-\frac{Q}{4\pi\varepsilon_0 r^2}\right) dr = \frac{Q}{4\pi\varepsilon_0}\int_R^{4R} \frac{1}{r^2} dr$ (1 pt: substituting the correct E expression). Integrating with the correct limits:
$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^{4R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{4-1}{4R}$ (1 pt: correctly

Solution 1

integrating with correct limits), so $\Delta V = \dfrac{3Q}{16\pi\varepsilon_0 R}$. (Alternate solution earning the same 3 points: treat the inner sphere as a point charge Q by symmetry, $\Delta V = \dfrac{Q}{4\pi\varepsilon_0 R} - \dfrac{Q}{4\pi\varepsilon_0 (4R)} = \dfrac{3Q}{16\pi\varepsilon_0 R}$, using charge Q — not $-Q$ or $3Q$ — on both terms and the correct r values.)

Solution 1

(e)(i)

Mark scheme

- Sketch of E vs. r (electric field, taking outward as positive): (1 pt) E is negative and its magnitude increases linearly from 0 at the center to a maximum magnitude at $r = R$ (region I); (1 pt) the curve is continuous at R and asymptotically approaches zero across $R < r < 4R$ (region II, negative and shrinking in magnitude); (1 pt) for $r > 4R$ (region III) the curve is positive, concave-up, and decreasing (falling off with distance from the now-enclosed total charge $+2Q$).

Solution 1

(e)(ii)

Mark scheme

- Sketch of V vs. r (electric potential): (1 pt) V is always increasing from $r = 0$ to $r = 4R$ (across regions I and II); (1 pt) V is decreasing for $r > 4R$ (region III); (1 pt) the graph is continuous across all three regions. (Scoring note: the vertical-axis intercept and where the curve crosses the horizontal axis are irrelevant — the curve may lie entirely below the axis as long as the shape criteria above are met.)

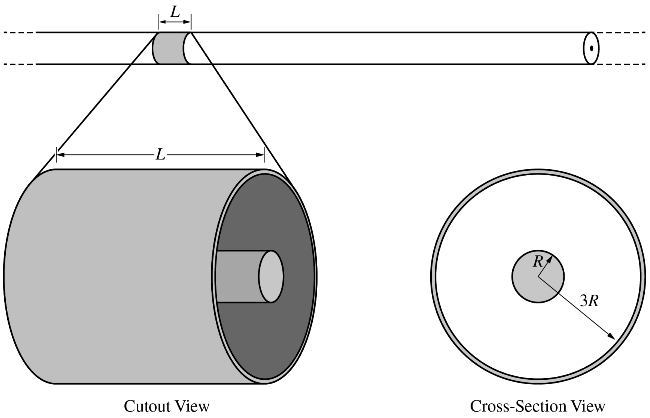
Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view. [Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

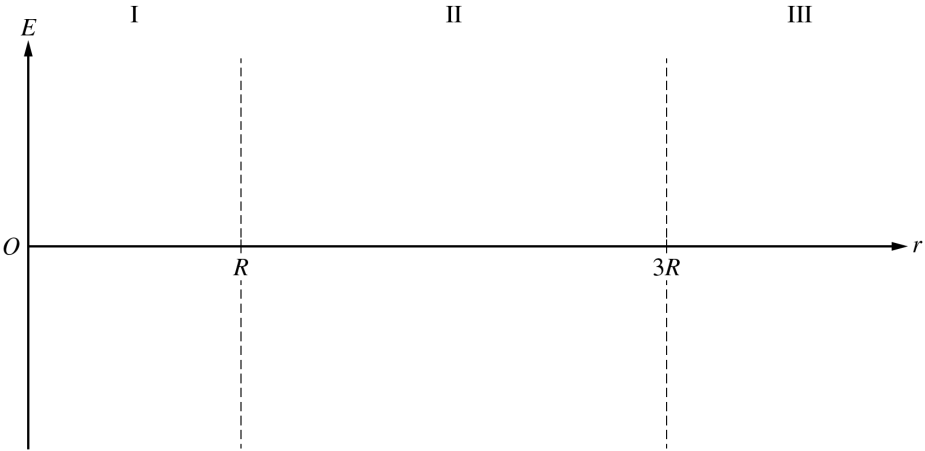
(a) Determine the charge on the outer surface of the cylindrical shell within length L.

Question 1

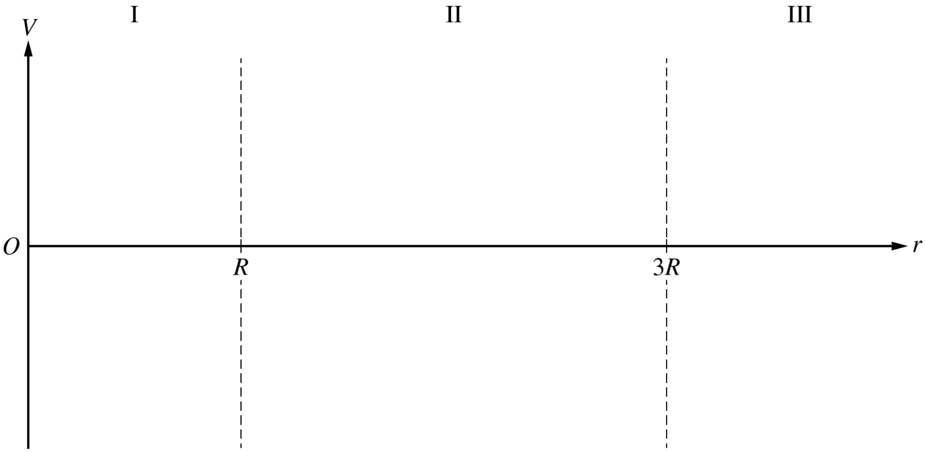


Note: Figures not drawn to scale.

Question 1



Question 1



Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(c) The magnitude of the electric field at $r = R$ is 12 N/C. Calculate the value of the electric field at $r = 2R$.

Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(e)(i) On the following axes that include regions **I**, **II**, and **III**, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.

Question 1

[Graph axes: vertical axis E, horizontal axis r; three regions labeled I, II, III are marked above the axis, with vertical dashed reference lines at $r = R$ (boundary between I and II) and $r = 3R$ (boundary between II and III); origin labeled O.]

(e)(ii) On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.

[Graph axes: vertical axis V, horizontal axis r; three regions labeled I, II, III are marked above the axis, with vertical dashed reference lines at $r = R$ (boundary between I and II) and $r = 3R$ (boundary between II and III); origin labeled O.]

Solution 1

(a)

Mark scheme

- By charge conservation on the shell: $q_{net} = q_{inner} + q_{outer}$, so $q_{outer} = q_{net} - q_{inner} = (+4Q) - (-Q) = +5Q$. The outer surface of the shell carries charge $+5Q$ within length L. (A correct final value may earn the point even with no work shown.)

Solution 1

(b) Mark scheme

- Apply Gauss's law on a coaxial cylindrical Gaussian surface of radius $r < R$, length L: $\frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA = E(2\pi rL)$. The enclosed charge scales with volume: $q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L}\right)(\pi r^2 L) = Q\frac{r^2}{R^2}$. Substituting: $E = q_{enc}\frac{1}{\varepsilon_0(2\pi rL)} = \left(Q\frac{r^2}{R^2}\right)\frac{1}{\varepsilon_0(2\pi rL)}$, giving $E = \frac{Qr}{2\pi\varepsilon_0 R^2 L}$ for $r < R$. Points: (1) using Gauss's law with the area or the enclosed charge substituted, (2) correct $q_{enc}(r)$, (3) correct area $2\pi rL$.

Solution 1

(c)

Mark scheme

- Outside the nonconducting cylinder (in the gap before the shell), $E \propto 1/r$. Since $E(R) = 12$ N/C and $2R$ is twice the distance, $E_{new} = E_{old}/2 = 12/2 = 6$ N/C. Points: (1) recognizing the reciprocal $E \propto 1/r$ dependence, (2) correct value with units, $E(2R) = 6$ N/C.

Solution 1

(d) Mark scheme

- Use $V_f - V_i = -\int_{s_i}^{s_f} E\,dr$ from $r = R$ to $r = 3R$: $\Delta V = -\int_R^{3R} E\,dr$. Substitute $E(r) = \dfrac{Q}{2\pi\varepsilon_0 rL}$ (from parts b/c): $\Delta V = -\dfrac{Q}{2\pi\varepsilon_0 L}\int_R^{3R}\dfrac{1}{r}\,dr = -\dfrac{Q}{2\pi\varepsilon_0 L}[\ln r]_R^{3R} = -\dfrac{Q}{2\pi\varepsilon_0 L}[\ln(3R) - \ln(R)] = -\dfrac{Q}{2\pi\varepsilon_0 L}\ln(3)$. So the magnitude of the potential difference is $|\Delta V| = \dfrac{Q\ln 3}{2\pi\varepsilon_0 L}$. Points: (1) correct integral setup with limits or evaluation attempted, (2) correct $E(r)$ substituted (consistent with part c), (3) correctly integrating with the correct limits.

Solution 1

(e)(i) Mark scheme

- Sketch of E vs r: in Region I ($0 < r < R$), E is positive and increases linearly from 0 at the axis to a maximum at $r = R$ (uniformly charged nonconducting cylinder, $E \propto r$). In Region II ($R < r < 3R$), E is positive, continuous with the value at R, and decreases along a concave-up curve (the $1/r$ falloff from parts b-d). At $r = 3R$ there is a discontinuous JUMP UP in magnitude (crossing the shell's surface charge). In Region III ($r > 3R$), E continues to decrease from this higher value. Points: (1) E positive with a linear increase from 0 to R; (2) a positive, concave-up decreasing graph for $r > R$; (3) a discontinuous increase in magnitude at $r = 3R$.

Solution 1

(e)(ii) Mark scheme

- Sketch of V vs r: V decreases monotonically across all three regions (never increases). In Region I it is concave down and decreasing; in Regions II and III it is concave up and decreasing; the curve is CONTINUOUS across $r = R$ and $r = 3R$ (no jump, unlike the E-graph). The vertical-axis intercept and where the curve crosses the horizontal axis are both irrelevant to scoring (the curve may sit entirely below the axis). A response with all three segments reflected across the horizontal axis caps at 2 of the 3 points. Points: (1) concave down and decreasing in Region I; (2) concave up and decreasing in Regions II and III; (3) continuous across all three regions.

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.

[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

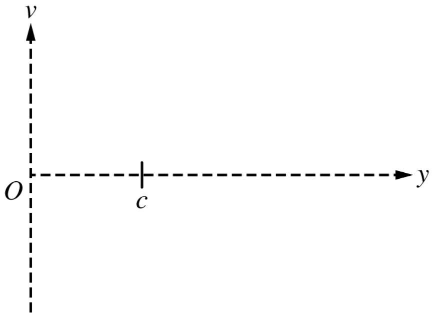
(a)(i) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

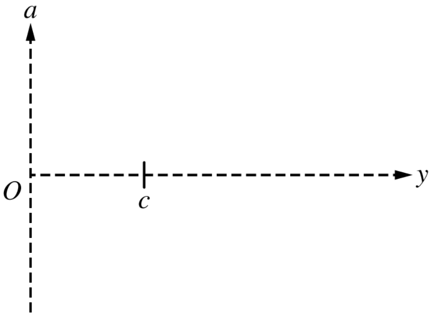
Question 1

[Figure: Point P shown as a dot on a vertical dashed line above a horizontal dashed line representing the cylinder's axis, with a small + segment marked at the axis below P.]
(a)(ii) Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.
(a)(iii) Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

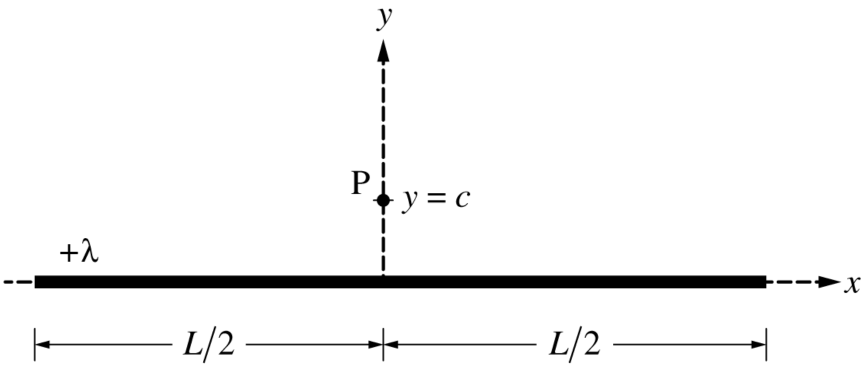
Question 1

position y and the acceleration a as a function of position y for the proton.



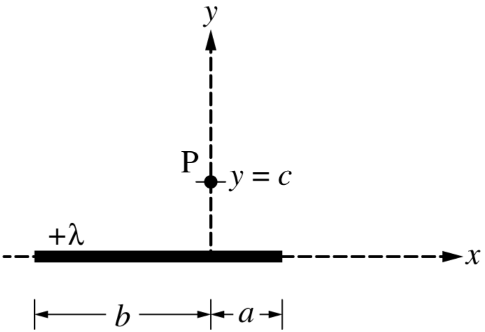


Question 1



Note: Figure not drawn to scale.

Question 1



Note: This figure is shown again for reference.

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.

[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

[Two blank graphs, side by side. Left graph: vertical axis v, horizontal axis y, origin O, with a tick mark labeled c on the y-axis. Right graph: vertical axis a, horizontal axis y,

Question 1

origin O, with a tick mark labeled c on the y-axis. Both axes have dashed guide lines at $y = c$; no curves are drawn — student sketches the curves.]

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.

[Figure: A short horizontal cylinder centered near the y-axis, carrying charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder. The cylinder extends a distance b to the left of the y-axis and a distance a to the right of the y-axis, with $a < b$.]

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

Question 1

[Figure: Point P shown as a dot on a vertical dashed line above a horizontal dashed line representing the shorter cylinder's axis, with a small + segment marked at the axis below P.]

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(d)(i) Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

_____ Yes _____ No

(d)(ii) If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain

Question 1

why Gauss's law cannot be applied to derive an expression for the electric field in this case.

[Figure: The shorter-cylinder figure shown again for reference — point P on the y-axis at $y = c$ above a cylinder of length b to the left of the y-axis and a to the right of the y-axis, carrying charge density $+\lambda$. Note: This figure is shown again for reference.]

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Question 1

Horizontal component:

$$|E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{(c^2 + x^2)^{3/2}}\, dx$$

Vertical component:

$$|E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{(c^2 + x^2)^{3/2}}\, dy$$

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(e)(i) One of the two expressions above is not correct. Which expression is not correct?

______ Horizontal component ______ Vertical component

(e)(ii) Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

Solution 1

(a)(i)

Mark scheme

- The field points straight up: draw the arrow starting at P and pointing in the $+y$ direction (directly away from the rod). By symmetry, for a point on the axis of a uniformly charged, symmetric rod, the field has no horizontal component, and since the charge is positive $(+\lambda)$, the field points away from the rod. Full credit (1 point) for an arrow at P pointing straight up.

Solution 1

(a)(ii)

Mark scheme

- Use a cylindrical Gaussian surface that is coaxial with the (thin, charged) cylinder – i.e., sharing the same central axis – with radius c so that its curved lateral surface passes through point P. (Credit is also earned if the correct surface is simply drawn on the figure.) 1 point.

Solution 1

(a)(iii) Mark scheme

- Apply Gauss's law: $\frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA \Rightarrow \frac{Q}{\varepsilon_0} = EA$ (1 pt for using Gauss's law). Substitute the enclosed charge $Q = \lambda L$ (1 pt). Substitute the lateral area of the Gaussian cylinder $A = 2\pi cL$ (1 pt). Then $\frac{\lambda L}{\varepsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\varepsilon_0 c} = \frac{2k\lambda}{c}$, directed away from the cylinder (the $+y$ direction at P).

Solution 1

(b) Mark scheme

- Near the long rod ($L \gg c$), the field behaves like that of an infinite line charge, $E(y) \approx \dfrac{2k\lambda}{y}$, so the acceleration of the proton is $a(y) = \dfrac{eE(y)}{m} \propto \dfrac{1}{y}$. Velocity-position graph: the proton starts at rest at $y = c$ (not at the origin), so $v = 0$ at $y = c$; as y increases, v increases but at an ever-decreasing rate (since the driving field/acceleration falls off), giving a concave-down curve that begins on the y-axis at $y = c$ rather than at the origin (1 point). Acceleration-position graph: $a \propto 1/y$ is large at $y = c$ and decreases toward zero as $y \to \infty$, giving a concave-up curve that approaches (is asymptotic to) the horizontal axis (1 point).

Solution 1

(c) Mark scheme

- The shorter cylinder is asymmetric about P: it extends a distance a to the right of the y-axis but b to the left, with $a < b$, so there is more (unbalanced) charge on the left. Charge elements in the 'extra' left segment (length $b - a$, with no symmetric partner on the right) lie to the lower-left of P, so their contribution to $\vec{E}$ at P points up and to the right (away from that charge). Superposed on the symmetric part's purely vertical field, the net field at P therefore points both upward and to the right. 1 point for showing a rightward horizontal component; 1 point for the full correct direction – up and to the right (not straight up, and not straight right).

Solution 1

(d)(i)

Mark scheme

- No – see the combined justification scored in part (d)(ii). In this Scoring Guidelines, parts (d)(i) and (d)(ii) are graded together as a single 1-point item ('For selecting No with a valid explanation'); that point is recorded on leaf 1dii so the question's points are not double-counted.

Solution 1

(d)(ii) Mark scheme

- No, there is not a single Gaussian surface that can be used. Claim: select 'No.' Evidence: the length of the (shorter) cylinder is not much greater than the distance from the cylinder to point P, and the charge distribution is asymmetric about the axis through P (length b to the left vs. a to the right, $a < b$). Reasoning: therefore the approximation that E has constant magnitude over a cylindrical surface (the symmetry assumption needed to pull E out of the Gauss's-law integral) does not hold, so Gauss's law cannot be used to derive an expression for the field here. 1 point total (covering both (d)(i) and (d)(ii)) for selecting 'No' with this valid justification.

Solution 1

(e)(i)

Mark scheme

- The Vertical component expression, $|E_y| = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_{-b}^{a}\dfrac{y}{(c^2+x^2)}\,dy$, is the incorrect one (the Horizontal component expression is correct). 1 point for correctly selecting 'Vertical component.'

Solution 1

(e)(ii) Mark scheme

- Two mistakes in the vertical-component integral, each worth 1 point to identify + 1 point to correct (4 points total). (1) Wrong integration variable: the integral is written with dy, but since the charge is distributed along the x-axis (from $x = -b$ to $x = a$), the integration must run along the length of the cylinder, i.e., over dx. Claim: change dy to dx; Reasoning: this makes the integral valid (it must sum contributions from charge elements along the rod's length). (2) Wrong power in the denominator: the denominator power should be $3/2$, not 1 (matching the correct horizontal-component integral's $(c^2 + x^2)^{3/2}$). Claim: change the power to $3/2$; Reasoning: the units/dimensions of the integrand are only valid (consistent

Solution 1

with an electric field) with the $3/2$ power. Corrected form (using c, the fixed height of P, in place of the erroneous y): $|E_y| = \dfrac{\lambda c}{4\pi\varepsilon_0} \displaystyle\int_{-b}^{a} \dfrac{dx}{(c^2 + x^2)^{3/2}}$.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

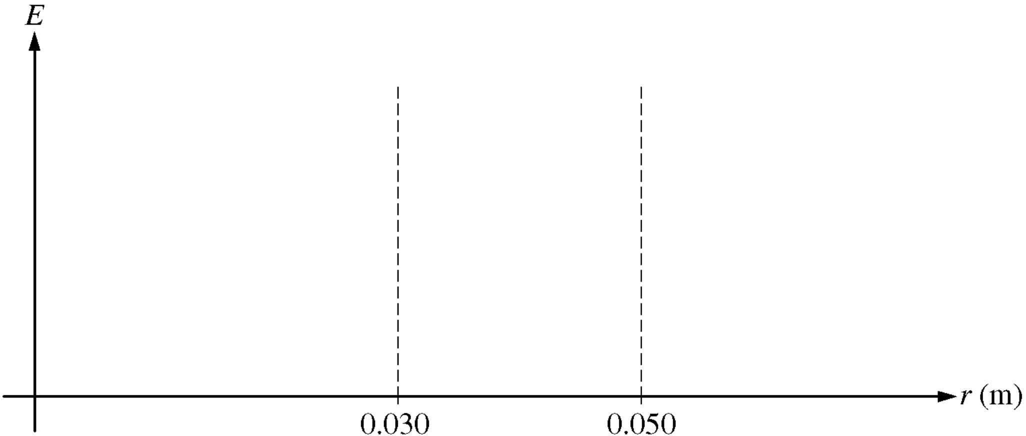
Question 2

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

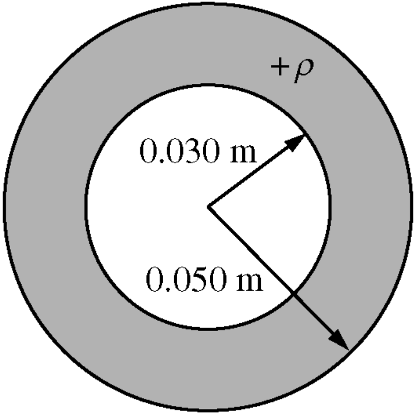
$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

Question 2



Question 2



Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050\ \text{m}: \quad \rho = 0$$

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

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$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.
[Graph with vertical axis labeled E and horizontal axis labeled r (m); two vertical dashed reference lines are marked at $r = 0.030$ and $r = 0.050$; the axes are otherwise blank for the student to sketch on.]

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(e)(i) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s. Calculate the magnitude of the initial acceleration of the proton.

(e)(ii) Calculate the speed of the proton after a long time.

Solution 2

(a) Mark scheme

- $Q = \int \rho \, dV = \int_{0.03}^{0.05} \frac{b}{r}(4\pi r^2)\, dr = 4\pi b \int_{0.03}^{0.05} r\, dr = 4\pi b \left[\frac{r^2}{2}\right]_{0.03}^{0.05} = 2\pi b \left(0.05^2 - 0.03^2\right)$. With $b = 1.6 \times 10^{-6}$ C/m^2: $Q = 2\pi(1.6 \times 10^{-6})(0.0016) = 1.61 \times 10^{-8}$ C. (1 pt: recognizing the need to integrate the charge density expression, $Q = \int \rho \, dV$, with the spherical shell volume element $4\pi r^2 \, dr$; 1 pt: correct substitution of $\rho = b/r$ into the integral; 1 pt: using the correct limits of integration and obtaining the correct numeric answer.)

Solution 2

(b) Mark scheme

- By symmetry, Gauss's law gives $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$. At the outer surface ($r = 0.05$ m) all the charge is enclosed, so $Q_{enc} = Q = 1.61 \times 10^{-8}$ C (from part (a)). $E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{1.61 \times 10^{-8}}{4\pi(8.85 \times 10^{-12})(0.05)^2} = 5.79 \times 10^4$ N/C. (1 pt: correctly evaluating the surface integral in Gauss's law; 1 pt: substituting the answer from part (a) and the correct radius into the equation; 1 pt: correct final answer with correct units, consistent with part (a).)

Solution 2

(c) Mark scheme

- $E = 0$ for $r < 0.030$ m (no enclosed charge). For 0.030 m $< r < 0.050$ m, E rises continuously from zero, concave down, increasing up to the value found in part (b) at $r = 0.050$ m (as Q_{enc} grows with r inside the shell). For $r > 0.050$ m, all the charge is enclosed and E falls off as $1/r^2$, decreasing asymptotically toward the horizontal (r) axis. (1 pt: clearly showing $E = 0$ for $r < 0.030$ m; 1 pt: a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$; 1 pt: a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m.)

Solution 2

(d) Mark scheme

- With $V = 0$ at infinity, and since at $r = 0.05$ m all the charge is enclosed (the sphere behaves as a point charge from that radius outward): $V_R = \frac{Q_{tot}}{4\pi\varepsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8}\ \text{C})}{0.05\ \text{m}} \approx 2900$ V. (Equivalent alternate method: integrate $\Delta V_R = -\int_\infty^r E\,dr = -\int_\infty^{0.05} \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}\,dr = 2900$ V.) (1 pt: substituting the total charge from part (a) into a correct expression for electric potential; 1 pt: substituting $r = 0.05$ m (or the correct limits, if integrating) into that expression.)

Solution 2

(e)(i)

Mark scheme

- Newton's second law with the electric force: $F = ma \Rightarrow qE = ma \Rightarrow a = \dfrac{qE}{m}$. Using the proton's charge $q = 1.6 \times 10^{-19}$ C, mass $m = 1.67 \times 10^{-27}$ kg, and $E = 5.79 \times 10^{4}$ N/C from part (b): $a = \dfrac{(1.6 \times 10^{-19})(5.79 \times 10^{4})}{1.67 \times 10^{-27}} = 5.55 \times 10^{12}$ m/s^2. (1 pt: using a correct expression of Newton's second law in terms of the electric field; 1 pt: correctly substituting values into that equation.)

Solution 2

(e)(ii) Mark scheme

- Using the work-energy theorem with electric potential: $-q\Delta V = \frac{1}{2}mv^2$, with $\Delta V = V_\infty - V_R = 0 - 2900$ V (the proton is repelled outward, converting electric potential energy into kinetic energy as it moves to infinity).
  $v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19})(0 - 2900)}{1.67 \times 10^{-27}}} = 7.45 \times 10^5$ m/s. (1 pt: a correct expression of kinetic energy in terms of the electric potential difference; 1 pt: correctly substituting into the equation and obtaining the correct final answer.)

Question 1

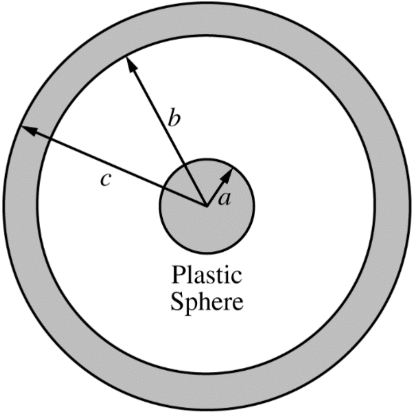
2018 ▪ Q1

A solid plastic sphere of radius *a* and a conducting spherical shell of inner radius *b* and outer radius *c* are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and *r* is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, *r*, *a*, and physical constants, as appropriate.
[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius *a* at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius *b* and outer radius *c*. Arrows from the center label the radius *a* to the edge of the plastic sphere, radius *b* to the inner edge of the shell, and radius *c* to the outer edge of the shell.]

Question 1

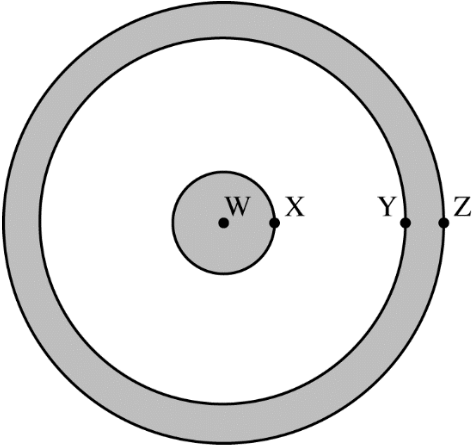
(a)(i) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following region.
$r < a$
(a)(ii) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following region.
$a < r < b$

Question 1



Conducting Spherical Shell

Question 1



Question 1

Question 1

2018 ▪ Q1

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

(b)(i) Use Gauss's law to derive an expression for the magnitude of the electric field in the following region.

$r < a$

Question 1

(b)(ii) Use Gauss's law to derive an expression for the magnitude of the electric field in the following region.
$a < r < b$

Question 1

2018 ▪ Q1

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

(c)(i) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

Determine the charge on the inner surface of the conducting shell.

Question 1

Justify your answer.

(c)(ii) Determine the charge on the outer surface of the conducting shell.

Question 1

2018 ▪ Q1

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

(d)(i) On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

Question 1

[Graph: Axes with vertical axis labeled E and horizontal axis labeled r, origin at O. Horizontal axis shows tick marks at a, b, c (with b and c close together near the right side, and a dashed segment continuing past c). No data plotted — blank axes for the student to sketch on.]

(d)(ii) The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

______ W ______ X ______ Y ______ Z

[Figure: A repeat of the cross-sectional diagram (plastic sphere inside conducting spherical shell), with four labeled points marked: W at the center of the sphere, X on

Question 1

the surface of the sphere (just outside W), Y on the inner surface of the shell, and Z on the outer surface of the shell.]

Solution 1

(a)(i)

Mark scheme

- Set up the enclosed-charge integral using $\rho(r) = \beta r$: $q_{enc} = \int \rho\, dV = \int \beta r\, dV$. Substitute the spherical shell volume element $dV = 4\pi r'^2\, dr'$: $q_{enc} = \int_0^r (4\pi r'^2)(\beta r')\, dr' = 4\beta\pi \int_0^r r'^3\, dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_0^r$. Final answer: $q_{enc} = \beta\pi r^4$. Points: 1 for setting up the integral with $\rho = \beta r$, 1 for correctly substituting dV, 1 for the correct final expression.

Solution 1

(a)(ii)

Mark scheme

- For $a < r < b$ the Gaussian sphere encloses the entire plastic sphere (charge stops at $r = a$), so the enclosed charge is just the part (a)(i) result evaluated at $r = a$: $q_{enc} = \beta\pi a^4$.

Solution 1

(b)(i)

Mark scheme

- Apply Gauss's law with spherical symmetry: $\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = E(4\pi r^2)$ (1 point). Substitute $q_{enc} = \beta\pi r^4$ from part (a)(i): $\frac{\beta\pi r^4}{\varepsilon_0} = E(4\pi r^2)$, giving $E = \dfrac{\beta r^2}{4\varepsilon_0}$ (1 point).

Solution 1

(b)(ii)

Mark scheme

- Substitute $q_{enc} = \beta\pi a^4$ from part (a)(ii) into Gauss's law: $\dfrac{\beta\pi a^4}{\varepsilon_0} = E(4\pi r^2)$, giving $E = \dfrac{\beta a^4}{4\varepsilon_0 r^2}$.

Solution 1

(c)(i)

Mark scheme

- $q_{inner} = -\beta\pi a^4$ (opposite sign of, and consistent with, the part (a)(ii)/(b)(ii) sphere charge $\beta\pi a^4$) (1 point). Justification: since $E = 0$ everywhere outside the shell, $\frac{q_{enc}}{\varepsilon_0} = 0$ for a Gaussian surface enclosing the whole shell, so $q_{enc} = 0 = q_{sphere} + q_{inner}$, giving $q_{inner} = -q_{sphere} = -\beta\pi a^4$ (1 point).

Solution 1

(c)(ii)

Mark scheme

- $q_{outer} = 0$. Since $E = 0$ outside the shell, the shell's total charge $Q = q_{inner} + q_{outer}$ must equal $-q_{sphere}$ (to cancel the sphere's field beyond the shell). Because $q_{inner} = -q_{sphere}$ already accounts for all of that required charge, the outer surface carries none: $q_{outer} = 0$.

Solution 1

(d)(i) Mark scheme

- Sketch: for $0 \leq r < a$, $E = \dfrac{\beta r^2}{4\varepsilon_0}$ is a concave-up curve starting at $E = 0$ at the origin and increasing up to $r = a$ (1 point). For $a < r < b$, $E = \dfrac{\beta a^4}{4\varepsilon_0 r^2}$ is a concave-up curve ($\propto 1/r^2$) that continues from the $r = a$ value and decreases to a nonzero value at $r = b$ (1 point). For $r > b$ (inside the conducting shell material, where $E = 0$ always, and beyond the shell, given $E = 0$ there), the curve sits at $E = 0$ (1 point). A discontinuity in the curve exactly at $r = b$ is not required due to scale.

Solution 1

(d)(ii)

Mark scheme

- Ranking ($1 =$ largest potential): $W = 1$, $X = 2$, $Y = 3$, $Z = 3$ (Y and Z tied). Reasoning: potential decreases moving outward from the center in this configuration, so W (center) $>$ X (sphere surface) $>$ shell surfaces; the conducting shell is an equipotential ($E=0$ inside the conductor), so Y (inner shell surface) and Z (outer shell surface) are at the same potential and share the ranking 3. 1 point for any ranking with Y and Z tied, 1 point for the order $W > X > Y$ & Z.

Question 1

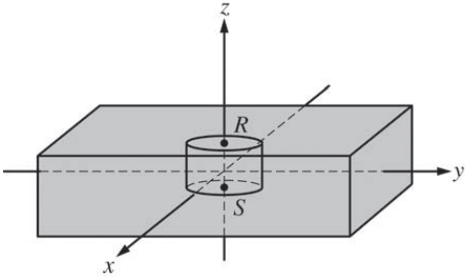
2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.
Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.
[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the z-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

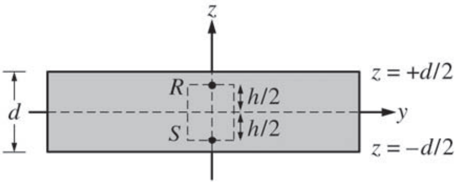
Question 1

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.
[Two blank diagrams, each showing a single dot: the first labeled i. with a dot marked $\bullet R$, the second labeled ii. with a dot marked $\bullet S$. The student is to draw a field-direction vector at each dot or write $E = 0$.]

Question 1



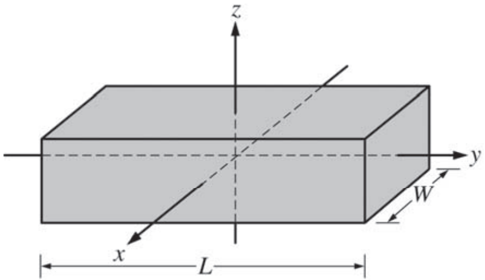
Perspective View



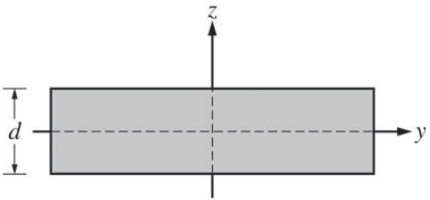
Side View

Note: Figures not drawn to scale.

Question 1



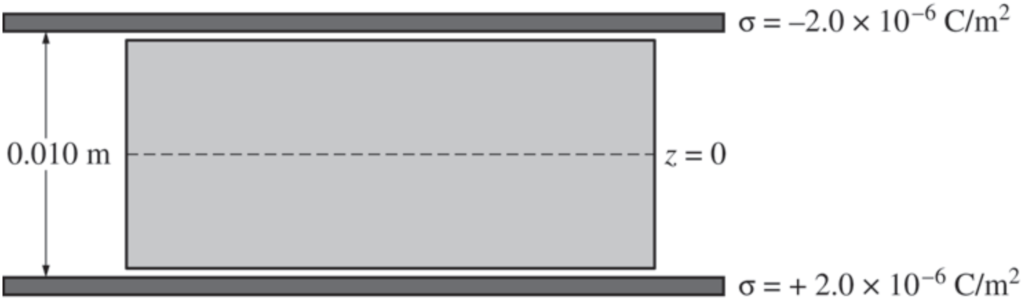
Perspective View



Side View

Note: Figures not drawn to scale.

Question 1



Note: Figure not drawn to scale.

Question 1

2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.

[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the z-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

(b)(i) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.
Derive an expression for the total flux Φ through the Gaussian surface shown.
(b)(ii) Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.

Question 1

2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.

[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the z-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

(c) [Figure: two large horizontal metal plates separated by a distance of 0.010 m, approximately the thickness of the slab, with the charged nonconducting slab centered between them (not touching either plate); $z = 0$ marks the mid-plane. The top plate carries surface charge density $\sigma = -2.0 \times 10^{-6}$ C/m^2; the bottom plate carries surface charge density $\sigma = +2.0 \times 10^{-6}$ C/m^2.]
The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}$ C/m^2, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}$ C/m^3.
(c)(i) The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

Question 1

$\underline{\qquad}$ $z_0 > 0$ $\underline{\qquad}$ $z_0 = 0$ $\underline{\qquad}$ $z_0 < 0$

Justify your answer.

(c)(ii) Calculate the value z_0.

Question 1

2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.

[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the *z*-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

Solution 1

(a)

Mark scheme

- Two vectors, 1 point each. At point R: draw a single vector pointing upward (away from the slab) — R lies above the positively charged slab ($\rho_0 > 0$), and the field of a positively charged slab points outward from it. At point S: draw a single vector pointing downward (away from the slab) — S lies below the slab. (If a point is inside the slab where the field is zero by symmetry, "$E = 0$" should be written instead, but here both R and S are outside the slab so nonzero vectors pointing away from the slab are required.)

Solution 1

(b)(i) Mark scheme

- Use Gauss's law to get the flux through the Gaussian surface: $\Phi = \dfrac{Q}{\varepsilon_0} = \dfrac{\rho_0 V}{\varepsilon_0}$ (1 point, for using Gauss's law to calculate the electric flux), where $V = Ah$ is the enclosed volume (A = cross-sectional area, h = height of the Gaussian surface within the slab). Correct final answer: $\Phi = \dfrac{\rho_0 Ah}{\varepsilon_0}$ (1 point).

Solution 1

(b)(ii) Mark scheme

- Apply Gauss's law to the slab, with flux only through the top and bottom faces by symmetry: $\frac{Q}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = EA_{top} + EA_{bottom}$ (1 point, for correctly applying Gauss's law to the slab). Substitute the flux from part (b)(i) and include both top and bottom contributions: $\frac{\rho_0 Ah}{\varepsilon_0} = 2EA$ (1 point). Relate the Gaussian-surface height h to the position z by $h = 2z$: $\frac{\rho_0 A(2z)}{\varepsilon_0} = 2EA \Rightarrow E = \frac{\rho_0 z}{\varepsilon_0}$ (1 point, for correctly relating h to 2z and reaching the given result).

Solution 1

(c)

Mark scheme

- This is the introductory stem/setup text for part (c), not a separately scored question — it describes the new configuration: the charged slab is now placed between two large horizontal metal plates separated by 0.010 m (about the slab's thickness), with $z = 0$ at the slab's mid-plane; the top plate carries surface charge density $\sigma = -2.0 \times 10^{-6}$ C/m^2 and the bottom plate carries $\sigma = +2.0 \times 10^{-6}$ C/m^2. No points are allocated to this stem; the scored sub-parts are (c)i and (c)ii.

Solution 1

(c)(i)

Mark scheme

- Correct selection: $z_0 < 0$. Justification (1 point): the electric field due to the plates alone (given the stated charge signs) points toward the top of the page between the plates. Therefore the net field can be zero only where the slab's own field points toward the bottom of the page — this occurs below the slab's mid-plane, i.e. where $z_0 < 0$.

Solution 1

(c)(ii) Mark scheme

- Set the net field between the plates to zero, as the sum of the plates' field and the slab's field: $E_{total} = 0 = E_{plate} + E_{slab}$ (1 point, for setting the net field to zero and relating it to the sum of the two contributions). Use the correct field expressions (regardless of sign convention): $\dfrac{\sigma}{\varepsilon_0} = -\dfrac{\rho_0 z_0}{\varepsilon_0} \Rightarrow z_0 = -\dfrac{\sigma}{\rho_0}$ (1 point). Substituting values: $z_0 = -\dfrac{2.0 \times 10^{-6}\ \mathsf{C/m^2}}{1.0 \times 10^{-3}\ \mathsf{C/m^3}} = -2.0 \times 10^{-3}$ m (1 point, for an answer consistent with the student's own formula above; units must be included).

Solution 1

(d) Mark scheme

- Total potential difference from the slab's center to its top is the sum of the slab's and plates' contributions:
  $\Delta V = \Delta V_{slab} + \Delta V_{plate} = \int E_{total}\, dr = \int E_{slab}\, dr + \int E_{plate}\, dr$ (1 point, for using the sum of the two potential contributions as the total). Integrate with the correct limits (z = 0 to z = 0.005 m, the top of the slab):
  $\Delta V = \int_0^{0.005} \frac{\rho_0 z}{\varepsilon_0}\, dz + \int_0^{0.005} \frac{\sigma}{\varepsilon_0}\, dz$ (1 point). Carry out the integration:
  $\Delta V = \frac{\rho_0 z^2}{2\varepsilon_0} + \frac{\sigma z}{\varepsilon_0}$ evaluated from 0 to 0.005 (1 point, for correctly integrating each term). Substituting numbers ($\rho_0 = 1.0 \times 10^{-3}$ C/m^3, $\sigma = 2.0 \times 10^{-6}$ C/m^2,

Solution 1

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N $\cdot$ m^2) gives $\Delta V = 2540$ V (1 point, for the correct final numerical answer).

Question 1

2015 ▪ Q1

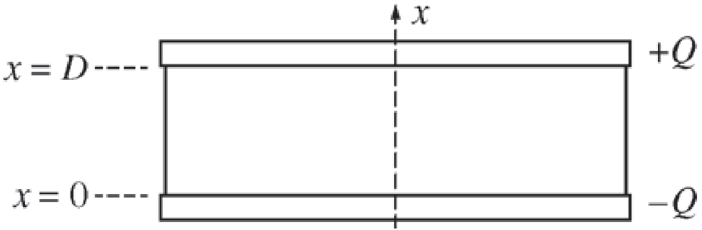
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.
[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]
(a)(i) On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
(a)(ii) On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

Question 1

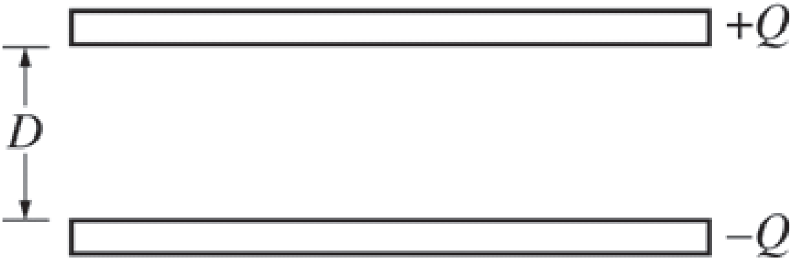
(a)(iii) Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.
[Figure: A vertical x-axis points upward. The bottom plate is at $x = 0$, labeled $-Q$; the top plate is at $x = D$, labeled $+Q$. A dashed vertical center line runs between the plates.]
The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

Question 1

of the electric field E between the plates. Express your answer in term
constants, as appropriate.



Question 1



Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(b) Determine an expression for the dielectric constant κ as a function of x.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(c)(i) Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

(c)(ii) Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(d) Determine the capacitance of the capacitor.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

_____ $U_V < U_C$ _____ $U_V > U_C$ _____ $U_V = U_C$

Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The electric field between two oppositely charged parallel plates points from the positive plate toward the negative plate. Since the $+Q$ plate is at $x = D$ (top) and the $-Q$ plate is at $x = 0$ (bottom), the field points downward, from the top plate to the bottom plate. Credit (1 point) requires at least one arrow between the plates pointing downward (from $+Q$ toward $-Q$) with no extraneous arrows pointing in any other direction.

Solution 1

(a)(ii)

Mark scheme

- Draw a Gaussian pillbox (a small rectangular box with flat faces parallel to the plates) straddling one of the plates so that it encloses at least the inner edge of that plate, with its flat end-faces parallel to the plate surfaces (perpendicular to $\vec{E}$). Credit (1 point) requires an appropriate Gaussian surface enclosing at least the inner edge of one plate that can be used to determine E between the plates.

Solution 1

(a)(iii) Mark scheme

- Start from Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\varepsilon_0}$ (1 point for a correct statement of Gauss's law). Apply it to the pillbox from part (a)-ii: only the flat end-face between the plates carries flux (the field is uniform and perpendicular to that face, and zero elsewhere on the surface), so $EA_{GS} = \dfrac{q_{enc}}{\varepsilon_0}$, giving $E = \dfrac{q_{enc}}{\varepsilon_0 A_{GS}}$; writing the surface charge density as $\sigma = \dfrac{q_{enc}}{A_{GS}} = \dfrac{Q}{A}$ (enclosed charge and Gaussian area consistent with the plate) gives $E = \dfrac{\sigma}{\varepsilon_0}$ (1 point for applying Gauss's law with enclosed charge and area consistent with the surface drawn in part (a)-ii). The

Solution 1

final result, with work shown, is $E = \dfrac{Q}{\varepsilon_0 A}$ (1 point for a correct answer with work shown).

Solution 1

(b)

Mark scheme

- Compare the standard parallel-plate field with a uniform dielectric, $E = \dfrac{Q}{\kappa\varepsilon_0 A}$, to the given expression for the field with the varying dielectric in place, $E = \dfrac{Q}{\varepsilon_0\kappa_0 e^{-x/D}A}$. Matching the two expressions term by term gives $\kappa = \kappa_0 e^{-x/D}$ (1 point; the answer must be consistent with the expression for E found in part (a)-iii).

Solution 1

(c)(i) Mark scheme

- Use the relation between electric field and potential, $E = -\frac{dV}{dx}$, and substitute the field from part (b): $\frac{dV}{dx} = -\left(-\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\right) = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$ (1 point for a correct differential equation; it does not need to be solved yet). An equivalent alternate approach accepted by the scoring guidelines uses $\Delta V = -\int \vec{E} \cdot d\vec{r}$ directly, giving $\Delta V = \int \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\, dx$, which earns the same 1 point.

Solution 1

(c)(ii) Mark scheme

- Separate variables in the differential equation from part (c)-i: $dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) e^{x/D}\, dx$. Integrate using the correct limits (V runs from V_0 to V_D as x runs from 0 to D): $\int_{V_0}^{V_D} dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \int_0^D e^{x/D}\, dx$ (1 point for using the correct limits of integration). Carrying out the integration: $[V]_{V_0}^{V_D} = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \left[De^{x/D}\right]_0^D$, so $(V_D - V_0) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) \left(e^{D/D} - e^0\right) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) (e - 1)$ (1 point for correctly

Solution 1

integrating the equation). Taking the magnitude gives $\Delta V = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)$ (1 point for the correct absolute value of the potential difference between the plates), and this value must be reported as positive (1 point for having the potential difference be positive). Numerically, $\Delta V = \frac{1.72\, QD}{\varepsilon_0 \kappa_0 A}$.

Solution 1

(d) Mark scheme

- Use the definition of capacitance, $C = \dfrac{Q}{\Delta V}$, and substitute the result from part (c)-ii: $C = \dfrac{Q}{\left(\dfrac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)} = \dfrac{\varepsilon_0 \kappa_0 A}{D(e-1)} = \dfrac{\varepsilon_0 \kappa_0 A}{1.72D}$ (1 point; the answer must be consistent with part (c)-ii).

Solution 1

(e) Mark scheme

- Select $U_V > U_C$ (1 point). Justification: from part (d), a capacitor with the uniform dielectric constant κ_0 throughout has a larger capacitance than the varying-dielectric capacitor, i.e. $C_C > C_V$, because $\kappa = \kappa_0 e^{-x/D} \leq \kappa_0$ everywhere reduces the effective capacitance compared to a full-κ_0 fill (1 point for correctly comparing the capacitance, or equivalently the potential difference, of the varying-dielectric capacitor to that of the constant-dielectric capacitor). Since both capacitors are given the same charge Q and $U = \dfrac{Q^2}{2C}$, the smaller capacitance stores more energy; because $C_C > C_V$, it follows that $U_V > U_C$ (1 point for correctly comparing the two stored energies consistent with the

Solution 1

capacitance/potential-difference comparison). Example from the scoring guidelines: 'According to the equation from part (d), $C_C > C_V$. Since $U = Q^2/2C$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.'

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

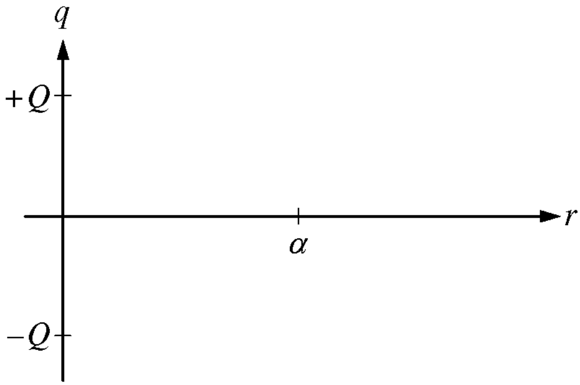
where α and β are positive constants and r is the distance from the center of the atom. **(a)(i)** On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

Question 3

i. The nuclear charge only
[Graph: axes with vertical axis q (labeled $+Q$ and $-Q$ as tick marks above and below the origin) and horizontal axis r (with a tick mark labeled α); blank, for the student to sketch.]
(a)(ii) ii. The electron charge only
[Graph: axes with vertical axis q (labeled $+Q$ and $-Q$ as tick marks above and below the origin) and horizontal axis r (with a tick mark labeled α); blank, for the student to sketch.]

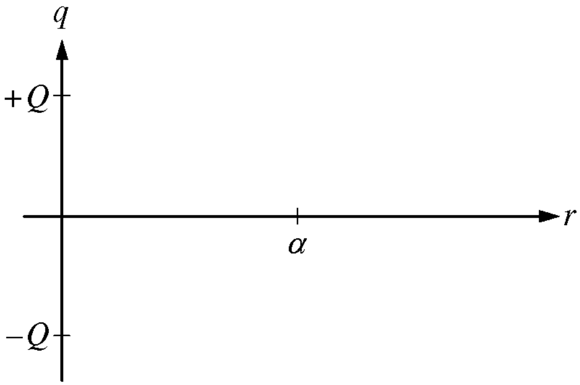
Question 3

ii. The <u>electron charge</u> only

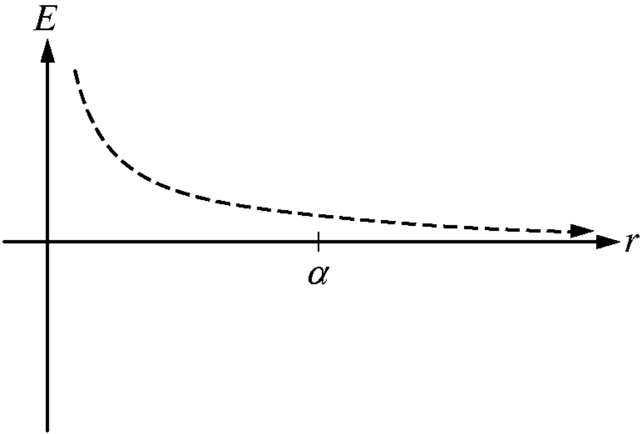


Question 3

i. The nuclear charge only



Question 3



Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric

Question 3

field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it. [Graph: axes with vertical axis E and horizontal axis r (with a tick mark labeled α); a dashed curve is shown starting high near $r = 0$ and decreasing smoothly (following an inverse-square-like shape) toward the r-axis as r increases past α, approaching zero for large r. The student is to add their own solid curve for the neutral-atom field on the same axes.]

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\frac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(c)(i) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of Q, α, β, r, and fundamental constants, as appropriate.

i. $r > \alpha$

(c)(ii) ii. $r < \alpha$

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α?