Past-paper walk-through

Circuits with Resistors and Inductors (LR Circuits) ▪ 13.5

eXercise

Questions in this set

- 2026 Q3
- 2024 Set 1 Q2
- 2024 Set 2 Q2
- 2019 Set 1 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

[Figure 1: A circuit diagram showing a charged capacitor (parallel-plate symbol) on the left connected via a switch (initially open, shown as a break with two contact dots) to an inductor L_1 (coil symbol) on the right, forming a closed loop.]

The following information applies to parts C and D.

Question 3

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a 12 V battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.
[Figure 2: A circuit diagram showing a 12 V battery on the left connected via a switch (initially open) at the top to a loop containing an inductor L_2 (coil symbol) on the right side and a $10\ \Omega$ resistor along the bottom, all forming a single closed loop.]
After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\dfrac{dI}{dt}$, is determined. The collected data are shown in Table 1.

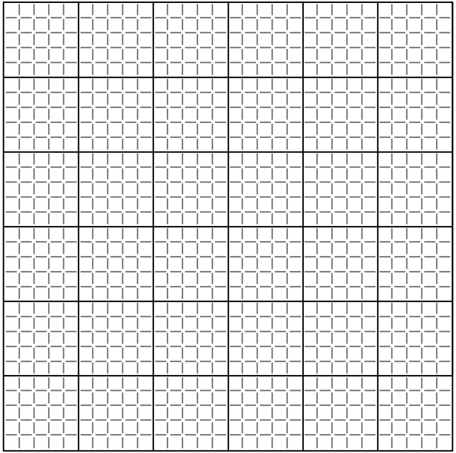
Table 1

I (A)	$\dfrac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Question 3

(a)(i) Indicate quantities that could be measured by the students that would allow them to determine L_1 using a linear graph.
(a)(ii) Briefly **describe** a method to reduce experimental uncertainty.

Question 3



Question 3

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Question 3

2026 ▪ Q3

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In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

[Figure 1: A circuit diagram showing a charged capacitor (parallel-plate symbol) on the left connected via a switch (initially open, shown as a break with two contact dots) to an inductor L_1 (coil symbol) on the right, forming a closed loop.]

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a 12 V battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Question 3

[Figure 2: A circuit diagram showing a 12 V battery on the left connected via a switch (initially open) at the top to a loop containing an inductor L_2 (coil symbol) on the right side and a $10\ \Omega$ resistor along the bottom, all forming a single closed loop.]

After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

(b)(i) Indicate what quantities the students could graph on the horizontal and vertical axes to create a linear graph that can be used to determine L_1.

(b)(ii) Briefly **describe** the relationship between L_1 and a feature of the graph from part B(i). Your answer may include an equation that relates L_1 and the chosen feature of the graph.

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

[Figure 1: A circuit diagram showing a charged capacitor (parallel-plate symbol) on the left connected via a switch (initially open, shown as a break with two contact dots) to an inductor L_1 (coil symbol) on the right, forming a closed loop.]

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a 12 V battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Question 3

[Figure 2: A circuit diagram showing a 12 V battery on the left connected via a switch (initially open) at the top to a loop containing an inductor L_2 (coil symbol) on the right side and a $10\ \Omega$ resistor along the bottom, all forming a single closed loop.]

After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

(c)(i) Label the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine L_2.

(c)(ii) On the grid provided, create a graph of the quantities indicated in part C(i).

- Clearly **label** each axis with a numerical scale.

Question 3

- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

[A blank labeled grid is provided for the student to plot the graph, with blank axis-label lines above/below reading "Quantity (units, if appropriate)" for both the horizontal and vertical axes.]

(c)(iii) Draw a best-fit line for the data plotted in part C(ii).

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

[Figure 1: A circuit diagram showing a charged capacitor (parallel-plate symbol) on the left connected via a switch (initially open, shown as a break with two contact dots) to an inductor L_1 (coil symbol) on the right, forming a closed loop.]

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a 12 V battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Question 3

[Figure 2: A circuit diagram showing a 12 V battery on the left connected via a switch (initially open) at the top to a loop containing an inductor L_2 (coil symbol) on the right side and a $10\ \Omega$ resistor along the bottom, all forming a single closed loop.]

After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

(d) Using the best-fit line that you drew in part C(iii), **calculate** an experimental value for L_2.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a

Question 2

function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)(i) On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that <u>decreases</u> with time.

[Voltmeter Symbol: a circle containing the letter “V”, with two connection leads.]

(a)(ii) Describe a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that <u>decreases</u> with time. Provide enough detail so that another student could replicate the experiment.

Question 2

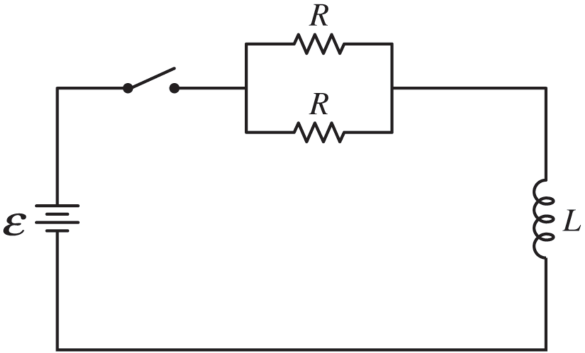

Figure 1

Question 2

Figure 2

Question 2

Voltmeter Symbol

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

Question 2

(b)(i) [Figure 2: A blank graph with a vertical axis (unlabeled) and a horizontal axis (unlabeled, shown as a dashed line), both with arrowheads, meeting at the origin.]
On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

(b)(ii) Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

Question 2

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

Question 2

(d) After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.
Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.
$\_\_\_\_\_ |\Delta V_2| > |\Delta V_1|$ $\_\_\_\_\_ |\Delta V_2| < |\Delta V_1|$ $\_\_\_\_\_ |\Delta V_2| = |\Delta V_1|$
Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Draw the voltmeter symbol connected in parallel across the two terminals of the inductor L, so that it measures the potential difference across the inductor — the quantity that decreases with time as the circuit approaches steady state after the switch is closed.

Solution 2

(a)(ii) Mark scheme

- Describe a data-collection procedure: after placing the voltmeter across the inductor, close the switch and use the voltmeter to record the potential difference across the inductor as a function of time (1 pt, for a procedure using the voltmeter to measure the potential difference at least once). The measurements must span from immediately after the switch is closed until steady-state conditions are established (or otherwise over a time interval that would allow the time constant to be determined) (1 pt). Example: 'Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.'

Solution 2

(b)(i) Mark scheme

- Sketch the voltmeter reading ΔV (potential difference across the inductor) versus time t, 4 points total (1 pt each): (1) correctly label the vertical axis as potential difference and the horizontal axis as time; (2) draw a concave-up, decreasing curve; (3) the curve should asymptotically approach zero (or start at the origin and approach a horizontal asymptote), consistent with the voltmeter placement across the inductor from part (a)(i); (4) include a vertical intercept (or horizontal asymptote) correctly labeled as $\mathcal{E}$ — the curve begins at $\Delta V = \mathcal{E}$ when $t = 0$ and decays smoothly toward the time axis.

Solution 2

(b)(ii) Mark scheme

- (1) Indicate that the graph should be fit with an exponential function, $V_L = \mathcal{E}\, e^{-tR/2L}$, or correctly relate the area under the curve to the current in the circuit (1 pt). (2) Indicate that the coefficient in front of t in that exponential fit equals $\dfrac{R}{2L}$, or equivalently correctly relate R to L and the current — i.e. that the time constant equals $\dfrac{2L}{R}$ (1 pt). Since $\mathcal{E}$ and L are known, this coefficient/time-constant lets R be calculated. Alternate: identify the time on the graph where $\Delta V \approx 0.37\mathcal{E}$ as the time constant $2L/R$, from which R follows because L is known.

Solution 2

(c) Mark scheme

- Derive the differential equation for the inductor current $I(t)$ using Kirchhoff's loop rule, 3 points: (1) a multi-step derivation that begins with an attempt at the loop rule, e.g. $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ (1 pt); (2) indicate that the potential difference across the parallel combination of the two resistors R is $I\left(\frac{R}{2}\right)$ (1 pt); (3) indicate that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$ (1 pt). Combining: $\mathcal{E} - I\left(\frac{R}{2}\right) - L\frac{dI}{dt} = 0$, which rearranges to $-\frac{R}{L}\left(\frac{I}{2} - \frac{\mathcal{E}}{R}\right) = \frac{dI}{dt}$.

Solution 2

(d)

Mark scheme

- Select $|\Delta V_2| > |\Delta V_1|$, with an attempt at a relevant justification (1 pt). The justification must indicate that the potential difference across the original, ideal inductor is zero once steady-state conditions are established (1 pt), and that the potential difference across the new inductor (which has nonnegligible resistance) is nonzero at steady state — equal to the product of the steady-state current and the inductor's resistance (1 pt).

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

Question 2

(a)(i) On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.
[Voltmeter Symbol: a circle containing the letter "V", with a wire lead extending from each side, labeled "Voltmeter Symbol".]
(a)(ii) **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

Question 2

long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

Question 2

Figure 2

Question 2

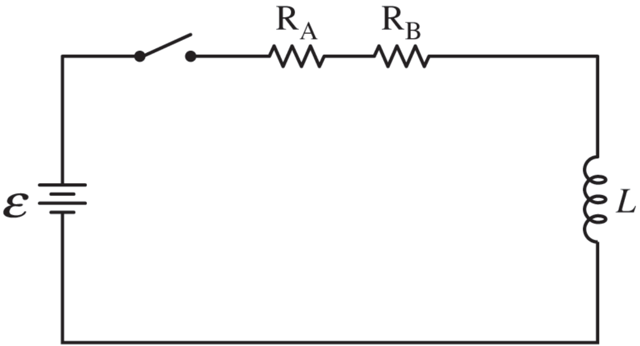

Figure 1

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

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(b)(i) On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

Question 2

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

[Figure 2: A blank graph with a vertical axis (upward arrow, unlabeled) and a horizontal axis (rightward arrow, unlabeled, drawn dashed). No scale or curve shown; axes are empty for the student to label and sketch.]

(b)(ii) Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

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(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the

Question 2

inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(d) After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has

Question 2

nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.
Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.
$|\Delta V_2| > |\Delta V_1|$ $|\Delta V_2| < |\Delta V_1|$ $|\Delta V_2| = |\Delta V_1|$
Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Place the voltmeter in parallel with Resistor A, Resistor B, or the series combination of Resistors A and B (not across the inductor) — this location's potential difference increases with time as current builds up after the switch closes, unlike the inductor voltage which decreases with time. 1 point for correctly placing the voltmeter across one of these three valid options.

Solution 2

(a)(ii)

Mark scheme

- Procedure: Close the switch, then use the voltmeter to record the potential difference across the chosen resistor(s) as a function of time, from immediately after the switch is closed until steady-state conditions are established (or until enough time has elapsed that the time constant can be determined from the data). 1 point for a procedure indicating the voltmeter is used to measure the potential difference for at least one time, 1 point for measuring from immediately after the switch closes through steady state (or until the time constant is determinable).

Solution 2

(b)(i) Mark scheme

- Sketch potential difference (vertical axis) vs. time (horizontal axis): a curve that starts at the origin $(0,0)$, is concave-down and increasing, and asymptotically approaches a nonzero potential-difference value. The labeled horizontal asymptote must be consistent with the voltmeter placement chosen in part (a)(i) — e.g. $\varepsilon/2$ if measuring across one resistor (A or B alone), or ε if measuring across the series combination of A and B. 1 point for correctly labeling potential difference vertically and time horizontally, 1 point for a concave-down increasing curve, 1 point for a curve starting at the origin and asymptotically approaching a nonzero value, 1 point for correctly labeling the horizontal asymptote consistent with part (a)(i).

Solution 2

(b)(ii) Mark scheme

- Fit the data to an exponential function of the form $V_R = \varepsilon\left(1 - e^{-t2R/L}\right)$ (for the series-combination case; for a single resistor the analogous form applies). Since ε and L are known, the coefficient multiplying t in the fit equals $2R/L$, from which R can be calculated. Alternate: read off the time at which the potential difference reaches $\approx 0.63\varepsilon/2$ (one resistor) or $\approx 0.63\varepsilon$ (combination); this time equals the time constant $L/2R$, and since L is known, R can be calculated. 1 point for indicating the curve fit is an exponential function (or, alternately, indicating the 0.63-value time), 1 point for indicating the fitted coefficient of t equals $2R/L$ (or, alternately, that the time constant equals $L/2R$).

Solution 2

(c) Mark scheme

- Apply Kirchhoff's loop rule: $\varepsilon - \Delta V_R - \Delta V_L = 0$. The potential difference across the series combination of the two resistors is $I(2R)$, and the magnitude of the potential difference across the inductor is $L\, dI/dt$. Substituting: $\varepsilon - I(2R) - L\dfrac{dI}{dt} = 0$, which rearranges to $\dfrac{dI}{dt} = \dfrac{\varepsilon}{L} - \dfrac{2IR}{L} = -\dfrac{R}{L}\left(2I - \dfrac{\varepsilon}{R}\right)$ (do not solve further). 1 point for a multi-step derivation beginning with an attempt at Kirchhoff's loop rule, 1 point for indicating the resistor-combination potential difference is $I(2R)$, 1 point for indicating the inductor's potential-difference magnitude is $L\, dI/dt$.

Solution 2

(d) Mark scheme

- $|\Delta V_2| < |\Delta V_1|$. Justification: because the new inductor has nonnegligible resistance, the total resistance of the new circuit is greater than in the original circuit, so at steady state the current is smaller than before, and the potential difference across R_A (which equals IR_A) decreases. (Equivalently: the increased total resistance means more of the source emf is dropped across the inductor's internal resistance at steady state, increasing the potential difference across the inductor and thereby decreasing the potential difference across R_A.) 1 point for selecting $|\Delta V_2| < |\Delta V_1|$ with an attempted justification, 1 point for indicating the total resistance has increased, 1 point for indicating the current decreases due

Solution 2

to the increased resistance (or, alternately, that the inductor's potential difference increases so R_A's must decrease).

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)(i) Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

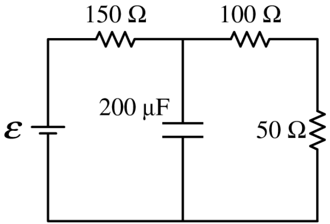
(a)(ii) Calculate the current in the $200\ \Omega$ resistor.

Question 2

(a)(iii) Calculate the power dissipated by the $200\ \Omega$ resistor.
[Figure 2: A circuit with a battery of emf $\mathcal{E}$, a $150\ \Omega$ resistor and $100\ \Omega$ resistor along the top in series from the battery's positive terminal, a $200\ \Omega$ resistor in the middle branch, a voltmeter V connected across the $200\ \Omega$ resistor, and a $50\ \Omega$ resistor in the right branch below the $100\ \Omega$ resistor.]
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

Question 2

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



Question 2

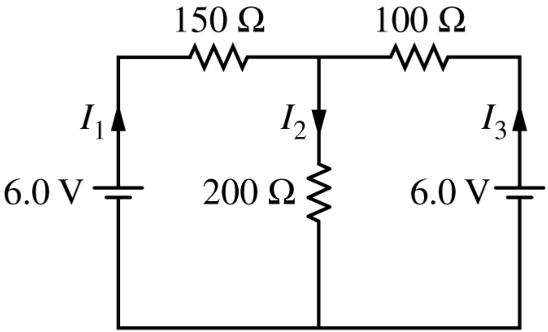


Figure 1

Question 2

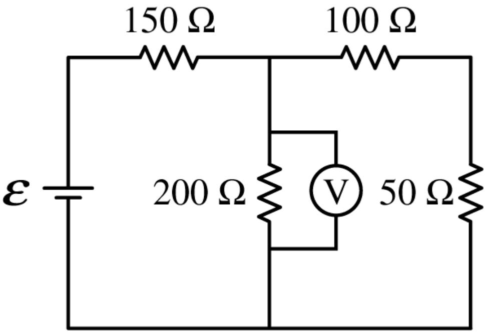


Figure 2

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(b) Calculate the current through the $50\ \Omega$ resistor.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(d)(i) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with a $200\ \mu$F capacitor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

Question 2

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu$F capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

(d)(ii) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with an ideal 50 mH inductor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?

$G^{reaterthan}{}_{L}{}^{essthan}{}_{E}{}^{qualto}$

Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Using Kirchhoff's rules (do not solve): Junction rule at the node joining all three branches: $I_1 - I_2 + I_3 = 0$ (1 pt). Loop rule, left loop (through the left 6.0 V battery, the 150 ohm resistor, and the 200 ohm resistor): $6 - 150I_1 - 200I_2 = 0$ (1 pt). Loop rule, right loop (through the right 6.0 V battery, the 100 ohm resistor, and the 200 ohm resistor): $6 - 100I_3 - 200I_2 = 0$ (1 pt). (Equivalently, the outer loop $6 - 150I_1 + 100I_3 - 6 = 0$ may replace one of these; full credit is also earned for two correct loop equations written using loop currents.)

Solution 2

(a)(ii)

Mark scheme

- Combine the three equations from part (a)(i) (1 pt for combining them / using a calculator's system-solver, e.g. $I_1 - I_2 + I_3 = 0$, $-I_1 - 1.33 I_2 = -0.04$, $-I_3 - 2I_2 = -0.06 \Rightarrow -4.33 I_2 = -0.10$). Solving: $I_2 = \dfrac{-0.10}{-4.33} = 0.023$ A (1 pt for the correct answer with correct units).

Solution 2

(a)(iii)

Mark scheme

- $P = I_2^2 R = (0.023\ \text{A})^2(200\ \Omega) = 0.107\ \text{W}$. 1 point for using a correct power equation and computing this value.

Solution 2

(b) Mark scheme

- In the modified circuit (Figure 2), the 100 ohm and 50 ohm resistors are in series in the right-hand branch, so their combined resistance is $R = 100\ \Omega + 50\ \Omega = 150\ \Omega$ (1 pt). This branch is in parallel with the 200 ohm resistor, across which the voltmeter reads 4.4 V, so that same 4.4 V is the potential difference across the 150 ohm series combination: $I = \dfrac{V}{R} = \dfrac{4.4\ \text{V}}{100\ \Omega + 50\ \Omega} = 0.029$ A (1 pt for a correct answer using the correct potential difference).

Solution 2

(c) Mark scheme

- Using $I_1 = I_2 + I_3$ (1 pt for the correct equation determining current through the 150 ohm resistor) and substituting $I_3 = 0.029$ A from part (b) with $I_2 = \dfrac{4.4 \text{ V}}{200\ \Omega}$ (1 pt for correct substitution): $I_1 = \dfrac{4.4 \text{ V}}{200\ \Omega} + 0.029 \text{ A} = 0.051 \text{ A}$. Then apply the loop equation through the battery/150-ohm branch (1 pt):
$\mathcal{E} = I_1 R_1 + V_{200\Omega \text{ branch}} = (0.051 \text{ A})(150\ \Omega) + 4.4 \text{ V} = 12.1 \text{ V}$. (Equivalent alternate method: find the circuit's total equivalent resistance
$R_T = 150\ \Omega + \left(\frac{1}{200} + \frac{1}{150}\right)^{-1} \Omega = 236\ \Omega$, giving
$\mathcal{E} = I_1 R_T = (0.051 \text{ A})(236\ \Omega) \approx 12.0\text{-}12.1 \text{ V}$ – also full credit.)

Solution 2

(d)(i) Mark scheme

- At steady state, the fully-charged 200 uF capacitor carries no current, so the 150 ohm, 100 ohm, and 50 ohm resistors form a single series loop carrying the same current (1 pt for correctly finding the total series resistance $R_{tot} = 150 + 100 + 50 = 300\ \Omega$). Using the battery voltage found in part (c), $\mathcal{E} = 12.1$ V (1 pt for substituting this consistent voltage into Ohm's law): $I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{12.1\text{ V}}{300\ \Omega} = 40.3$ mA.

Solution 2

(d)(ii)

Mark scheme

- Less than (1 pt for correctly selecting 'Less than' with an attempt at a relevant justification). Justification (1 pt): at steady state an ideal inductor acts as a short circuit (zero resistance / zero back-emf), so essentially all the current from the source flows through the (now zero-resistance) inductor branch and none flows through the 100-ohm + 50-ohm branch – so the current through the 50 ohm resistor is less than the 0.029 A found in part (b) (in fact it approaches zero).