Past-paper walk-through

Electric Charge and Electric Force ▪ 8.1

eXercise

Questions in this set

- 2026 Q2
- 2024 Set 2 Q1
- 2023 Set 1 Q1
- 2023 Set 2 Q1
- 2021 Q2
- 2019 Set 1 Q1
- 2019 Set 2 Q2
- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2026 ▪ Q2

Thin, nonconducting, charged Rods S and T are held fixed in the *xy*-plane, as shown in Figure 1. Rod S is oriented along the *x*-axis in the region $-3L \leq x \leq -L$. Rod T is oriented along the *y*-axis in the region $L \leq y \leq 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

[Figure 1: In the *xy*-plane, Rod S is a thick horizontal segment on the *x*-axis from $x = -3L$ to $x = -L$, labeled "Rod S" with charge density $+\lambda$. Rod T is a thick vertical segment on the *y*-axis from $y = L$ to $y = 3L$, labeled "Rod T" with charge density $+\lambda$. Point P is marked with a dot at $(-2L, 2L)$. Dashed gridlines mark $x = -3L, -2L, -L, O, L$ and $y = L, 2L, 3L$.]

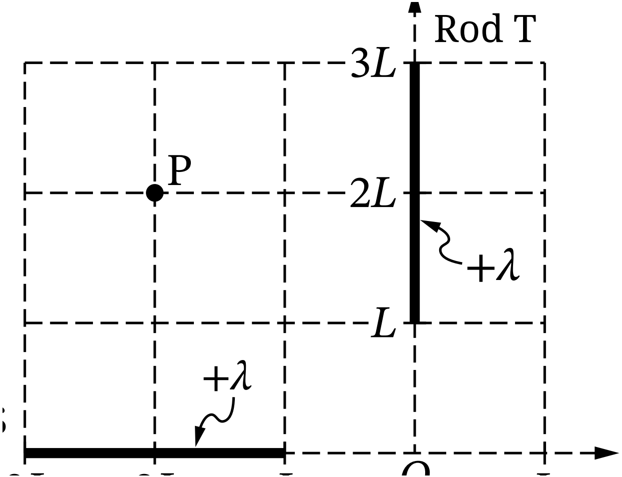
(a) Complete the following tasks in Figure 2.

Question 2

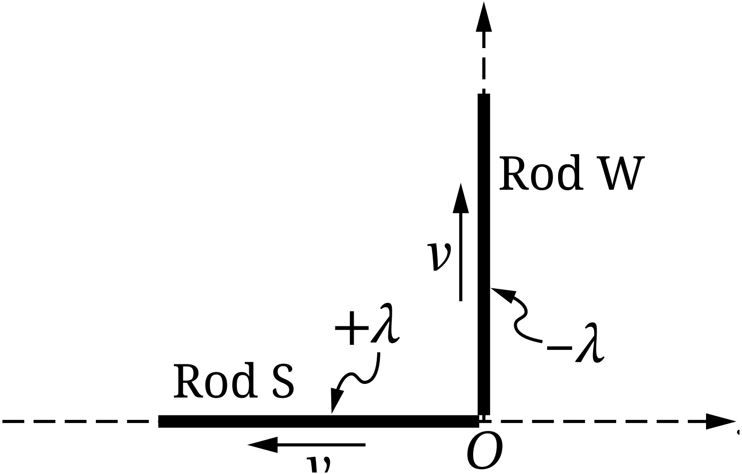
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_P$ at Point P due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_O$ at the origin O due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net acceleration $\vec{a}_O$ of a small, negatively charged sphere (not shown) immediately after the sphere is released from rest at the origin O.

[Figure 2: Three separate blank coordinate-axis boxes, each showing a dashed horizontal and vertical axis crossing at a labeled dot: the first labeled "Direction of $\vec{E}_P$" with the dot labeled P; the second labeled "Direction of $\vec{E}_O$" with the dot labeled O; the third labeled "Direction of $\vec{a}_O$" with the dot labeled O. Students draw an arrow from each dot in the appropriate direction.]

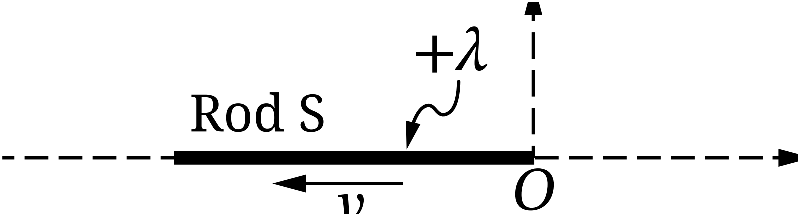
Question 2



Question 2



Question 2



Question 2

2026 ▪ Q2

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[Figure 1: In the *xy*-plane, Rod S is a thick horizontal segment on the *x*-axis from $x = -3L$ to $x = -L$, labeled "Rod S" with charge density $+\lambda$. Rod T is a thick vertical segment on the *y*-axis from $y = L$ to $y = 3L$, labeled "Rod T" with charge density $+\lambda$. Point P is marked with a dot at $(-2L, 2L)$. Dashed gridlines mark $x = -3L, -2L, -L, O, L$ and $y = L, 2L, 3L$.]

(b) **Derive** an expression for the magnitude E_O of the net electric field at the origin O due to Rods S and T in terms of L, λ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2026 ▪ Q2

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[Figure 1: In the *xy*-plane, Rod S is a thick horizontal segment on the *x*-axis from $x = -3L$ to $x = -L$, labeled "Rod S" with charge density $+\lambda$. Rod T is a thick vertical segment on the *y*-axis from $y = L$ to $y = 3L$, labeled "Rod T" with charge density $+\lambda$. Point P is marked with a dot at $(-2L, 2L)$. Dashed gridlines mark $x = -3L, -2L, -L, O, L$ and $y = L, 2L, 3L$.]

(c) Rod T is removed. Rod S is then moved such that the right end of Rod S is at the origin O. Rod S is then moved with speed v in the $-x$-direction, as shown in Figure 3. [Figure 3: Rod S is a thick horizontal segment lying along the negative *x*-axis with its right end at the origin O, labeled with charge density $+\lambda$ and moving with speed v in

Question 2

the $-x$-direction (arrow pointing left below the rod). The x-axis extends in both directions from O with a dashed line.]
On the axes shown in Figure 4, **sketch** a graph of the electric potential V_O at the origin O due to Rod S as a function of the distance r between the origin and the right end of Rod S.
[Figure 4: A blank graph with vertical axis V_O and horizontal axis r, both starting from the origin; axes shown with no scale marked, for the student to sketch a curve on.]

Question 2

2026 ▪ Q2

Thin, nonconducting, charged Rods S and T are held fixed in the *xy*-plane, as shown in Figure 1. Rod S is oriented along the x-axis in the region $-3L \leq x \leq -L$. Rod T is oriented along the y-axis in the region $L \leq y \leq 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

[Figure 1: In the *xy*-plane, Rod S is a thick horizontal segment on the x-axis from $x = -3L$ to $x = -L$, labeled "Rod S" with charge density $+\lambda$. Rod T is a thick vertical segment on the y-axis from $y = L$ to $y = 3L$, labeled "Rod T" with charge density $+\lambda$. Point P is marked with a dot at $(-2L, 2L)$. Dashed gridlines mark $x = -3L, -2L, -L, O, L$ and $y = L, 2L, 3L$.]

(d) A new thin, nonconducting, charged Rod W has the same length as Rod S. Rod W has uniform, negative linear charge density $-\lambda$ and is oriented along the y-axis. Rods S and W are initially held fixed with one end at the origin. Starting at the same time,

Question 2

Rod S is moved with speed v in the $-x$-direction and Rod W is moved with speed v in the $+y$-direction, as shown in Figure 5.
[Figure 5: Rod S is a thick horizontal segment along the negative x-axis with its right end at the origin O, labeled $+\lambda$, moving with speed v in the $-x$-direction. Rod W is a thick vertical segment along the positive y-axis with its bottom end at the origin O, labeled $-\lambda$, moving with speed v in the $+y$-direction.]
Indicate how a sketch of V_O at the origin O due to both Rods S and W as a function of r would be different from the sketch made in part C. Briefly **justify** your answer.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\text{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.
[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost

Question 1

equipotential loops on the lower left, with A below B. A horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

Question 1

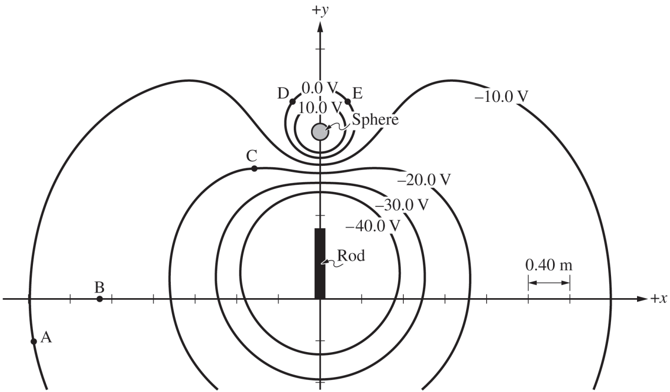

Figure 1

Question 1

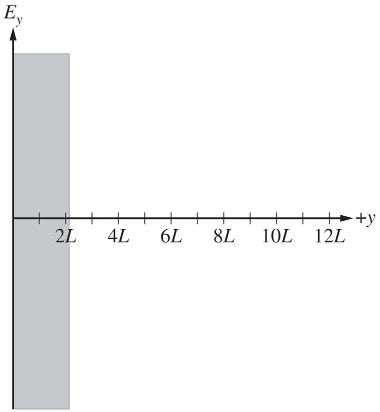

Figure 4

Question 1

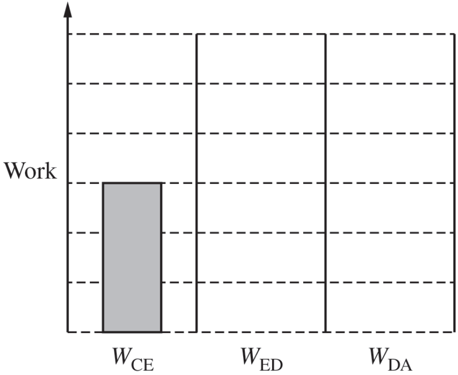

Figure 2

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the y-axis, in the xy-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(b)(i) A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

Draw a bar to represent the relative absolute value of the work W_{ED}, done by the external force on the test charge to move the test charge from Position E to Position D.

Draw a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.

Question 1

The height of each bar should be proportional to the value of W_{CE}. If $W_{\text{ED}} = 0$ and/or $W_{\text{DA}} = 0$, write a "0" in the corresponding columns, as appropriate.
[Figure 2: A bar chart with vertical axis labeled "Work" (unlabeled scale, dashed horizontal gridlines) and three columns on the horizontal axis labeled W_{CE}, W_{ED}, W_{DA}. A single shaded bar of medium height is already drawn above the W_{CE} column; the W_{ED} and W_{DA} columns are empty for the student to complete.]
(b)(ii) Calculate the approximate magnitude of the x-component of the electric field at Position B.

Question 1

2024 Set 2 ▪ Q1

A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\mathrm{nC}$. The rod and sphere are held at rest on the *y*-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the *x*-axis and on the *y*-axis are 0.40 m apart.

[Figure 1: A set of equipotential lines around a sphere and a rod on the *y*-axis, in the *xy*-plane with $+x$ to the right and $+y$ upward. The rod is a thick vertical bar below the origin, labeled "Rod". Above the origin sits a small circle labeled "Sphere". Equipotential lines are curved, roughly circular/oval loops surrounding the sphere and rod, labeled from innermost to outermost with potential values: -10.0 V (innermost, near sphere, marked with point D), 0.0 V (marked with point E, near the sphere), -20.0 V, -30.0 V, -40.0 V (marked with point C, to the left of the rod). Point P is marked near the sphere at the top. Points B and A lie further out along the outermost equipotential loops on the lower left, with A below B. A

Question 1

horizontal double-headed arrow near the *x*-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(c) The positive test charge is placed at Position C. The test charge is then released from rest.

Indicate the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_______ $+x$ _______ $+y$ _______ Directly away from the sphere

_______ $-x$ _______ $-y$ _______ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Question 1

2024 Set 2 ▪ Q1

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Question 1

horizontal double-headed arrow near the x-axis on the right indicates a spacing of 0.40 m between adjacent tick marks.]

(d)(i) [Figure 3: A vertical y-axis with tick marks labeled from bottom to top: $-\lambda$ at 0, then $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. A thick vertical bar labeled "Rod" spans from 0 to $2L$ on the axis, adjacent to the label $-\lambda$.]

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

Using integral calculus, **derive** the expression for V_P provided.

Question 1

(d)(ii) On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.
[Figure 4: A blank graph with vertical axis labeled E_y (no numeric scale shown) and horizontal axis labeled y with tick marks at $2L$, $4L$, $6L$, $8L$, $10L$, $12L$. Axes are otherwise empty for the student to sketch a curve.]

Solution 1

(a)

Mark scheme

- Use $\Phi_E = Q/\varepsilon_0$ (from Gauss's law, $\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$) with $Q = 1.0 \times 10^{-9}$ C enclosed by the 0.0 V equipotential surface:
$\Phi_E = \dfrac{1.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12}\ \mathrm{C}^2/(\mathrm{N} \cdot \mathrm{m}^2)} = 113\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}$. 1 point for using the correct flux equation, 1 point for the correct numerical answer (units not required for this point).

Solution 1

(b)(i)

Mark scheme

- Positions D and E lie on the same equipotential line, so no work is required to move the test charge between them: $W_{ED} = 0$ (bar of height 0). The bar representing W_{DA} should have a height of approximately one and a half units (about half the height of the given W_{CE} bar). 1 point for indicating $W_{ED} = 0$, 1 point for drawing the W_{DA} bar at roughly 1.5 units.

Solution 1

(b)(ii) Mark scheme

- Relate electric field to the potential gradient between adjacent equipotential lines: $E_y = -dV/dy$, so $|E_y| = |\Delta V/\Delta y|$. Substituting the potential values and spacing of the equipotential lines bracketing Position B: $|E_y| = \left|\frac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right| = 7.1\text{ V/m}$. 1 point for using a correct field-potential relationship (may start with numbers already substituted), 1 point for correctly substituting the potential values and equipotential-line spacing used to get the field magnitude at Position B.

Solution 1

(c) Mark scheme

- Direction: $-y$ (only). Justification: the electric field vector is perpendicular to the equipotential line (tangent) at Position C. Because the test charge is positive, it moves from higher to lower electric potential when a force acts on it; at Position C the field/force direction that is perpendicular to the equipotential line and points toward decreasing potential is $-y$ (directly toward the sphere). 1 point for selecting only $-y$ with an attempted justification, 1 point for stating the field is perpendicular to the equipotential line at C, 1 point for one of: the charge moves from higher to lower potential / the test charge and rod have opposite-sign charge / the test charge and sphere have same-sign charge.

Solution 1

(d)(i) Mark scheme

- Set up the potential from a continuous line charge: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$, i.e. $V_P = k\int\left(\frac{1}{y_P - y}\right)dq$, where $y_P - y$ is the distance from a charge element at height y on the rod to point P. Substitute the charge element of the negatively-charged rod, $dq = -\lambda\, dy$: $V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$. Integrate over the rod's extent with limits $y = 0$ to $y = 2L$: $V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$. 1 point for an appropriate potential-from-line-charge equation, 1 point for correctly determining $r = y_P - y$,

Solution 1

1 point for the correct integral with $\lambda\, dy$ substituted for dq, 1 point for the correct limits of integration (0 to 2L).

Solution 1

(d)(ii)

Mark scheme

- For $2L < y < 12L$, sketch E_y as a curve that is always negative (field points toward the negatively-charged rod, i.e. in $-y$) and continually approaches (but never reaches) the horizontal axis as y increases — i.e. a concave-down curve that asymptotically rises toward zero from below, steepest just above $y = 2L$ and flattening out toward $y = 12L$. 1 point for a curve/line that continually approaches the horizontal axis as position increases, 1 point for a concave-down curve that stays always negative.

Question 1

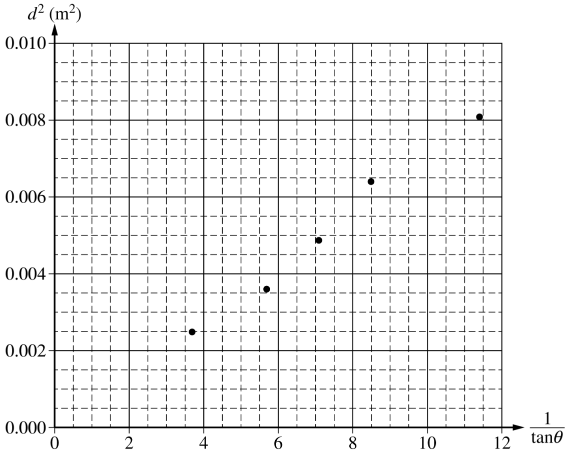
2023 Set 1 ▪ Q1

Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical. The experiment is repeated.
[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

Question 1

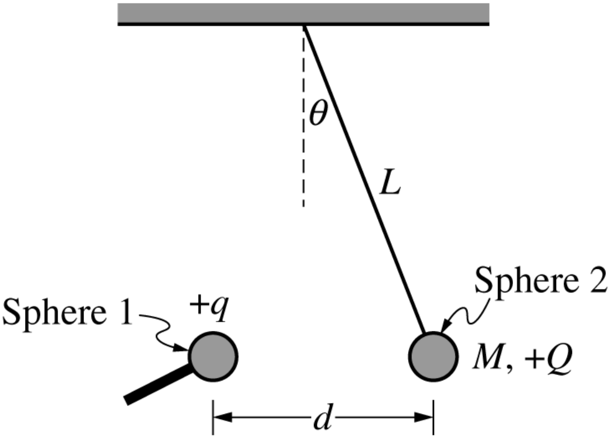
(a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. [Diagram: A single dot with a dashed horizontal line and a dashed vertical line through it, representing Sphere 2 at its equilibrium position, for the student to draw force vectors on.]

Question 1

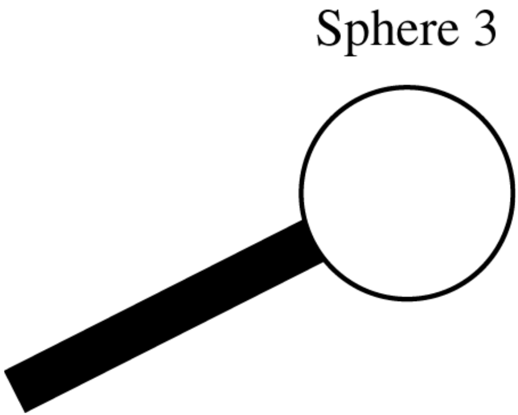


d^2 (m^2)
0.010
0.008
0.006
0.004
0.002
0.000
0
2
4
6
8
10
12
$\frac{1}{\tan\theta}$

Question 1



Question 1



Question 1

2023 Set 1 ▪ Q1

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[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

Question 1

(b) Derive the relationship between the distance d and the angle θ to show that

$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}.$$

Question 1

2023 Set 1 ▪ Q1

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[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

(c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8}$ C, $\theta = 12^\circ$, and $d = 0.057$ m. Calculate the expected force of tension exerted on Sphere 2 by the string.

Question 1

2023 Set 1 ▪ Q1

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[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

(d)(i) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\dfrac{1}{\tan\theta}$.

Question 1

[Graph: y-axis d^2 (m^2) from 0.000 to 0.010 in increments of 0.002; x-axis $\dfrac{1}{\tan\theta}$ from 0 to 12 in increments of 2. Data points (approximate grid positions): (1.5, 0.0015), (2.3, 0.0022), (4.3, 0.0038), (6.2, 0.0050), (8.0, 0.0068), (11.3, 0.0087).]

Draw the best-fit line for the data.

(d)(ii) Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $M = 0.0050\,\text{kg}$ and $Q = q = 6.0 \times 10^{-8}\,\text{C}$.

Question 1

2023 Set 1 ▪ Q1

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[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

(e)(i) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

Question 1

The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single "+" sign to represent the location of highest concentration of the excess positive charges.
[Diagram: A circle labeled "Sphere 3" mounted on a thick insulating rod (shown as a solid black wedge), positioned as Sphere 1 was in the original figure, for the student to mark the "+" location.]
(e)(ii) Briefly explain your reasoning for the sketch drawn in part (e)(i).
(e)(iii) In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.
Is θ_2 greater than, less than, or equal to θ_1?
_____ $\theta_2 > \theta_1$ _____ $\theta_2 < \theta_1$ _____ $\theta_2 = \theta_1$

Question 1

Briefly justify your answer.

Solution 1

(a)

Mark scheme

- Three forces act on Sphere 2: the electrostatic (Coulomb) force F_E directed horizontally away from Sphere 1 (to the right, since both spheres carry like positive charge and repel), the string tension F_T directed up and to the left along the string, and gravity F_g directed straight down. 1 point: correctly draw and label F_E pointing to the right. 1 point: draw F_T up-and-to-the-left and F_g straight down. Scoring note: a maximum of 1 point may be earned if extraneous (extra, non-existent) forces are included in the diagram.

Solution 1

(b) Mark scheme

- Using $\Sigma F_y = 0$: $F_{Ty} = F_g \Rightarrow F_T \cos\theta = Mg$ (1 pt, equating vertical tension component to gravity). Using $\Sigma F_x = 0$: $F_{Tx} = F_E \Rightarrow F_T \sin\theta = F_E$ (1 pt, equating horizontal tension component to the electrostatic force). Coulomb's law gives $F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}$, so $F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}$. Substituting into the vertical equation and solving simultaneously (1 pt, attempt to simultaneously solve the two force equations): $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{\cos\theta}{\sin\theta} = Mg \Rightarrow d^2 = \frac{Qq\cos\theta}{4\pi\varepsilon_0 Mg\sin\theta} \Rightarrow d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$, as required.

Solution 1

(c) Mark scheme

- From Coulomb's law and the horizontal force balance, $F_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{d^2} = F_T \sin\theta$ (1 pt, applying Coulomb's law to determine tension; this point may be earned using the vertical component of tension instead). So $F_T = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{d^2\sin\theta}$ (1 pt, correct substitution into an expression for tension consistent with part (b), or a correct tension expression). Substituting $Q = q = 6.0 \times 10^{-8}\,\mathrm{C}$, $d = 0.057\,\mathrm{m}$, $\theta = 12^\circ$: $F_T = \dfrac{1}{4\pi(8.85\times10^{-12})}\dfrac{(6.0\times10^{-8})^2}{(0.057)^2\sin(12^\circ)} = 0.048\,\mathrm{N}$.

Solution 1

(d)(i)

Mark scheme

- Draw a single straight best-fit line through the cluster of six data points on the d^2 vs. $\dfrac{1}{\tan\theta}$ graph, approximating the overall linear trend (passing near the data points across their full range, roughly extrapolating toward the origin). 1 point for a line that reasonably approximates the trend of the data (does not need to pass through every point or through the origin exactly).

Solution 1

(d)(ii) Mark scheme

- Using two points that lie directly on the student's drawn best-fit line (not raw data points unless they fall exactly on the line): slope $= \dfrac{\Delta(d^2)}{\Delta(1/\tan\theta)} = \dfrac{0.0075\,\mathrm{m}^2 - 0.001\,\mathrm{m}^2}{10.5 - 2} = 7.647 \times 10^{-4}\,\mathrm{m}^2$ (1 pt, using two points from the trend line to calculate the slope). Relating the slope to the theoretical equation $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$: squaring gives $d^2 = \left(\dfrac{Qq}{4\pi\varepsilon_0 Mg}\right)\dfrac{1}{\tan\theta}$, so slope $= \dfrac{Qq}{4\pi\varepsilon_0 Mg}$ (1 pt, correctly relating slope to the equation). Solving for and substituting to get $\varepsilon_0 = \dfrac{Qq}{4\pi Mg(\mathrm{slope})} =$

Solution 1

$$\frac{(6.0\times 10^{-8})^2}{4\pi(0.0050\,\mathsf{kg})(9.8\,\mathsf{m/s}^2)(7.647\times 10^{-4}\,\mathsf{m}^2)} \approx 7.6\times 10^{-12}\,\frac{\mathsf{C}^2}{\mathsf{N}\cdot\mathsf{m}^2}$$ (1 pt, substituting the slope value to calculate an experimental ε_0).

Solution 1

(e)(i)

Mark scheme

- Draw a single "+" sign on the left side of the circle representing conducting Sphere 3 — the side farther from (facing away from) Sphere 2 — indicating where the excess positive charge concentrates. 1 point for the "+" placed on the correct (left) side of the sphere.

Solution 1

(e)(ii)

Mark scheme

- Because Sphere 3 is a conductor, its free (mobile) negative charges are attracted toward the positive charges on Sphere 2 and migrate to the side of Sphere 3 nearest Sphere 2 (the right side), leaving a net excess of positive charge concentrated on the far side of Sphere 3, away from Sphere 2 (the left side). 1 point for a statement that correctly indicates this charge rearrangement on Sphere 3 due to the electric force from Sphere 2's charges.

Solution 1

(e)(iii) Mark scheme

- $\theta_2 < \theta_1$ (1 pt, for selecting $\theta_2 < \theta_1$ with an attempt at a relevant justification). Justification (1 pt, for indicating either of the following, consistent with the charge location drawn in part (e)(i)): on Sphere 3 the excess like (positive) charges concentrate on the side farther from Sphere 2, so the average separation between the concentrated charges on the two spheres is greater than it would be for point charges at the spheres' centers — equivalently, the electrostatic repulsive force between the spheres is smaller than in the original point-charge case. A smaller repulsive force at the same horizontal separation d_1 requires a smaller angle from vertical to balance gravity, so $\theta_2 < \theta_1$.

Question 1

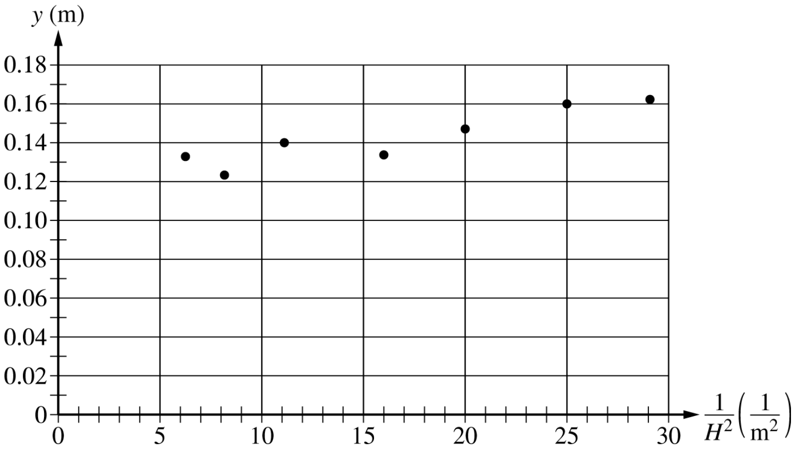
2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.] Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Question 1

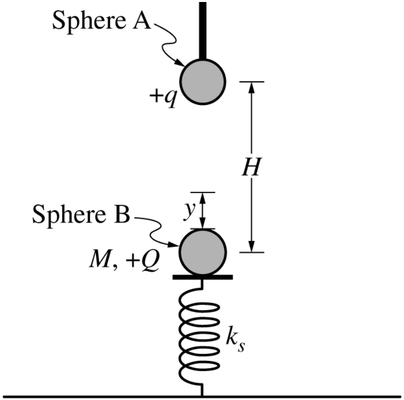
Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.
(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. [Diagram: a single dot with a short dashed horizontal line and a short dashed vertical line crossing through it, indicating axes for drawing force vectors. No forces are pre-drawn; the student draws them.]

Question 1



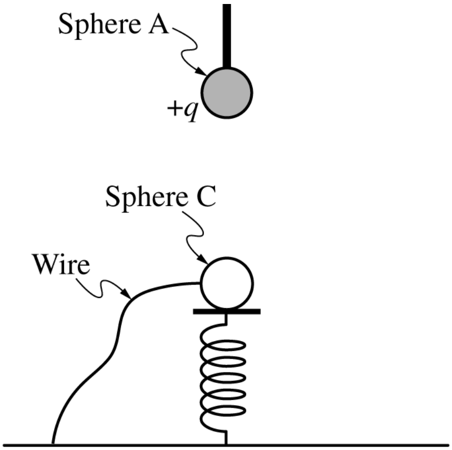
y (m)
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
$\frac{1}{H^2}\left(\frac{1}{\mathrm{m}^2}\right)$

Question 1



Note: Figure not drawn to scale.

Question 1



Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(b) Derive the relationship between y and H to show that

$$y = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{k_s H^2} + \frac{Mg}{k_s}.$$

Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(c)(i) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph. [Graph: y-axis labeled "y (m)" ranging from 0 to 0.18 in increments of 0.02 (gridlines at 0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18); x-axis labeled "$\frac{1}{H^2}\left(\frac{1}{\mathrm{m}^2}\right)$" ranging from 0 to 30 in increments of 5. Data points (approximate): (0, 0.10), (7, 0.125), (10, 0.12), (13, 0.135), (17, 0.14), (20, 0.135), (23, 0.16), (27, 0.165), (29, 0.165).]
Draw the best-fit line for the data.
(c)(ii) Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.
(c)(iii) Using the best-fit line, calculate an experimental value for the mass of Sphere B.

Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(d)(i) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.
In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.
Is y_2 greater than, less than, or equal to y_1?
______ $y_2 > y_1$ ______ $y_2 < y_1$ ______ $y_2 = y_1$
Justify your answer.
(d)(ii) [Figure: Sphere A (charge $+q$) hangs above Sphere C. Sphere C rests on the platform above the spring, with a wire connected from Sphere C down to the ground/base.]

Question 1

Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write “does not move.”

Solution 1

(a)

Mark scheme

- Free-body diagram on Sphere B (dot), three vectors all starting at the dot: electrostatic force F_E pointing DOWNWARD (repulsion from Sphere A), normal force F_N pointing UPWARD (platform/spring pushing up on B; label F_S is also acceptable), and gravitational force F_g pointing DOWNWARD. 1 point for correctly drawing and labeling F_E downward. 1 point for drawing F_N upward AND F_g downward together. Scoring note: a maximum of 1 point may be earned if extraneous forces are included.

Solution 1

(b) Mark scheme

- Derive $y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{k_s H^2} + \dfrac{Mg}{k_s}$ from equilibrium of Sphere B. Start from the equilibrium equation consistent with the part (a) diagram: $F_N - F_E - F_g = 0$ (1 point). Substitute the spring/normal force $F_N = k_s y$ (1 point). Substitute the electrostatic (Coulomb) force $F_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{H^2}$ (1 point). Substitute the gravitational force $F_g = Mg$ (1 point). Full derivation: $F_N = F_E + F_g \Rightarrow k_s y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{H^2} + Mg \Rightarrow y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{k_s H^2} + \dfrac{Mg}{k_s}$, matching the given result.

Solution 1

(c)(i)

Mark scheme

- Draw a straight line (best-fit/trend line) through the plotted y vs $1/H^2$ data that reasonably approximates the trend of the scattered points (data run roughly from $y \approx 0.10$–0.12 m at $1/H^2 = 0$ up to $y \approx 0.16$–0.165 m at $1/H^2 = 30\ \mathrm{m}^{-2}$). The line need not pass through the origin or any specific data point; it must simply approximate the overall upward trend of the data (e.g. rising from about (0, 0.12) to about (30, 0.16)).

Solution 1

(c)(ii) Mark scheme

- Calculate an experimental value of ε_0 from the graph, given $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m. (1) Use two points ON the student's trend line to compute the slope: slope $= \dfrac{\Delta y}{\Delta(1/H^2)}$ (e.g. using (0, 0.12 m) and (20, 0.15 m): slope $= \dfrac{0.15 - 0.12}{20 - 0} = 0.0015 \text{ m}^3$). (2) Relate the slope to ε_0 from $y = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Qq}{k_s H^2} + \dfrac{Mg}{k_s}$: slope $= \dfrac{Qq}{4\pi\varepsilon_0 k_s}$. (3) Substitute the slope and given quantities: $\varepsilon_0 = \dfrac{Qq}{4\pi k_s(\text{slope})} = \dfrac{(2 \times 10^{-6}\,\text{C})^2}{4\pi(25\,\text{N/m})(0.0015\,\text{m}^3)} \approx 8.5 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$. A

Solution 1

numerical answer without units may earn the third point; exact value depends on the student's own trend line and chosen points.

Solution 1

(c)(iii) Mark scheme

- Calculate an experimental value of the mass M of Sphere B from the graph's y-intercept. (1) Identify that the y-intercept equals $\frac{Mg}{k_s}$. (2) Substitute the graph's y-intercept (approximately 0.12 m) and given quantities: $M = \frac{\text{(y-intercept)}\, k_s}{g} = \frac{(0.12\,\mathrm{m})(25\,\mathrm{N/m})}{9.8\,\mathrm{m/s^2}} \approx 0.30\ \mathrm{kg}$. A numerical answer without units may earn the second point; exact value depends on the student's own trend line's intercept.

Solution 1

(d)(i) Mark scheme

- Compare y_2 (conducting Sphere C, modified experiment) to y_1 (nonconducting Sphere B, original experiment) at the same separation H_1: the correct answer is $y_2 < y_1$ (1 point for selecting this with an attempted relevant justification). Justification (1 point): because Sphere C is conducting, its charges are free to redistribute. Electrons in Sphere C are attracted toward the excess positive charge on Sphere A and shift toward A, leaving the far side of C more positively charged. This charge rearrangement (polarization) results in LESS net electric repulsion between the spheres than in the nonconducting case, so the spring compresses less, giving $y_2 < y_1$.

Solution 1

(d)(ii) Mark scheme

- Draw an arrow indicating the platform moves UPWARD immediately after Sphere C is grounded (1 point; the SG's example response shows a single upward-pointing arrow at the spring/platform). Reasoning: grounding forces Sphere C's electric potential to zero; since C sits in the positive potential produced by nearby charged Sphere A, reaching zero net potential requires C's own net charge to decrease (effectively negative charge flows onto C from ground, or positive charge flows off to ground). This reduces the electrostatic repulsion between A and C (and can even become attractive), decreasing the downward force on C, so the spring relaxes and the platform rises.

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance d, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation d between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

Question 2

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\text{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(a)(i) Draw a line that represents the best fit to the points shown.

(a)(ii) Use the graph to calculate the charge Q.

Question 2

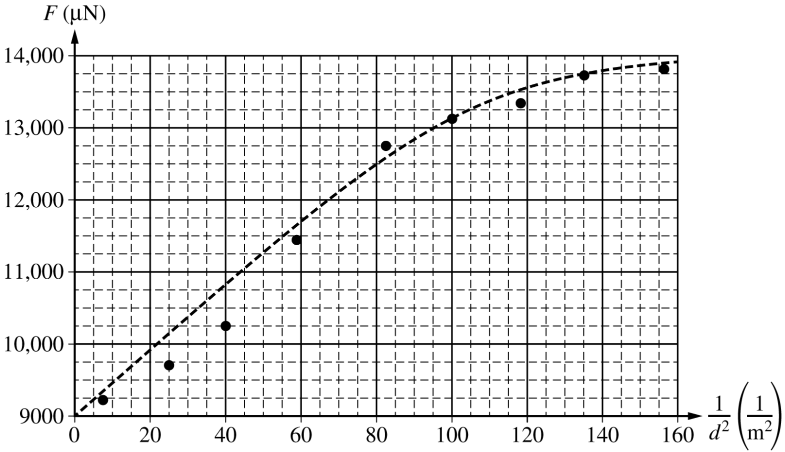
(a)(iii) [Second, extended graph: vertical axis F (μN) ranging from 9000 to 14,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\text{m}^2}\right)$ ranging from 0 to 160 in increments of 20; a dashed best-fit line/curve is drawn through data points from the origin region up through about $(140, 13, 700)$; plotted data points (approximate): $(10, 9100)$, $(30, 9700)$, $(50, 10, 000)$, $(65, 11, 500)$, $(90, 12, 700)$, $(105, 13, 000)$, $(120, 13, 300)$, $(140, 13, 700)$, $(155, 13, 800)$.]

On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

(a)(iv) Determine the distance between the centers of the spheres for the data point indicated above.

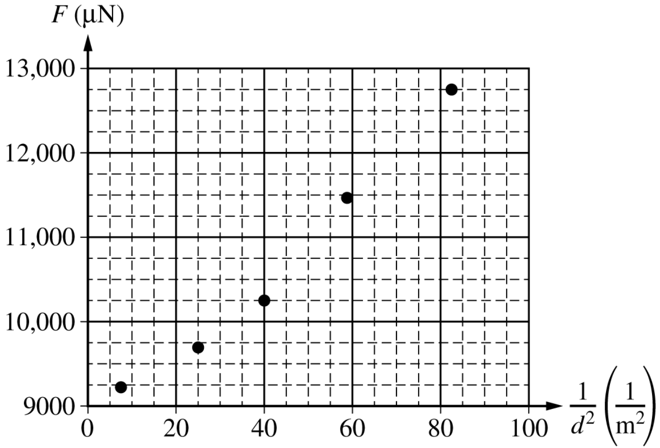
(a)(v) What physical quantity does the vertical intercept represent? Justify your answer.

Question 2



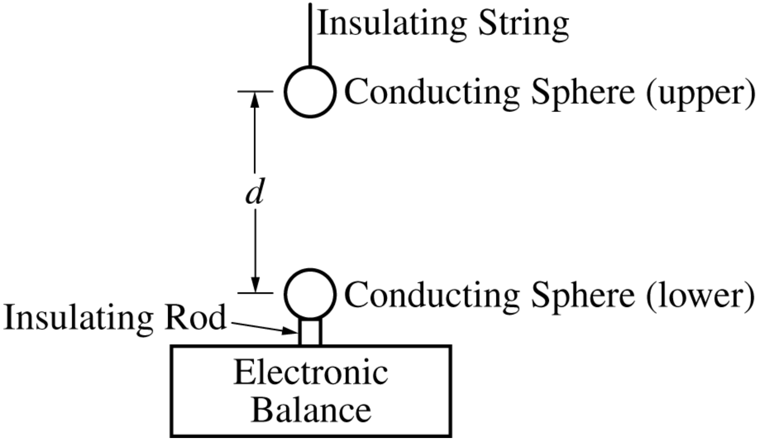
F (µN)
14,000
13,000
12,000
11,000
10,000
9000
0
20
40
60
80
100
120
140
160
1/d² (1/m²)

Question 2



F (µN)
13,000
12,000
11,000
10,000
9000
0
20
40
60
80
100
$\frac{1}{d^2}\left(\frac{1}{m^2}\right)$

Question 2


Insulating String
Conducting Sphere (upper)
d
Conducting Sphere (lower)
Insulating Rod
Electronic
Balance

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance d, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation d between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and

Question 2

the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\text{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(b) The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

_______ Yes _______ No

Question 2

Justify your answer.

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance d, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation d between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and

Question 2

the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\mathrm{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(c)(i) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

On the circles representing the spheres below, use a single "$+$" sign on each sphere to represent the locations of highest concentration of the excess positive charges.

[Two blank circles are shown, stacked vertically, representing the upper and lower spheres, for the student to mark.]

(c)(ii) Explain how this rearrangement could be responsible for the discrepancy.

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance *d*, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation *d* between their centers.]
Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.
For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and

Question 2

the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\text{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(d)(i) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

On the circles representing the spheres below, use a single "$+$" sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single "$-$" sign on the top sphere to represent the location of the highest concentration of the excess negative charges.

Question 2

[Two blank circles are shown, stacked vertically, representing the upper (top) and lower (bottom) spheres, for the student to mark.]

(d)(ii) For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

______ Greater than ______ Less than ______ Equal to

Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Draw a single straight best-fit line through the plotted data points (approximately through the trend from the lower-left points near $(10, 9100)$ up through the upper-right points near $(140, 13700)$), balancing points above and below the line rather than connecting the dots.

Solution 2

(a)(ii) Mark scheme

- Using two points on the best-fit line, the slope is $m = \frac{\Delta y}{\Delta x} = \frac{(12{,}000 - 10{,}500)\ \mu\text{N}}{(72 - 40)\ \text{m}^{-2}} = 4.7 \times 10^{-5}\ \text{N} \cdot \text{m}^2$. Since $F = \frac{kQ^2}{r^2} = kQ^2 \cdot \frac{1}{d^2}$, the slope of F vs $1/d^2$ equals kQ^2. Solving for the charge: $Q = \sqrt{\frac{\text{slope}}{k}} = \sqrt{\frac{4.7 \times 10^{-5}\ \text{N} \cdot \text{m}^2}{9 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2}} = 7.2 \times 10^{-8}\ \text{C}$. Points: (1) correctly calculating the slope from two points on the best-fit line, (2) correctly relating the slope to the charge (slope $= kQ^2$), (3) correctly calculating $Q = 7.2 \times 10^{-8}$ C consistent with that slope.

Solution 2

(a)(iii)

Mark scheme

- Circle the data point corresponding to $1/d^2 \approx 82.5$ m^{-2} (the point near $F \approx 12{,}900\ \mu$N that is used in part (a)(iv) to determine the separation distance d).

(a)(iv)

Mark scheme

- From the graph, the circled data point corresponds to $1/d^2 = 82.5$ m^{-2}, so $d = \sqrt{1/82.5 \text{ m}^{-2}} = 0.11$ m.

Solution 2

(a)(v) Mark scheme

- The vertical (F) intercept represents the weight of the (lower) conducting sphere and insulating rod, i.e. the balance/scale reading with no electric force present. Justification: When the spheres are infinitely far apart ($1/d^2 \to 0$), there is no electric force between them, so the only force recorded by the balance is the weight of the lower sphere and insulating rod — equivalently, the scale is zeroed before the rod and lower sphere are placed on it, so the nonzero y-intercept is their weight. Points: (1) indicating the intercept is the weight of the sphere/rod, (2) a correct justification.

Solution 2

(b)

Mark scheme

- Yes. Justification: Because the later force measurements fall below the values predicted by extrapolating the best-fit line, a decrease in charge (charge slowly leaking off the top sphere over the course of the experiment, so Q — and hence $F \propto Q^2$ — decreases) could explain the discrepancy seen in the data.

Solution 2

(c)(i) Mark scheme

- On each sphere, mark a single '+' at the point farthest from the other sphere (e.g. at the top of the upper sphere and the bottom of the lower sphere). Since both spheres carry excess positive charge, the mutual repulsion between the two spheres' charge distributions pushes each sphere's own excess positive charge toward the side away from the other sphere. Points: (1) correctly indicating the position of the excess positive charges (concentrated away from the other sphere on each), (2) correctly indicating the net charge on each sphere is positive (a single '+' marked, not a mix of signs).

Solution 2

(c)(ii)

Mark scheme

- On a conductor, excess charges are free to move and repel each other; the excess (same-sign) charges on each sphere rearrange until they are as far apart as possible from the charge on the other sphere. This polarization/redistribution increases the effective separation between the charge concentrations (beyond simply using the center-to-center distance d), which reduces the actual force below the $F = kQ^2/d^2$ prediction — explaining why the closer-spacing data points fall below the best-fit line.

Solution 2

(d)(i)

Mark scheme

- Mark a single '$-$' on the top sphere at the point closest to the bottom sphere, and a single '$+$' on the bottom sphere at the point closest to the top sphere. Because the spheres now carry opposite charges, they attract; unlike charges concentrate on the sides nearest each other.

Solution 2

(d)(ii)

Mark scheme

- Less than. Justification: The oppositely charged spheres attract each other, and this attractive force pulls the lower sphere upward (toward the upper sphere), decreasing the upward normal force the scale must exert to support it; thus the force (scale) reading with opposite charges is less than the reading obtained with spheres of the same (like) charge, which repelled and increased the reading. Points: (1) selecting 'Less than' and attempting a relevant justification, (2) a fully correct justification.

Question 1

2019 Set 1 ▪ Q1

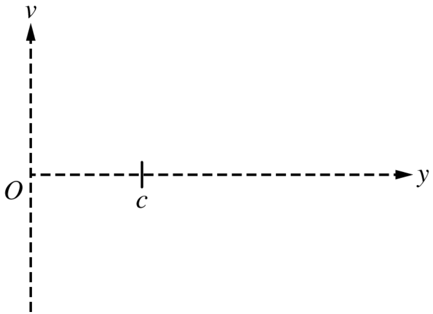
Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]
A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.
(a)(i) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

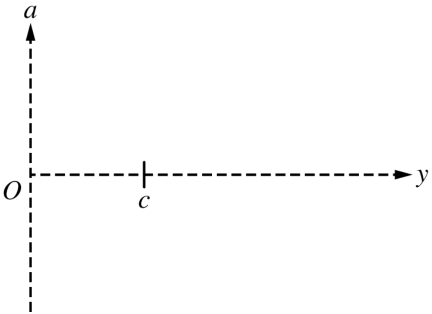
Question 1

[Figure: Point P shown as a dot on a vertical dashed line above a horizontal dashed line representing the cylinder's axis, with a small + segment marked at the axis below P.]
(a)(ii) Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.
(a)(iii) Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

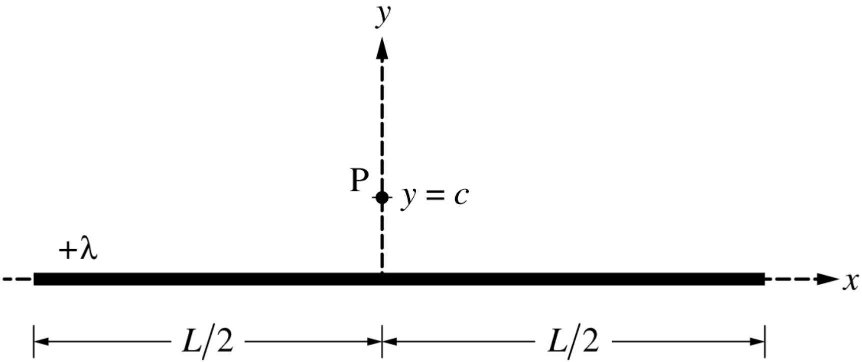
Question 1

<ins>position</ins> y and the acceleration a as a function of <ins>position</ins> y for the proton.



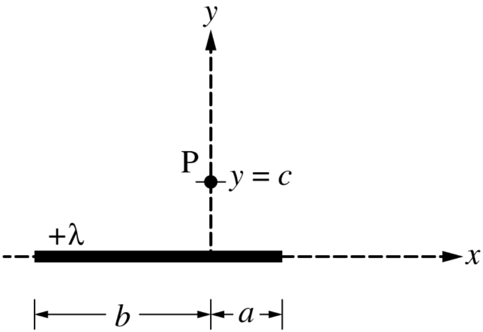


Question 1



Note: Figure not drawn to scale.

Question 1



Note: This figure is shown again for reference.

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

[Two blank graphs, side by side. Left graph: vertical axis v, horizontal axis y, origin O, with a tick mark labeled c on the y-axis. Right graph: vertical axis a, horizontal axis y,

Question 1

origin O, with a tick mark labeled c on the y-axis. Both axes have dashed guide lines at $y = c$; no curves are drawn — student sketches the curves.]
The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.
[Figure: A short horizontal cylinder centered near the y-axis, carrying charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder. The cylinder extends a distance b to the left of the y-axis and a distance a to the right of the y-axis, with $a < b$.]

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

Question 1

[Figure: Point P shown as a dot on a vertical dashed line above a horizontal dashed line representing the shorter cylinder's axis, with a small + segment marked at the axis below P.]

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(d)(i) Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

______ Yes ______ No

(d)(ii) If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain

Question 1

why Gauss's law cannot be applied to derive an expression for the electric field in this case.
[Figure: The shorter-cylinder figure shown again for reference — point P on the y-axis at $y = c$ above a cylinder of length b to the left of the y-axis and a to the right of the y-axis, carrying charge density $+\lambda$. Note: This figure is shown again for reference.]
A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Question 1

Horizontal component:

$$|E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{(c^2 + x^2)^{3/2}}\, dx$$

Vertical component:

$$|E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{(c^2 + x^2)^{3/2}}\, dy$$

Question 1

2019 Set 1 ▪ Q1

Note: Figure not drawn to scale.
[Figure: A long horizontal cylinder lies along the x-axis, centered on the y-axis, carrying uniform linear charge density $+\lambda$. Point P is on the y-axis at height $y = c$ above the cylinder's center. The cylinder extends from $x = -L/2$ to $x = +L/2$, so its total length is L.]

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L \gg c$.

(e)(i) One of the two expressions above is not correct. Which expression is not correct?

_____ Horizontal component _____ Vertical component

(e)(ii) Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

Solution 1

(a)(i)

Mark scheme

- The field points straight up: draw the arrow starting at P and pointing in the $+y$ direction (directly away from the rod). By symmetry, for a point on the axis of a uniformly charged, symmetric rod, the field has no horizontal component, and since the charge is positive $(+\lambda)$, the field points away from the rod. Full credit (1 point) for an arrow at P pointing straight up.

Solution 1

(a)(ii)

Mark scheme

- Use a cylindrical Gaussian surface that is coaxial with the (thin, charged) cylinder – i.e., sharing the same central axis – with radius c so that its curved lateral surface passes through point P. (Credit is also earned if the correct surface is simply drawn on the figure.) 1 point.

Solution 1

(a)(iii) Mark scheme

- Apply Gauss's law: $\dfrac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA \Rightarrow \dfrac{Q}{\varepsilon_0} = EA$ (1 pt for using Gauss's law). Substitute the enclosed charge $Q = \lambda L$ (1 pt). Substitute the lateral area of the Gaussian cylinder $A = 2\pi cL$ (1 pt). Then $\dfrac{\lambda L}{\varepsilon_0} = E(2\pi cL) \therefore E = \dfrac{\lambda}{2\pi\varepsilon_0 c} = \dfrac{2k\lambda}{c}$, directed away from the cylinder (the $+y$ direction at P).

Solution 1

(b) Mark scheme

- Near the long rod ($L \gg c$), the field behaves like that of an infinite line charge, $E(y) \approx \dfrac{2k\lambda}{y}$, so the acceleration of the proton is $a(y) = \dfrac{eE(y)}{m} \propto \dfrac{1}{y}$. Velocity-position graph: the proton starts at rest at $y = c$ (not at the origin), so $v = 0$ at $y = c$; as y increases, v increases but at an ever-decreasing rate (since the driving field/acceleration falls off), giving a concave-down curve that begins on the y-axis at $y = c$ rather than at the origin (1 point). Acceleration-position graph: $a \propto 1/y$ is large at $y = c$ and decreases toward zero as $y \to \infty$, giving a concave-up curve that approaches (is asymptotic to) the horizontal axis (1 point).

Solution 1

(c) Mark scheme

- The shorter cylinder is asymmetric about P: it extends a distance a to the right of the y-axis but b to the left, with $a < b$, so there is more (unbalanced) charge on the left. Charge elements in the 'extra' left segment (length $b - a$, with no symmetric partner on the right) lie to the lower-left of P, so their contribution to $\vec{E}$ at P points up and to the right (away from that charge). Superposed on the symmetric part's purely vertical field, the net field at P therefore points both upward and to the right. 1 point for showing a rightward horizontal component; 1 point for the full correct direction – up and to the right (not straight up, and not straight right).

Solution 1

(d)(i)

Mark scheme

- No – see the combined justification scored in part (d)(ii). In this Scoring Guidelines, parts (d)(i) and (d)(ii) are graded together as a single 1-point item ('For selecting No with a valid explanation'); that point is recorded on leaf 1dii so the question's points are not double-counted.

Solution 1

(d)(ii) Mark scheme

- No, there is not a single Gaussian surface that can be used. Claim: select 'No.' Evidence: the length of the (shorter) cylinder is not much greater than the distance from the cylinder to point P, and the charge distribution is asymmetric about the axis through P (length b to the left vs. a to the right, $a < b$). Reasoning: therefore the approximation that E has constant magnitude over a cylindrical surface (the symmetry assumption needed to pull E out of the Gauss's-law integral) does not hold, so Gauss's law cannot be used to derive an expression for the field here. 1 point total (covering both (d)(i) and (d)(ii)) for selecting 'No' with this valid justification.

Solution 1

(e)(i)

Mark scheme

- The Vertical component expression, $|E_y| = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_{-b}^{a}\dfrac{y}{(c^2+x^2)}\,dy$, is the incorrect one (the Horizontal component expression is correct). 1 point for correctly selecting 'Vertical component.'

Solution 1

(e)(ii) Mark scheme

- Two mistakes in the vertical-component integral, each worth 1 point to identify + 1 point to correct (4 points total). (1) Wrong integration variable: the integral is written with dy, but since the charge is distributed along the x-axis (from $x = -b$ to $x = a$), the integration must run along the length of the cylinder, i.e., over dx. Claim: change dy to dx; Reasoning: this makes the integral valid (it must sum contributions from charge elements along the rod's length). (2) Wrong power in the denominator: the denominator power should be $3/2$, not 1 (matching the correct horizontal-component integral's $(c^2 + x^2)^{3/2}$). Claim: change the power to $3/2$; Reasoning: the units/dimensions of the integrand are only valid (consistent

Solution 1

with an electric field) with the $3/2$ power. Corrected form (using c, the fixed height of P, in place of the erroneous y): $|E_y| = \dfrac{\lambda c}{4\pi\varepsilon_0}\displaystyle\int_{-b}^{a}\dfrac{dx}{(c^2+x^2)^{3/2}}$.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]
A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

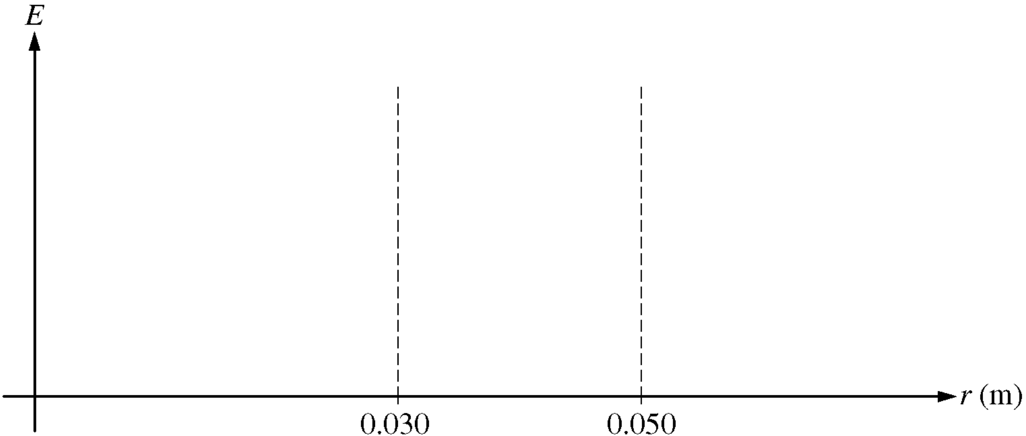
Question 2

$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

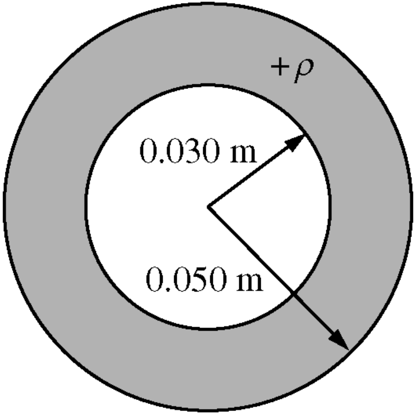
$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

Question 2



Question 2



Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]
A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030\ \text{m} : \quad \rho = 0$$

$$0.030\ \text{m} < r < 0.050\ \text{m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6}\ \text{C/m}^2$$

Question 2

$$r > 0.050 \text{ m} : \quad \rho = 0$$

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

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$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.
[Graph with vertical axis labeled E and horizontal axis labeled r (m); two vertical dashed reference lines are marked at $r = 0.030$ and $r = 0.050$; the axes are otherwise blank for the student to sketch on.]

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

Question 2

2019 Set 2 ▪ Q2

[Figure: A cross-sectional diagram of a nonconducting hollow sphere shown as an annulus (shaded gray ring) with an unshaded circular hole in the center. The inner radius is labeled 0.030 m and the outer radius is labeled 0.050 m, both measured from the center along a radial line with an arrowhead pointing outward. The shaded region is labeled $+\rho$.]
A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m} : \quad \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m} : \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

Question 2

$$r > 0.050 \text{ m} : \quad \rho = 0$$

(e)(i) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s. Calculate the magnitude of the initial acceleration of the proton.

(e)(ii) Calculate the speed of the proton after a long time.

Solution 2

(a) Mark scheme

- $Q = \int \rho \, dV = \int_{0.03}^{0.05} \frac{b}{r}(4\pi r^2)\, dr = 4\pi b \int_{0.03}^{0.05} r\, dr = 4\pi b \left[\frac{r^2}{2}\right]_{0.03}^{0.05} =$ $2\pi b \left(0.05^2 - 0.03^2\right)$. With $b = 1.6 \times 10^{-6}$ C/m^2: $Q = 2\pi(1.6 \times 10^{-6})(0.0016) = 1.61 \times 10^{-8}$ C. (1 pt: recognizing the need to integrate the charge density expression, $Q = \int \rho \, dV$, with the spherical shell volume element $4\pi r^2 \, dr$; 1 pt: correct substitution of $\rho = b/r$ into the integral; 1 pt: using the correct limits of integration and obtaining the correct numeric answer.)

Solution 2

(b) Mark scheme

- By symmetry, Gauss's law gives $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$. At the outer surface ($r = 0.05$ m) all the charge is enclosed, so $Q_{enc} = Q = 1.61 \times 10^{-8}$ C (from part (a)). $E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{1.61 \times 10^{-8}}{4\pi(8.85 \times 10^{-12})(0.05)^2} = 5.79 \times 10^4$ N/C. (1 pt: correctly evaluating the surface integral in Gauss's law; 1 pt: substituting the answer from part (a) and the correct radius into the equation; 1 pt: correct final answer with correct units, consistent with part (a).)

Solution 2

(c) Mark scheme

- $E = 0$ for $r < 0.030$ m (no enclosed charge). For 0.030 m $< r < 0.050$ m, E rises continuously from zero, concave down, increasing up to the value found in part (b) at $r = 0.050$ m (as Q_{enc} grows with r inside the shell). For $r > 0.050$ m, all the charge is enclosed and E falls off as $1/r^2$, decreasing asymptotically toward the horizontal (r) axis. (1 pt: clearly showing $E = 0$ for $r < 0.030$ m; 1 pt: a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$; 1 pt: a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m.)

Solution 2

(d) Mark scheme

- With $V = 0$ at infinity, and since at $r = 0.05$ m all the charge is enclosed (the sphere behaves as a point charge from that radius outward): $V_R = \frac{Q_{tot}}{4\pi\varepsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8}\text{ C})}{0.05\text{ m}} \approx 2900$ V. (Equivalent alternate method: integrate $\Delta V_R = -\int_{\infty}^{r} E\,dr = -\int_{\infty}^{0.05} \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}\,dr = 2900$ V.) (1 pt: substituting the total charge from part (a) into a correct expression for electric potential; 1 pt: substituting $r = 0.05$ m (or the correct limits, if integrating) into that expression.)

Solution 2

(e)(i)

Mark scheme

- Newton's second law with the electric force: $F = ma \Rightarrow qE = ma \Rightarrow a = \dfrac{qE}{m}$.
  Using the proton's charge $q = 1.6 \times 10^{-19}$ C, mass $m = 1.67 \times 10^{-27}$ kg, and $E = 5.79 \times 10^{4}$ N/C from part (b):
  $a = \dfrac{(1.6 \times 10^{-19})(5.79 \times 10^{4})}{1.67 \times 10^{-27}} = 5.55 \times 10^{12}$ m/s^2. (1 pt: using a correct expression of Newton's second law in terms of the electric field; 1 pt: correctly substituting values into that equation.)

Solution 2

(e)(ii) Mark scheme

- Using the work-energy theorem with electric potential: $-q\Delta V = \frac{1}{2}mv^2$, with $\Delta V = V_\infty - V_R = 0 - 2900$ V (the proton is repelled outward, converting electric potential energy into kinetic energy as it moves to infinity).
  $v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19})(0 - 2900)}{1.67 \times 10^{-27}}} = 7.45 \times 10^5$ m/s. (1 pt: a correct expression of kinetic energy in terms of the electric potential difference; 1 pt: correctly substituting into the equation and obtaining the correct final answer.)

Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

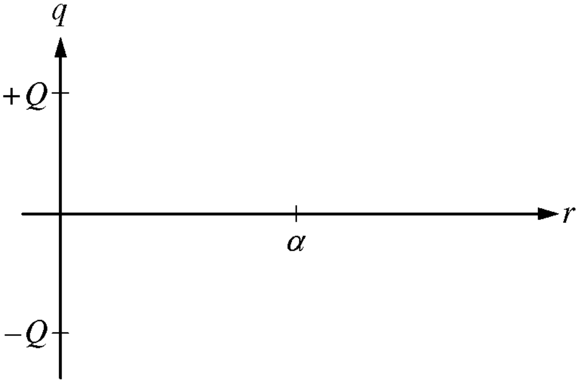
where α and β are positive constants and r is the distance from the center of the atom. **(a)(i)** On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

Question 3

i. The nuclear charge only
[Graph: axes with vertical axis q (labeled $+Q$ and $-Q$ as tick marks above and below the origin) and horizontal axis r (with a tick mark labeled α); blank, for the student to sketch.]
(a)(ii) ii. The electron charge only
[Graph: axes with vertical axis q (labeled $+Q$ and $-Q$ as tick marks above and below the origin) and horizontal axis r (with a tick mark labeled α); blank, for the student to sketch.]

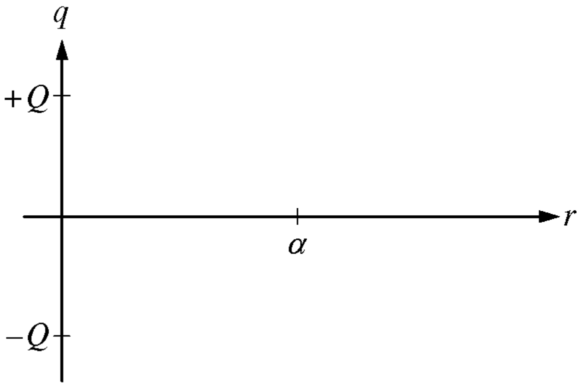
Question 3

ii. The <u>electron charge</u> only

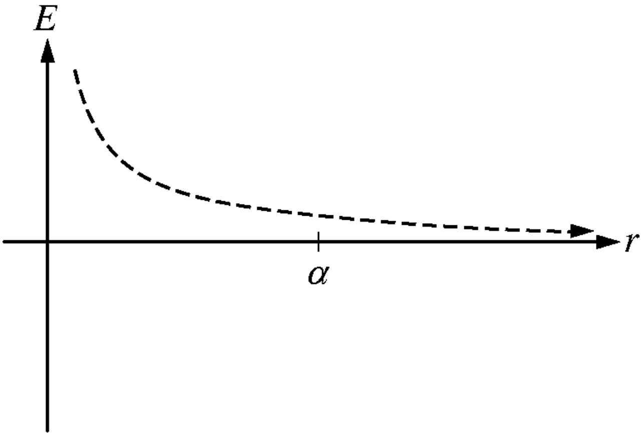


Question 3

i. The <u>nuclear charge</u> only



Question 3



Question 3

2014 ▪ Q3

A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

where α and β are positive constants and r is the distance from the center of the atom.

(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric

Question 3

field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it. [Graph: axes with vertical axis E and horizontal axis r (with a tick mark labeled α); a dashed curve is shown starting high near $r = 0$ and decreasing smoothly (following an inverse-square-like shape) toward the r-axis as r increases past α, approaching zero for large r. The student is to add their own solid curve for the neutral-atom field on the same axes.]

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(c)(i) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of Q, α, β, r, and fundamental constants, as appropriate.

i. $r > \alpha$

(c)(ii) ii. $r < \alpha$

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(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α?