Exercise walk-through

Circuits with Capacitors and Inductors (LC Circuits) ▪ 13.6

eXercise

You must be able to

- describe the **oscillation** 振荡 of charge and current in an LC circuit
- explain the **energy exchange** between the capacitor's electric field and the inductor's magnetic field
- relate the **angular frequency** 角频率 to the elements, $\omega = \frac{1}{\sqrt{LC}}$
- compare LC oscillations with the **simple harmonic motion** of a mass on a spring

Method — LC oscillation

With no resistance, the charge and current **oscillate**: energy sloshes back and forth between the capacitor's electric field and the inductor's magnetic field.

Method — Angular frequency

$$\omega = \frac{1}{\sqrt{LC}}.$$

Method — Like a mass on a spring

The LC circuit is the electrical analogue of simple harmonic motion (L plays the role of mass, $1/C$ the role of stiffness). Conservation of energy links the maximum charge to the maximum current: $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}LI^2$.

Practice 2.1 ▪ 2 marks

Find the angular frequency of an LC circuit with $L = 2.0$ H and $C = 8.0\ \mu$F.

Solution 2.1

Method: Angular frequency

Worked steps

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.0)(8.0 \times 10^{-6})}} = 250 \text{ rad s}^{-1}$$ M1 A1.

Practice 2.2 ▪ 1 mark

State what oscillates in an LC circuit.

Solution 2.2

Worked steps

the charge on the capacitor and the current in the inductor B1.

Practice 2.3 ▪ 1 mark

State the mechanical system that an LC circuit is analogous to.

Solution 2.3

Worked steps

a mass on a spring (simple harmonic motion) B1.

Exam-style 3.1 ▪ 1 mark

The angular frequency of an LC circuit is

A $\frac{1}{LC}$

B $\frac{1}{\sqrt{LC}}$

C $\sqrt{LC}$

D LC

Solution 3.1

Method: Angular frequency

A $\frac{1}{LC}$

B $\frac{1}{\sqrt{LC}}$ ✓

C $\sqrt{LC}$

D LC

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

In an LC circuit, energy is exchanged between the

A resistor and the battery

B capacitor and the inductor

C two resistors

D two batteries

Solution 3.2

A resistor and the battery

B capacitor and the inductor ✓

C two resistors

D two batteries

Worked steps

B.

Exam-style 3.3 ▪ 3 marks

An LC circuit has $L = 0.50$ H and $C = 20\ \mu$F.
(a) Find the angular frequency of oscillation.
(b) State the mechanical system this behaves like.

Solution 3.3

Worked steps

(a) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.50)(20 \times 10^{-6})}} = \dfrac{1}{\sqrt{1.0 \times 10^{-5}}} = 316 \text{ rad s}^{-1}$ M1 M1 A1.

(b) a mass oscillating on a spring B1.