Past-paper walk-through

Electric Fields ▪ 8.3

eXercise

Questions in this set

- 2024 Set 1 Q1
- 2024 Set 2 Q1
- 2017 Q1
- 2016 Q1
- 2015 Q1
- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the *x*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and the *y*-axis are 0.40 m apart.
[Figure 1: An *x*-*y* coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the *x*-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the *x*-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and

Question 1

between the rod and sphere. A horizontal double-headed arrow near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

Question 1

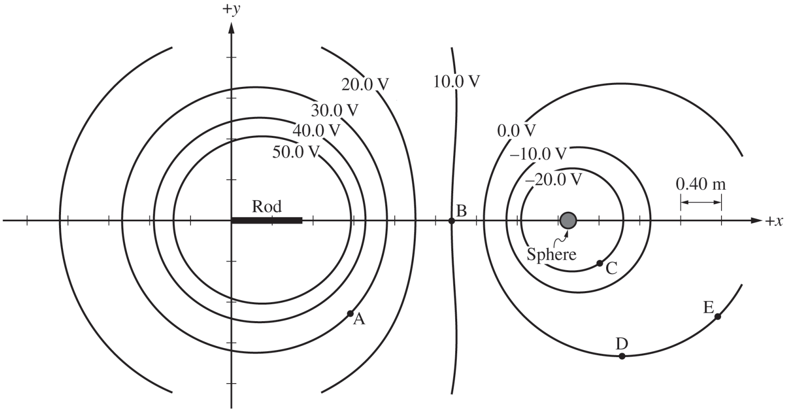


Figure 1

Question 1

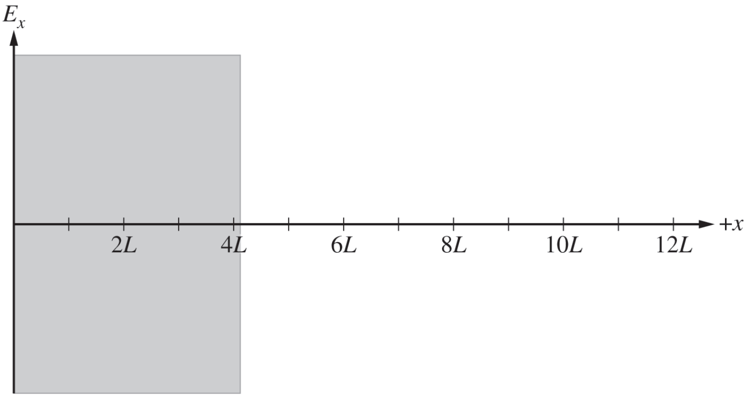


Figure 4

Question 1

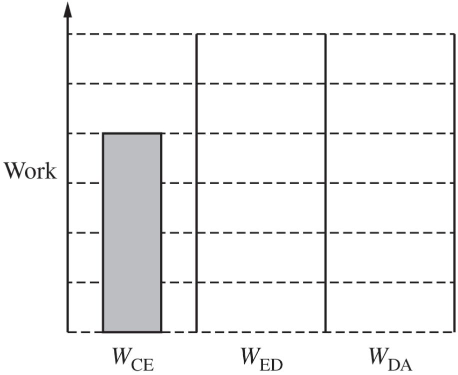


Figure 2

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

[Figure 1: An x-y coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the x-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the x-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and between the rod and sphere. A horizontal double-headed arrow

Question 1

near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C$\rightarrow$E$\rightarrow$D$\rightarrow$A. The test charge is momentarily held at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E. **Complete** the following bar graph in Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D. - **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to

Question 1

Position A. - The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate. [Figure 2: A bar-chart grid with a vertical axis labeled "Work" (unmarked scale, gridlines shown) and three labeled columns along the horizontal axis: W_{CE}, W_{ED}, W_{DA}. The bar for W_{CE} is already drawn at a fixed reference height (shaded); the bars for W_{ED} and W_{DA} are to be drawn by the student.]

(b)(ii) Calculate the approximate magnitude of the x-component of the electric field at Position B.

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

[Figure 1: An x-y coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the x-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the x-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and between the rod and sphere. A horizontal double-headed arrow

Question 1

near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(c) The positive test charge is placed at Position D. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 1 ▪ Q1

A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

[Figure 1: An x-y coordinate system (origin at lower left, $+y$ up, $+x$ to the right). On the left side, a horizontal rod lies along the x-axis, surrounded by concentric circular equipotential lines labeled (from innermost/rod outward) 50.0 V, 40.0 V, 30.0 V, 20.0 V, 10.0 V. Position A is marked near the rod, and Position B is marked further out among the rod's equipotential circles. On the right side, a small sphere sits on the x-axis, surrounded by concentric circular equipotential lines labeled (from outermost inward toward the sphere) 0.0 V, -10.0 V, -20.0 V, -30.0 V. Position C is at the sphere; Position D is below and between the rod and sphere; Position E is above and between the rod and sphere. A horizontal double-headed arrow

Question 1

near the sphere is labeled 0.40 m, spanning from the sphere to a point along $+x$. Tick marks along the x-axis and y-axis are spaced 0.40 m apart.]

(d)(i) [Figure 3: A horizontal rod labeled "Rod" lies along the x-axis from $x = 0$ to $x = 4L$, with linear charge density $+\lambda$ indicated by a bracket labeled $+\lambda$ above the rod near $x = 0$. The x-axis is marked with tick labels $0, 2L, 4L, 6L, 8L, 10L, 12L$.]

The sphere and the test charge are removed. The rod remains. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

The electric potential V_P at x_P is $V_P = k\lambda \ln\left[\dfrac{x_P}{x_P - 4L}\right]$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) [Figure 4: A blank graph with vertical axis E_x (unlabeled scale) and horizontal axis x marked with tick labels $2L, 4L, 6L, 8L, 10L, 12L$; a shaded region spans from $x = 4L$ to $x = 12L$ indicating the region to be graphed.]
On Figure 4, **sketch** a graph of the x-component of the electric field from the rod as a function of x in the region $4L < x < 12L$.

Solution 1

(a)

Mark scheme

- Use Gauss's law: $\Phi_E = Q/\varepsilon_0$ (1 pt for the correct equation). With enclosed charge $Q = -2.0 \times 10^{-9}$ C (the charge inside the -20.0 V equipotential Gaussian surface) and $\varepsilon_0 = 8.85 \times 10^{-12}$ $\mathsf{C}^2/(\mathsf{N} \cdot \mathsf{m}^2)$:
  $\Phi_E = \dfrac{-2.0 \times 10^{-9}\ \mathsf{C}}{8.85 \times 10^{-12}\ \mathsf{C}^2/(\mathsf{N} \cdot \mathsf{m}^2)} = -226\ \mathsf{N} \cdot \mathsf{m}^2/\mathsf{C}$. So $|\Phi_E| = 226\ \mathsf{N} \cdot \mathsf{m}^2/\mathsf{C}$
  (1 pt for the correct numerical answer; a negative sign or missing/incorrect units still earns this point).

Solution 1

(b)(i)

Mark scheme

- $W_{ED} = 0$ (1 pt), because Positions E and D lie on the same equipotential line so no work is done by the external force moving the test charge between them. On the bar graph, the bar representing W_{DA} should be drawn with a height of six units, using the same vertical scale as the given W_{CE} bar (1 pt).

Solution 1

(b)(ii) Mark scheme

- Start from the field-potential relation $E_x = -\dfrac{dV}{dx}$, so $|E_x| = \left|\dfrac{\Delta V}{\Delta x}\right|$ (1 pt for using a correct relating equation). Substitute the potential values and separation of the equipotential lines bracketing Position B: $|E_x| = \left|-\dfrac{20.0\ \text{V} - 0.0\ \text{V}}{0.65\ \text{m}}\right|$ (1 pt for correct substitution of values). This gives $|E_x| = 31\ \text{V/m}$.

Solution 1

(c) Mark scheme

- Select only $+y$, with an attempt at justification (1 pt). The justification must state that the electric field vector is perpendicular to the line tangent to the equipotential line at Position D (1 pt), and must also indicate one of: the positive test charge moves from a higher electric potential to a lower electric potential, or the test charge and the sphere have charges of opposite sign, or the test charge moves in the direction of the electric field, which is directed upward (1 pt). Model justification: the field is perpendicular to the equipotential line at D; since the test charge is positive it accelerates from higher to lower potential, so the net force (and field) at D points in $+y$, perpendicular to the equipotential line and in the direction of decreasing potential.

Solution 1

(d)(i) Mark scheme

- Derive V_P for the uniformly charged rod (length $4L$, density $+\lambda$, from $x = 0$ to $x = 4L$), 4 points total (1 pt each step): (1) start from an appropriate equation for potential from a charge distribution, $V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$ or $V_P = k\int \left(\frac{1}{x_P - x}\right) dq$; (2) correctly identify $r = x_P - x$, the distance from Point P (at x_P) to a point on the rod; (3) substitute $dq = \lambda\, dx$ to get $V_P = k\lambda \int \left(\frac{1}{x_P - x}\right) dx$; (4) use the correct limits of integration, 0 to $4L$: $V_P = k\lambda \int_0^{4L} \left(\frac{1}{x_P - x}\right) dx$. Evaluating

Solution 1

gives the final result $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$ (equivalently obtainable by integrating the field $E(x) = k\lambda\left(\frac{1}{x-4L} - \frac{1}{x}\right)$ from ∞ to x_P via $\Delta V = -\int \vec{E}\cdot d\vec{r}$).

Solution 1

(d)(ii)

Mark scheme

- Sketch the x-component of the rod's electric field for $4L < x < 12L$: the curve must continually approach (get closer to, without necessarily reaching) the horizontal axis as x increases (1 pt), and it must be a concave-up curve that stays always positive — falling steeply just past $x = 4L$ and flattening out toward (but not crossing) zero by $x = 12L$ (1 pt), consistent with the field weakening with distance from the charged rod.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.
[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost

Question 1

equipotential loops on the lower left, with A below B. A horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

Question 1

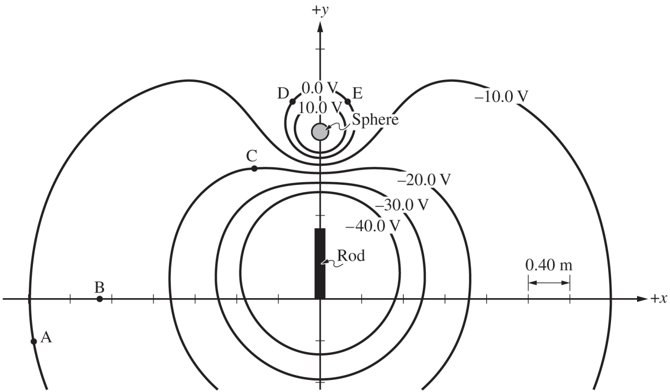


Figure 1

Question 1

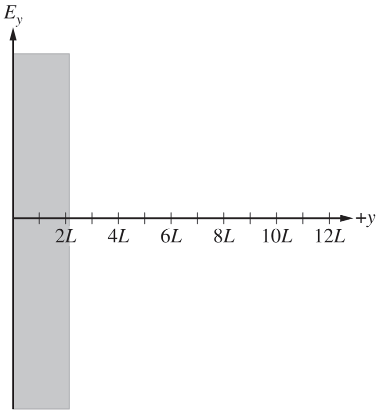


Figure 4

Question 1

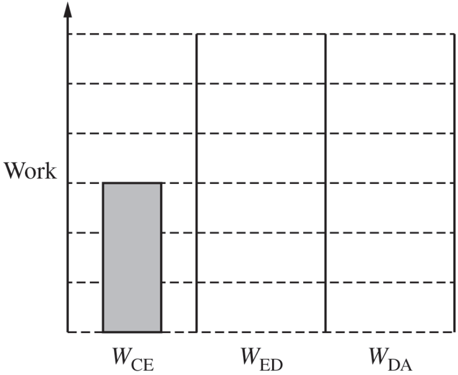


Figure 2

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

Draw a bar to represent the relative absolute value of the work W_{ED}, done by the external force on the test charge to move the test charge from Position E to Position D.

Draw a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.

Question 1

The height of each bar should be proportional to the value of W_{CE}. If $W_{\mathrm{ED}} = 0$ and/or $W_{\mathrm{DA}} = 0$, write a "0" in the corresponding columns, as appropriate.
[Figure 2: A bar chart with vertical axis labeled "Work" (unlabeled scale, dashed horizontal gridlines) and three columns on the horizontal axis labeled W_{CE}, W_{ED}, W_{DA}. A single shaded bar of medium height is already drawn above the W_{CE} column; the W_{ED} and W_{DA} columns are empty for the student to complete.]
(b)(ii) Calculate the approximate magnitude of the x-component of the electric field at Position B.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(c) The positive test charge is placed at Position C. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

______ $+x$ ______ $+y$ ______ Directly away from the sphere

______ $-x$ ______ $-y$ ______ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(d)(i) [Figure 3: A vertical y-axis with tick marks labeled from bottom to top: $-\lambda$ at 0, then $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. A thick vertical bar labeled "Rod" spans from 0 to $2L$ on the axis, adjacent to the label $-\lambda$.]

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.
[Figure 4: A blank graph with vertical axis labeled E_y (no numeric scale shown) and horizontal axis labeled y with tick marks at $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. Axes are otherwise empty for the student to sketch a curve.]

Solution 1

(a)

Mark scheme

- Use $\Phi_E = Q/\varepsilon_0$ (from Gauss's law, $\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$) with $Q = 1.0 \times 10^{-9}$ C enclosed by the 0.0 V equipotential surface:
  $\Phi_E = \dfrac{1.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12}\ \mathrm{C}^2/(\mathrm{N} \cdot \mathrm{m}^2)} = 113\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}$. 1 point for using the correct flux equation, 1 point for the correct numerical answer (units not required for this point).

Solution 1

(b)(i)

Mark scheme

- Positions D and E lie on the same equipotential line, so no work is required to move the test charge between them: $W_{ED} = 0$ (bar of height 0). The bar representing W_{DA} should have a height of approximately one and a half units (about half the height of the given W_{CE} bar). 1 point for indicating $W_{ED} = 0$, 1 point for drawing the W_{DA} bar at roughly 1.5 units.

Solution 1

(b)(ii) Mark scheme

- Relate electric field to the potential gradient between adjacent equipotential lines: $E_y = -dV/dy$, so $|E_y| = |\Delta V/\Delta y|$. Substituting the potential values and spacing of the equipotential lines bracketing Position B: $|E_y| = \left|\dfrac{-20.0\ \text{V} - (-10.0\ \text{V})}{1.4\ \text{m}}\right| = 7.1\ \text{V/m}$. 1 point for using a correct field-potential relationship (may start with numbers already substituted), 1 point for correctly substituting the potential values and equipotential-line spacing used to get the field magnitude at Position B.

Solution 1

(c) Mark scheme

- Direction: $-y$ (only). Justification: the electric field vector is perpendicular to the equipotential line (tangent) at Position C. Because the test charge is positive, it moves from higher to lower electric potential when a force acts on it; at Position C the field/force direction that is perpendicular to the equipotential line and points toward decreasing potential is $-y$ (directly toward the sphere). 1 point for selecting only $-y$ with an attempted justification, 1 point for stating the field is perpendicular to the equipotential line at C, 1 point for one of: the charge moves from higher to lower potential / the test charge and rod have opposite-sign charge / the test charge and sphere have same-sign charge.

Solution 1

(d)(i) Mark scheme

- Set up the potential from a continuous line charge: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$, i.e. $V_P = k\int\left(\frac{1}{y_P - y}\right)dq$, where $y_P - y$ is the distance from a charge element at height y on the rod to point P. Substitute the charge element of the negatively-charged rod, $dq = -\lambda\, dy$: $V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$. Integrate over the rod's extent with limits $y = 0$ to $y = 2L$: $V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$. 1 point for an appropriate potential-from-line-charge equation, 1 point for correctly determining $r = y_P - y$,

Solution 1

1 point for the correct integral with $\lambda\, dy$ substituted for dq, 1 point for the correct limits of integration (0 to 2L).

Solution 1

(d)(ii)

Mark scheme

- For $2L < y < 12L$, sketch E_y as a curve that is always negative (field points toward the negatively-charged rod, i.e. in $-y$) and continually approaches (but never reaches) the horizontal axis as y increases — i.e. a concave-down curve that asymptotically rises toward zero from below, steepest just above $y = 2L$ and flattening out toward $y = 12L$. 1 point for a curve/line that continually approaches the horizontal axis as position increases, 1 point for a concave-down curve that stays always negative.

Question 1

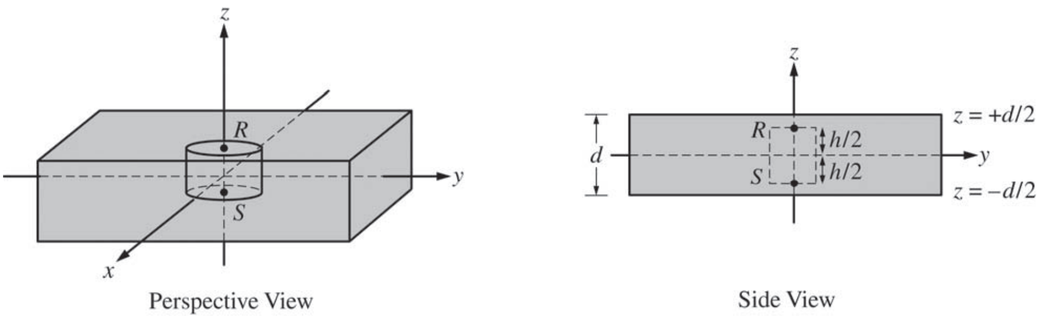
2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.
Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.
[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the z-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

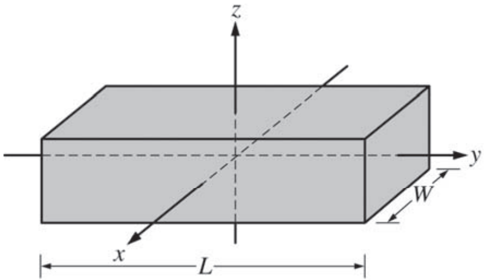
(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.
[Two blank diagrams, each showing a single dot: the first labeled i. with a dot marked $\bullet R$, the second labeled ii. with a dot marked $\bullet S$. The student is to draw a field-direction vector at each dot or write $E = 0$.]

Question 1

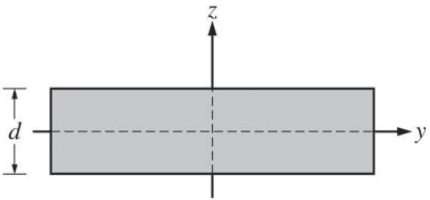


Note: Figures not drawn to scale.

Question 1



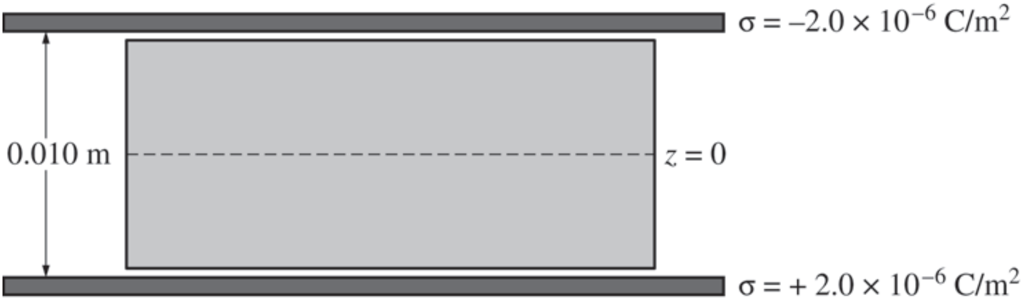
Perspective View



Side View

Note: Figures not drawn to scale.

Question 1



Note: Figure not drawn to scale.

Question 1

2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.
Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.
[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the z-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

(b)(i) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.
Derive an expression for the total flux Φ through the Gaussian surface shown.
(b)(ii) Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.

Question 1

2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.

[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the *z*-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

(c) [Figure: two large horizontal metal plates separated by a distance of 0.010 m, approximately the thickness of the slab, with the charged nonconducting slab centered between them (not touching either plate); $z = 0$ marks the mid-plane. The top plate carries surface charge density $\sigma = -2.0 \times 10^{-6}$ C/m^2; the bottom plate carries surface charge density $\sigma = +2.0 \times 10^{-6}$ C/m^2.]
The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}$ C/m^2, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}$ C/m^3.
(c)(i) The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

Question 1

______ $z_0 > 0$ ______ $z_0 = 0$ ______ $z_0 < 0$

Justify your answer.

(c)(ii) Calculate the value z_0.

Question 1

2017 ▪ Q1

A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the *xy*-plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.

[Perspective view and side view of the nonconducting slab (thickness d, length L, width W) centered at the origin, with a Gaussian cylinder of cross-sectional area A and height h positioned along the z-axis through the slab. In the side view, the slab extends from $z = -d/2$ to $z = +d/2$; the Gaussian cylinder has its top circular cap labeled R at height $h/2$ above the $z = 0$ plane and its bottom circular cap labeled S at height $h/2$ below the $z = 0$ plane, so the cylinder has total height h centered on $z = 0$.]

Question 1

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

Solution 1

(a)

Mark scheme

- Two vectors, 1 point each. At point R: draw a single vector pointing upward (away from the slab) — R lies above the positively charged slab ($\rho_0 > 0$), and the field of a positively charged slab points outward from it. At point S: draw a single vector pointing downward (away from the slab) — S lies below the slab. (If a point is inside the slab where the field is zero by symmetry, "$E = 0$" should be written instead, but here both R and S are outside the slab so nonzero vectors pointing away from the slab are required.)

Solution 1

(b)(i) Mark scheme

- Use Gauss's law to get the flux through the Gaussian surface: $\Phi = \dfrac{Q}{\varepsilon_0} = \dfrac{\rho_0 V}{\varepsilon_0}$ (1 point, for using Gauss's law to calculate the electric flux), where $V = Ah$ is the enclosed volume (A = cross-sectional area, h = height of the Gaussian surface within the slab). Correct final answer: $\Phi = \dfrac{\rho_0 A h}{\varepsilon_0}$ (1 point).

Solution 1

(b)(ii) Mark scheme

- Apply Gauss's law to the slab, with flux only through the top and bottom faces by symmetry: $\frac{Q}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = EA_{top} + EA_{bottom}$ (1 point, for correctly applying Gauss's law to the slab). Substitute the flux from part (b)(i) and include both top and bottom contributions: $\frac{\rho_0 Ah}{\varepsilon_0} = 2EA$ (1 point). Relate the Gaussian-surface height h to the position z by $h = 2z$: $\frac{\rho_0 A(2z)}{\varepsilon_0} = 2EA \Rightarrow E = \frac{\rho_0 z}{\varepsilon_0}$ (1 point, for correctly relating h to 2z and reaching the given result).

Solution 1

(c)

Mark scheme

- This is the introductory stem/setup text for part (c), not a separately scored question — it describes the new configuration: the charged slab is now placed between two large horizontal metal plates separated by 0.010 m (about the slab's thickness), with $z = 0$ at the slab's mid-plane; the top plate carries surface charge density $\sigma = -2.0 \times 10^{-6}$ C/m^2 and the bottom plate carries $\sigma = +2.0 \times 10^{-6}$ C/m^2. No points are allocated to this stem; the scored sub-parts are (c)i and (c)ii.

Solution 1

(c)(i)

Mark scheme

- Correct selection: $z_0 < 0$. Justification (1 point): the electric field due to the plates alone (given the stated charge signs) points toward the top of the page between the plates. Therefore the net field can be zero only where the slab's own field points toward the bottom of the page — this occurs below the slab's mid-plane, i.e. where $z_0 < 0$.

Solution 1

(c)(ii) Mark scheme

- Set the net field between the plates to zero, as the sum of the plates' field and the slab's field: $E_{total} = 0 = E_{plate} + E_{slab}$ (1 point, for setting the net field to zero and relating it to the sum of the two contributions). Use the correct field expressions (regardless of sign convention): $\dfrac{\sigma}{\varepsilon_0} = -\dfrac{\rho_0 z_0}{\varepsilon_0} \Rightarrow z_0 = -\dfrac{\sigma}{\rho_0}$ (1 point). Substituting values: $z_0 = -\dfrac{2.0 \times 10^{-6}\ \mathsf{C/m^2}}{1.0 \times 10^{-3}\ \mathsf{C/m^3}} = -2.0 \times 10^{-3}$ m (1 point, for an answer consistent with the student's own formula above; units must be included).

Solution 1

(d) Mark scheme

- Total potential difference from the slab's center to its top is the sum of the slab's and plates' contributions:
  $\Delta V = \Delta V_{slab} + \Delta V_{plate} = \int E_{total}\, dr = \int E_{slab}\, dr + \int E_{plate}\, dr$ (1 point, for using the sum of the two potential contributions as the total). Integrate with the correct limits (z = 0 to z = 0.005 m, the top of the slab):
  $\Delta V = \int_0^{0.005} \frac{\rho_0 z}{\varepsilon_0}\, dz + \int_0^{0.005} \frac{\sigma}{\varepsilon_0}\, dz$ (1 point). Carry out the integration:
  $\Delta V = \frac{\rho_0 z^2}{2\varepsilon_0} + \frac{\sigma z}{\varepsilon_0}$ evaluated from 0 to 0.005 (1 point, for correctly integrating each term). Substituting numbers ($\rho_0 = 1.0 \times 10^{-3}$ C/m^3, $\sigma = 2.0 \times 10^{-6}$ C/m^2,

Solution 1

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N $\cdot$ m^2) gives $\Delta V = 2540$ V (1 point, for the correct final numerical answer).

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

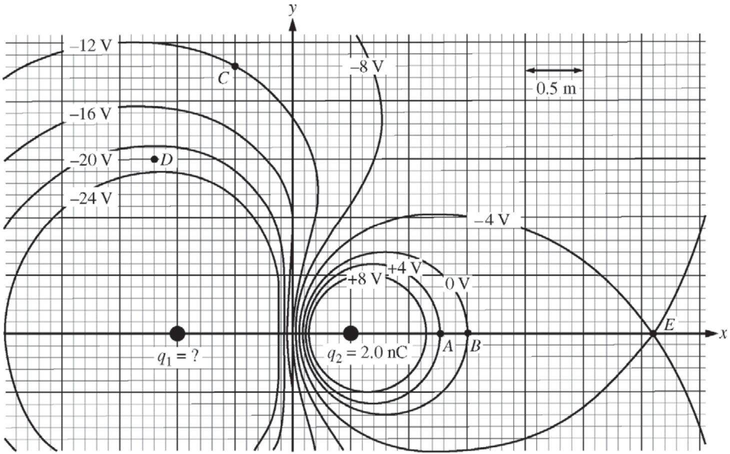
[Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside

Question 1

in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and (right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(a) Calculate the value of q_1.

Question 1



Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(e)(i) A proton is placed at point A and then released from rest. Calculate the work done by the electric field on the proton as it moves from point A to point E.

(e)(ii) Calculate the speed of the proton when it reaches point E.

Question 1

2016 ▪ Q1

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale. [Diagram: A set of equipotential curves in the xy-plane (drawn to scale, grid spacing 0.5 m per square as shown by the "0.5 m" scale bar). The x-axis runs horizontally with charge $q_1 = ?$ marked as a filled dot on the negative x-axis and charge $q_2 = 2.0$ nC marked as a filled dot at $x = +0.50$ m. Points A and B lie on the x-axis just to the right of q_2, with B to the right of A. Point E lies on the x-axis far to the right, at the crossing of the -4 V and 0 V equipotential curves. Point C is above and to the left of the y-axis, inside the -8 V curve near the top. Point D is to the left of the y-axis, between the -16 V and -20 V curves. The labeled equipotential curves, from outside in (left family) are -12 V, -16 V, -20 V, -24 V (nested curves opening around q_1 and extending to enclose part of the region left of the y-axis), and

Question 1

(right family, nested around q_2) -8 V, -4 V, $+8$ V, $+4$ V, 0 V. The 0 V curve is the innermost closed curve immediately surrounding q_2, passing near points A/B.]

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

______ Up ______ Down ______ Left ______ Right ______ Into the page ______ Out of the page ______ The direction is undefined since the acceleration is zero.

Justify your answer.

Solution 1

(a) Mark scheme

- Total potential is the sum of the potentials from the individual point charges. At point B, $V_B = 0 = V_1 + V_2$, so $-V_1 = V_2$, i.e. $-\dfrac{kq_1}{r_1} = \dfrac{kq_2}{r_2}$. Substituting $r_1 = 5 \times 0.5$ m, $r_2 = 2 \times 0.5$ m, $q_2 = 2.0$ nC: $-\dfrac{q_1}{(5 \times 0.5\text{ m})} = \dfrac{2.0\text{ nC}}{(2 \times 0.5\text{ m})}$. Solving gives $q_1 = -5.0$ nC. Points: 1 for indicating total potential = sum of individual potentials; 1 for correctly substituting into that equation (signs ignored at this step); 1 for the correct answer with correct sign and units, $q_1 = -5.0$ nC.

Solution 1

(b)

Mark scheme

- This is a drawing task: sketch, at point C, a vector for the electric field direction. The field must be drawn perpendicular to the equipotential line passing through C, and it must point in the direction of decreasing potential — i.e. from the -12 V line toward the -16 V line (field points from high to low potential). Points: 1 for drawing a vector perpendicular to the equipotential line for C; 1 for drawing the vector in the direction of the -16 V line.

Solution 1

(c) Mark scheme

- Use $E = -\dfrac{dV}{dx}$, so $|E| \approx \dfrac{\Delta V}{\Delta x}$. Reading values near point D from the equipotential diagram (lines -20 V and -24 V, separated by $\Delta x = 2 \times 0.1$ m): $E = \dfrac{(-20\text{ V}) - (-24\text{ V})}{2 \times 0.1\text{ m}} = 20$ N/C. Points: 1 for using the equation relating electric field to potential difference/gradient; 1 for correctly substituting the values from the figure to get $E = 20$ N/C.

Solution 1

(d)

Mark scheme

- By Gauss's law, $\Phi_E = \dfrac{q_{enc}}{\varepsilon_0}$, where $q_{enc} = 2.0$ nC is the charge enclosed by the 0 V equipotential surface. $\Phi_E = \dfrac{2.0 \text{ nC}}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 226 \text{ N} \cdot \text{m}^2/\text{C}$.

  Points: 1 for using the correct equation for electric flux (Gauss's law); 1 for the correct answer with units, $\Phi_E = 226 \text{ N} \cdot \text{m}^2/\text{C}$.

Solution 1

(e)(i)

Mark scheme

- Work equals the negative change in potential energy: $W = -q\Delta V$. With the proton's charge $q = 1.6 \times 10^{-19}$ C and $\Delta V = V_E - V_A = (-4\text{ V}) - (4\text{ V}) = -8$ V: $W = -(1.6 \times 10^{-19}\text{ C})(-8\text{ V}) = 1.28 \times 10^{-18}\text{ J} = 8.0\text{ eV}$. Points: 1 for using an equation relating work done to the change in potential energy ($W = -q\Delta V$); 1 for a correct answer including sign and units, $W = 1.28 \times 10^{-18}\text{ J} = 8.0\text{ eV}$.

Solution 1

(e)(ii)

Mark scheme

- The work done on the proton equals its kinetic energy gain: $W = \Delta K = \frac{1}{2}mv^2$. Substituting the part (e)(i) result and the proton mass $m = 1.67 \times 10^{-27}$ kg: 1.28×10^{-18} J $= \frac{1}{2}(1.67 \times 10^{-27}$ kg$)v^2 \Rightarrow v = 3.92 \times 10^4$ m/s. Points: 1 for indicating that the proton's kinetic energy at E equals the work done; 1 for correctly substituting the part (e)(i) answer to get $v = 3.92 \times 10^4$ m/s.

Solution 1

(f)

Mark scheme

- Correct choice: Left. Justification: an electron (negative charge) accelerates in the direction perpendicular to the equipotential surfaces, toward higher potential (opposite the electric field direction). At point B this direction is toward the left. Points: 1 for correctly selecting 'Left'; 1 for a correct justification along these lines. No points are earned if the wrong direction is selected.

Question 1

2015 ▪ Q1

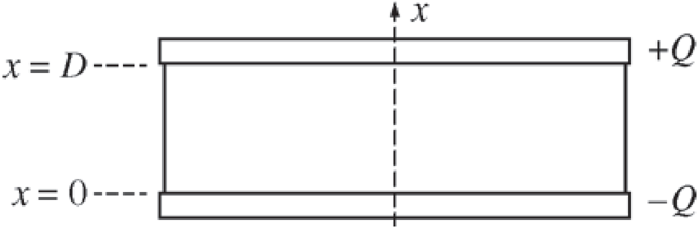
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.
[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]
(a)(i) On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
(a)(ii) On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

Question 1

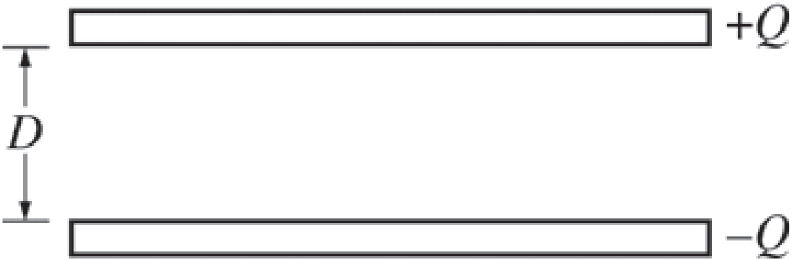
(a)(iii) Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.
[Figure: A vertical x-axis points upward. The bottom plate is at $x = 0$, labeled $-Q$; the top plate is at $x = D$, labeled $+Q$. A dashed vertical center line runs between the plates.]
The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

Question 1

of the electric field E between the plates. Express your answer in term
constants, as appropriate.



Question 1



Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(b) Determine an expression for the dielectric constant κ as a function of x.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(c)(i) Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

(c)(ii) Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(d) Determine the capacitance of the capacitor.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

______ $U_V < U_C$ ______ $U_V > U_C$ ______ $U_V = U_C$

Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The electric field between two oppositely charged parallel plates points from the positive plate toward the negative plate. Since the $+Q$ plate is at $x = D$ (top) and the $-Q$ plate is at $x = 0$ (bottom), the field points downward, from the top plate to the bottom plate. Credit (1 point) requires at least one arrow between the plates pointing downward (from $+Q$ toward $-Q$) with no extraneous arrows pointing in any other direction.

Solution 1

(a)(ii)

Mark scheme

- Draw a Gaussian pillbox (a small rectangular box with flat faces parallel to the plates) straddling one of the plates so that it encloses at least the inner edge of that plate, with its flat end-faces parallel to the plate surfaces (perpendicular to $\vec{E}$). Credit (1 point) requires an appropriate Gaussian surface enclosing at least the inner edge of one plate that can be used to determine E between the plates.

Solution 1

(a)(iii) Mark scheme

- Start from Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\varepsilon_0}$ (1 point for a correct statement of Gauss's law). Apply it to the pillbox from part (a)-ii: only the flat end-face between the plates carries flux (the field is uniform and perpendicular to that face, and zero elsewhere on the surface), so $EA_{GS} = \dfrac{q_{enc}}{\varepsilon_0}$, giving $E = \dfrac{q_{enc}}{\varepsilon_0 A_{GS}}$; writing the surface charge density as $\sigma = \dfrac{q_{enc}}{A_{GS}} = \dfrac{Q}{A}$ (enclosed charge and Gaussian area consistent with the plate) gives $E = \dfrac{\sigma}{\varepsilon_0}$ (1 point for applying Gauss's law with enclosed charge and area consistent with the surface drawn in part (a)-ii). The

Solution 1

final result, with work shown, is $E = \dfrac{Q}{\varepsilon_0 A}$ (1 point for a correct answer with work shown).

Solution 1

(b)

Mark scheme

- Compare the standard parallel-plate field with a uniform dielectric, $E = \dfrac{Q}{\kappa\varepsilon_0 A}$, to the given expression for the field with the varying dielectric in place, $E = \dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$. Matching the two expressions term by term gives $\kappa = \kappa_0 e^{-x/D}$ (1 point; the answer must be consistent with the expression for E found in part (a)-iii).

Solution 1

(c)(i) Mark scheme

- Use the relation between electric field and potential, $E = -\frac{dV}{dx}$, and substitute the field from part (b): $\frac{dV}{dx} = -\left(-\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\right) = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$ (1 point for a correct differential equation; it does not need to be solved yet). An equivalent alternate approach accepted by the scoring guidelines uses $\Delta V = -\int \vec{E} \cdot d\vec{r}$ directly, giving $\Delta V = \int \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\, dx$, which earns the same 1 point.

Solution 1

(c)(ii) Mark scheme

- Separate variables in the differential equation from part (c)-i: $dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) e^{x/D}\, dx$. Integrate using the correct limits (V runs from V_0 to V_D as x runs from 0 to D): $\int_{V_0}^{V_D} dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \int_0^D e^{x/D}\, dx$ (1 point for using the correct limits of integration). Carrying out the integration: $[V]_{V_0}^{V_D} = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \left[De^{x/D}\right]_0^D$, so $(V_D - V_0) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) \left(e^{D/D} - e^0\right) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) (e - 1)$ (1 point for correctly

Solution 1

integrating the equation). Taking the magnitude gives $\Delta V = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)$ (1 point for the correct absolute value of the potential difference between the plates), and this value must be reported as positive (1 point for having the potential difference be positive). Numerically, $\Delta V = \frac{1.72\, QD}{\varepsilon_0 \kappa_0 A}$.

Solution 1

(d) Mark scheme

- Use the definition of capacitance, $C = \dfrac{Q}{\Delta V}$, and substitute the result from part (c)-ii: $C = \dfrac{Q}{\left(\dfrac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)} = \dfrac{\varepsilon_0 \kappa_0 A}{D(e-1)} = \dfrac{\varepsilon_0 \kappa_0 A}{1.72D}$ (1 point; the answer must be consistent with part (c)-ii).

Solution 1

(e) Mark scheme

- Select $U_V > U_C$ (1 point). Justification: from part (d), a capacitor with the uniform dielectric constant κ_0 throughout has a larger capacitance than the varying-dielectric capacitor, i.e. $C_C > C_V$, because $\kappa = \kappa_0 e^{-x/D} \leq \kappa_0$ everywhere reduces the effective capacitance compared to a full-κ_0 fill (1 point for correctly comparing the capacitance, or equivalently the potential difference, of the varying-dielectric capacitor to that of the constant-dielectric capacitor). Since both capacitors are given the same charge Q and $U = \dfrac{Q^2}{2C}$, the smaller capacitance stores more energy; because $C_C > C_V$, it follows that $U_V > U_C$ (1 point for correctly comparing the two stored energies consistent with the

Solution 1

capacitance/potential-difference comparison). Example from the scoring guidelines: 'According to the equation from part (d), $C_C > C_V$. Since $U = Q^2/2C$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.'

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

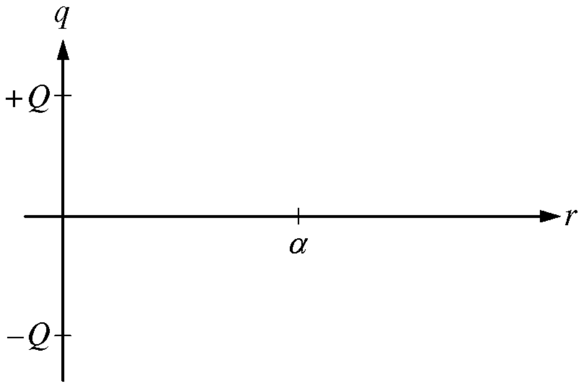
where α and β are positive constants and r is the distance from the center of the atom.
(a)(i) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

Question 3

i. The nuclear charge only
[Graph: axes with vertical axis q (labeled $+Q$ and $-Q$ as tick marks above and below the origin) and horizontal axis r (with a tick mark labeled α); blank, for the student to sketch.]
(a)(ii) ii. The electron charge only
[Graph: axes with vertical axis q (labeled $+Q$ and $-Q$ as tick marks above and below the origin) and horizontal axis r (with a tick mark labeled α); blank, for the student to sketch.]

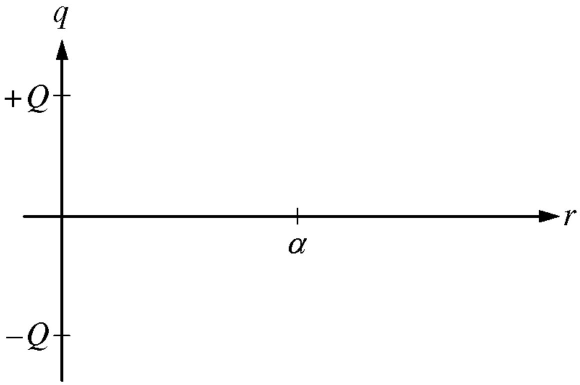
Question 3

ii. The electron charge only

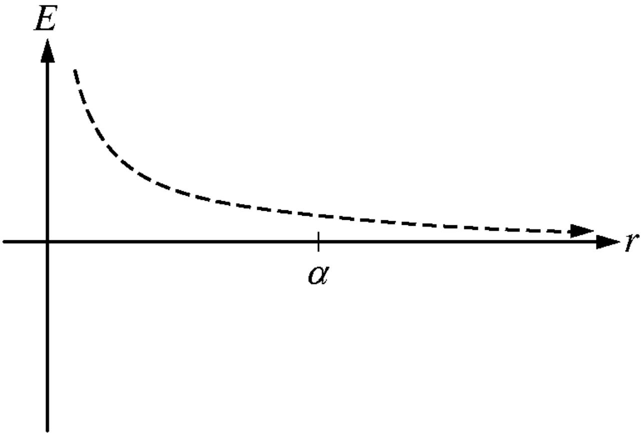


Question 3

i. The <u>nuclear charge</u> only



Question 3



Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric

Question 3

field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it. [Graph: axes with vertical axis E and horizontal axis r (with a tick mark labeled α); a dashed curve is shown starting high near $r = 0$ and decreasing smoothly (following an inverse-square-like shape) toward the r-axis as r increases past α, approaching zero for large r. The student is to add their own solid curve for the neutral-atom field on the same axes.]

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(c)(i) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of Q, α, β, r, and fundamental constants, as appropriate.

i. $r > \alpha$

(c)(ii) ii. $r < \alpha$

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α?