

# Past-paper walk-through

## Magnetism and Moving Charges ■ 12.2

eXercise

## Questions in this set

- 2026 Q1
- 2025 Q4
- 2023 Set 2 Q2
- 2019 Set 2 Q3
- 2018 Q3

## Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

## Question 1

2026 ■ Q1

Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the  $y$ -axis, as shown in Figure 1. The centers of the wires are in the  $xy$ -plane. The wires are described as follows.

- Wire S has radius  $2a$  and is centered at  $x = 0$ . Wire S has nonuniform current density and carries total current  $I_0$  in the  $+y$ -direction. The magnitude of the current density in Wire S as a function of the radial distance  $r$  from the center of the wire is described by  $J = Cr$ , where  $C$  is a positive constant.
- Wire T has radius  $a$  and is centered at  $x = 10a$ . Wire T has uniform current density and carries total current  $I_0$  in the  $-y$ -direction.

## Question 1

[Figure 1: Cross-section diagram in the  $xy$ -plane. Wire S is a shaded cylindrical bar centered at  $x = 0$  spanning from  $-2a$  to  $2a$ , with a  $y$ -axis arrow through its center pointing up (current  $I_0$  in  $+y$ -direction) and a label  $J = Cr$  pointing to the wire. Point P is marked inside Wire S, above the center axis, at horizontal position  $x = a$ . Wire T is a shaded cylindrical bar centered at  $x = 10a$  spanning from  $9a$  to  $11a$ , with a downward arrow through its center (current  $I_0$  in  $-y$ -direction). The  $x$ -axis is marked with gridlines at  $-2a, 0, 2a, 4a, 6a, 8a, 10a$ .]

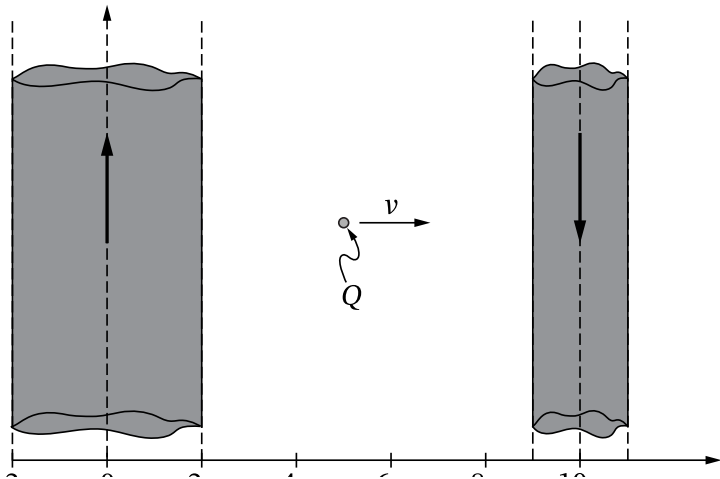
**(a)(i)** Point P is located inside of Wire S at horizontal position  $x = a$ . **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of  $C$ ,  $a$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

## Question 1

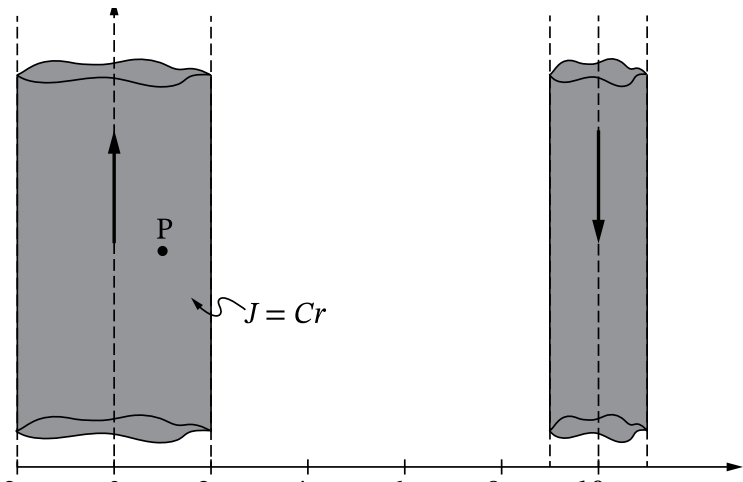
**(a)(ii)** On the axes shown in Figure 2, **\*\*sketch\*\*** a graph of the magnitude  $B$  of the magnetic field due to the currents in both wires as a function of  $x$  along the  $x$ -axis in the region  $2a \leq x \leq 9a$ . Sketches made in the shaded region will not be graded.

[Figure 2: A blank graph grid with vertical axis  $B$  and horizontal axis  $x$ , gridlines marked at  $O, 2a, 3a, 4a, 5a, 6a, 7a, 8a, 9a$ . The region from  $O$  to  $2a$  is shaded (not to be used); the region from  $2a$  to  $9a$  is the graded sketch area.]

## Question 1

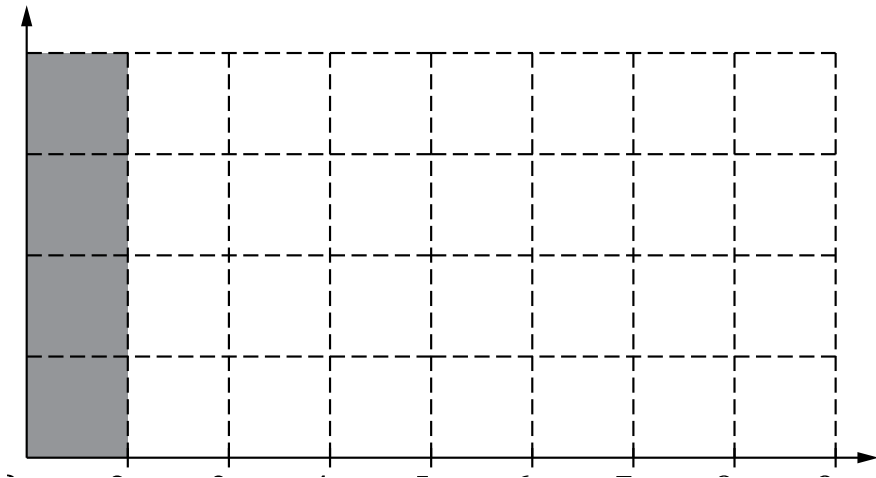


## Question 1





## Question 1



## Question 1

2026 ■ Q1

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- Wire S has radius  $2a$  and is centered at  $x = 0$ . Wire S has nonuniform current density and carries total current  $I_0$  in the  $+y$ -direction. The magnitude of the current density in Wire S as a function of the radial distance  $r$  from the center of the wire is described by  $J = Cr$ , where  $C$  is a positive constant.
- Wire T has radius  $a$  and is centered at  $x = 10a$ . Wire T has uniform current density and carries total current  $I_0$  in the  $-y$ -direction.

## Question 1

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**(b)** At the instant shown in Figure 3, a small sphere with charge  $Q$  is at horizontal position  $x = 5a$  and moving in the  $xy$ -plane with speed  $v$  in the  $+x$ -direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude  $F$ .

[Figure 3: Same wire configuration as Figure 1 (Wire S centered at  $x = 0$  with current  $I_0$  in  $+y$ -direction, Wire T centered at  $x = 10a$  with current  $I_0$  in  $-y$ -direction). A

## Question 1

small circle labeled  $Q$  is shown at  $x = 5a$ , midway between the wires, with an arrow labeled  $v$  pointing in the  $+x$ -direction.]

**Derive** an expression for  $F$  in terms of  $a$ ,  $Q$ ,  $I_0$ ,  $v$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

## Question 4

2025 ■ Q4

Long, parallel wires S and T are a distance  $2d$  apart. Both wires carry equal currents  $I$ , but the currents are in opposite directions. Both wires are parallel to the  $x$ -axis. At the instant shown in Figure 1, Sphere 1 is a distance  $d$  above Wire S, Sphere 2 is a distance  $d$  below Wire S, and both spheres are moving with speed  $v$  in the  $+x$ -direction. Each sphere has positive charge  $+Q$ . Gravitational effects are negligible. [Figure 1: Sphere 1 is shown a distance  $d$  above Wire S, moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . Wire S runs horizontally carrying current  $I$  to the right. A distance  $d$  below Wire S is Sphere 2, also moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . A further distance  $d$  below Sphere 2 is Wire T, carrying current  $I$  to the left. A small 3D axis to the right shows  $+y$  up,  $+x$  to the right, and  $+z$  out of the page (circle with a dot).]

## Question 4

(a)  $F_1$  is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T.  $F_2$  is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

**Indicate** whether  $F_2$  is greater than, less than, or equal to  $F_1$  by writing one of the following.

- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

**Justify** your answer.

## Question 4

2025 ■ Q4

Long, parallel wires S and T are a distance  $2d$  apart. Both wires carry equal currents  $I$ , but the currents are in opposite directions. Both wires are parallel to the  $x$ -axis. At the instant shown in Figure 1, Sphere 1 is a distance  $d$  above Wire S, Sphere 2 is a distance  $d$  below Wire S, and both spheres are moving with speed  $v$  in the  $+x$ -direction. Each sphere has positive charge  $+Q$ . Gravitational effects are negligible.

[Figure 1: Sphere 1 is shown a distance  $d$  above Wire S, moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . Wire S runs horizontally carrying current  $I$  to the right. A distance  $d$  below Wire S is Sphere 2, also moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . A further distance  $d$  below Sphere 2 is Wire T, carrying current  $I$  to the left. A small 3D axis to the right shows  $+y$  up,  $+x$  to the right, and  $+z$  out of the page (circle with a dot).]

## Question 4

**(b)** **\*\*Derive\*\*** an expression for the magnitude  $B_{\text{tot}}$  of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of  $d$ ,  $I$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



## Question 4

2025 ■ Q4

Long, parallel wires S and T are a distance  $2d$  apart. Both wires carry equal currents  $I$ , but the currents are in opposite directions. Both wires are parallel to the  $x$ -axis. At the instant shown in Figure 1, Sphere 1 is a distance  $d$  above Wire S, Sphere 2 is a distance  $d$  below Wire S, and both spheres are moving with speed  $v$  in the  $+x$ -direction. Each sphere has positive charge  $+Q$ . Gravitational effects are negligible.

[Figure 1: Sphere 1 is shown a distance  $d$  above Wire S, moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . Wire S runs horizontally carrying current  $I$  to the right. A distance  $d$  below Wire S is Sphere 2, also moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . A further distance  $d$  below Sphere 2 is Wire T, carrying current  $I$  to the left. A small 3D axis to the right shows  $+y$  up,  $+x$  to the right, and  $+z$  out of the page (circle with a dot).]

## Question 4

(c) Later, Wire T carries current  $3I$  in the  $+x$ -direction. At the instant shown in Figure 2, Sphere 2 is a distance  $d$  below Wire S and is moving with speed  $v$  in the  $+x$ -direction.  $F_{\text{new}}$  is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

[Figure 2: Wire S runs horizontally carrying current  $I$  to the right. A distance  $d$  below Wire S is Sphere 2, moving with velocity  $v$  in the  $+x$ -direction and carrying charge  $+Q$ . A further distance  $d$  below Sphere 2 is Wire T, now carrying current  $3I$  to the right (same direction as Wire S). A small 3D axis to the right shows  $+y$  up,  $+x$  to the right, and  $+z$  out of the page (circle with a dot).]

**\*\*Indicate\*\*** whether  $F_{\text{new}}$  is greater than, less than, or equal to  $F_2$  by writing one of the following. -  $F_{\text{new}} > F_2$  -  $F_{\text{new}} < F_2$  -  $F_{\text{new}} = F_2$

Briefly **\*\*justify\*\*** your answer by referencing your derivation in part B.

## Solution 4

(a) Mark scheme

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- $F_2 > F_1$ . The magnetic force on a moving charge has magnitude  $F = QvB$ , and  $Q$  and  $v$  are the same for both spheres, so the difference in force magnitudes comes entirely from  $B$ . Points: (1) indicating  $F_2 > F_1$ , (2) correctly relating force magnitude to field magnitude ( $F \propto B$ , via  $F = QvB$ ), (3) justifying that  $B$  is greater at Sphere 2's location than Sphere 1's, referencing relative distance

## Solution 4

(b) Mark scheme

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- Start from Ampere's law  $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$  (or the Biot – Savart law  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{\ell} \times \hat{r})}{r^2}$ ; vector notation not required). For a single long straight wire:  $B(2\pi d) = \mu_0 I$ , so  $B = \mu_0 I / (2\pi d)$  – the field at Sphere 2 due to one wire. Since both wires S and T carry current I and their fields add at Sphere 2,  $B_{tot} = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{\pi d}$ . Points: (1) multi-step derivation including Ampere's or Biot – Savart law (a derivation reaching  $B = \mu_0 I / (2\pi d)$  also earns this), (2) correct expression for the field at Sphere 2 due to one wire, (3) indicating the wire field.

## Solution 4

(c) Mark scheme

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■  $F_{\text{new}} =$

$F_2$ . Now Wire T carries  $3I$ ; the field at Sphere 2 due to Wire T is in the opposite direction to the field due to Wire S.

$$\frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{\pi d} \text{ (OR: )}$$

stated directly that the net field magnitude at Sphere 2 stays the same even though the field from Wire T changes.

$$F_{\text{new}} = F_2. \text{ Points: (1) indicating } F_{\text{new}} = F_2, \text{ (2) indicating } B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} -$$

$$\frac{\mu_0 I}{2\pi d} \text{ OR indicating that the net field magnitude at Sphere 2 remains the same even though the field from Wire T changes.}$$

## Question 2

2023 Set 2 ■ Q2

[Figure, Perspective View (at time  $t = 0$ ): A sloped section of two parallel conducting rails at height  $H$ , with a resistor  $R$  at the top of the slope. The rails transition to a horizontal section. A region of magnetic field  $B_1$  (dots, pointing out of the page,  $+y$ -direction) covers part of the horizontal rails near the bottom of the slope. Farther along, a separate region of magnetic field  $B_2$  (arrows pointing into the page,  $+z$ -direction, shown as downward-slanting field lines with arrowheads) covers another part of the rails. Axes shown:  $+x$ ,  $+y$ ,  $+z$  in a right-handed orientation.]

[Figure, Top View (at time  $t_B$ ): Two horizontal parallel rails separated by distance  $d$ , viewed from above. A resistor  $R$  is at the left end (position  $x_0$ ). The bar is shown as a vertical rectangle. Positions marked along the  $x$ -axis:  $x_0$ ,  $x_1$ ,  $x_B$ ,  $x_2$ ,  $x_3$ ,  $x_4$ . The region between  $x_1$  and  $x_2$  is shaded with dots labeled  $B_1$  (field out of the page). The region

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between  $x_3$  and  $x_4$  has downward arrows labeled  $B_2$  (field into the page). The bar is shown at position  $x_B$ , between  $x_1$  and  $x_2$ .]

Two parallel conducting rails are separated by distance  $d = 0.30$  m. A resistor of resistance  $R = 0.20\ \Omega$  connects the rails. A conducting bar is placed on a sloped section of the rails at height  $H$  above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time  $t = 0$ , the bar is released from rest from position  $x_0$  and slides down the sloped section of the rails, as shown in the Perspective View. - At time  $t_1$ , the bar reaches position  $x_1$  and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude  $B_1 = 0.40$  T that is directed in the  $+y$ -direction. - At time  $t_2$ , the bar reaches position  $x_2$  and enters a region with no magnetic field. - At time  $t_3$ , the bar reaches position  $x_3$  and enters a uniform magnetic

## Question 2

field of magnitude  $B_2 = 0.60 \text{ T}$  that is directed in the  $+z$ -direction. - At time  $t_4$ , the bar reaches position  $x_4$  and enters a region with no magnetic field.

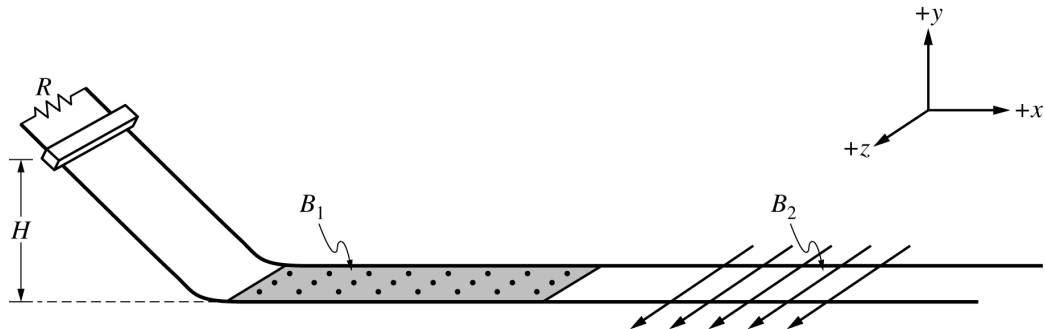
The bar is at position  $x_B$  (shown in Top View) at time  $t_B$  such that  $t_1 < t_B < t_2$ .

**(a)** [Diagram: a vertical rectangle representing the bar, viewed from the Top View perspective, with no arrows drawn.]

On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force  $F_{\text{net}}$  exerted on the bar at time  $t_B$ . If the net force is zero, write  $F_{\text{net}} = 0$ .

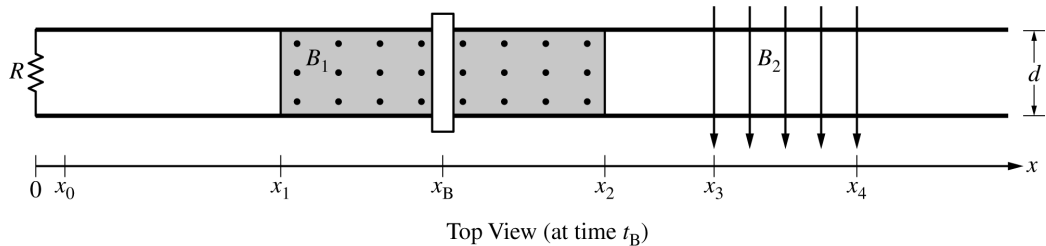


## Question 2

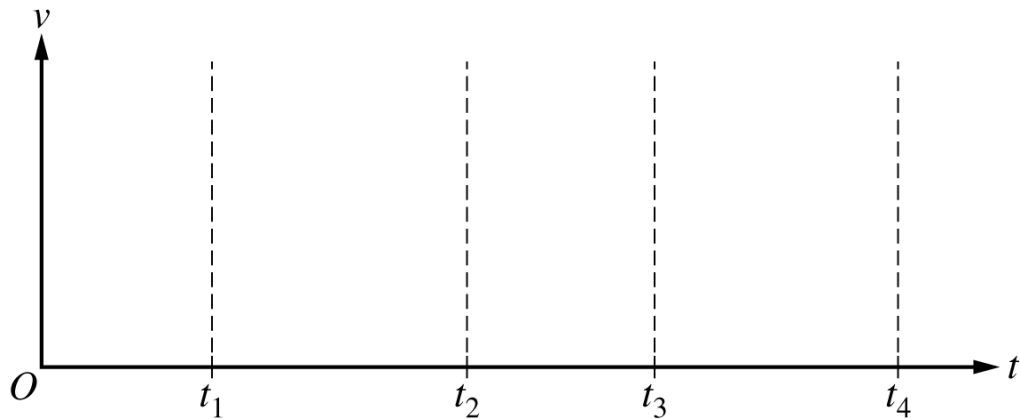


Perspective View (at time  $t = 0$ )

## Question 2



## Question 2



## Question 2

2023 Set 2 ■ Q2

[Figure, Perspective View (at time  $t = 0$ ): A sloped section of two parallel conducting rails at height  $H$ , with a resistor  $R$  at the top of the slope. The rails transition to a horizontal section. A region of magnetic field  $B_1$  (dots, pointing out of the page,  $+y$ -direction) covers part of the horizontal rails near the bottom of the slope. Farther along, a separate region of magnetic field  $B_2$  (arrows pointing into the page,  $+z$ -direction, shown as downward-slanting field lines with arrowheads) covers another part of the rails. Axes shown:  $+x$ ,  $+y$ ,  $+z$  in a right-handed orientation.]

[Figure, Top View (at time  $t_B$ ): Two horizontal parallel rails separated by distance  $d$ , viewed from above. A resistor  $R$  is at the left end (position  $x_0$ ). The bar is shown as a vertical rectangle. Positions marked along the  $x$ -axis:  $x_0$ ,  $x_1$ ,  $x_B$ ,  $x_2$ ,  $x_3$ ,  $x_4$ . The region between  $x_1$  and  $x_2$  is shaded with dots labeled  $B_1$  (field out of the page). The region between  $x_3$  and  $x_4$  has downward arrows labeled  $B_2$  (field into the page). The bar is shown at position  $x_B$ , between  $x_1$  and  $x_2$ .]

## Question 2

Two parallel conducting rails are separated by distance  $d = 0.30$  m. A resistor of resistance  $R = 0.20\ \Omega$  connects the rails. A conducting bar is placed on a sloped section of the rails at height  $H$  above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time  $t = 0$ , the bar is released from rest from position  $x_0$  and slides down the sloped section of the rails, as shown in the Perspective View. - At time  $t_1$ , the bar reaches position  $x_1$  and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude  $B_1 = 0.40$  T that is directed in the  $+y$ -direction. - At time  $t_2$ , the bar reaches position  $x_2$  and enters a region with no magnetic field. - At time  $t_3$ , the bar reaches position  $x_3$  and enters a uniform magnetic field of magnitude  $B_2 = 0.60$  T that is directed in the  $+z$ -direction. - At time  $t_4$ , the bar reaches position  $x_4$  and enters a region with no magnetic field.

The bar is at position  $x_B$  (shown in Top View) at time  $t_B$  such that  $t_1 < t_B < t_2$ .

**(b)(i)** At time  $t_B$ , the speed of the bar is  $v = 2.5$  m/s.

## Question 2

Calculate the magnitude of the current in the bar at time  $t_B$ .

**(b)(ii)** Calculate the magnitude of the net force  $F_{\text{net}}$  exerted on the bar at time  $t_B$ .

## Question 2

2023 Set 2 ■ Q2

[Figure, Perspective View (at time  $t = 0$ ): A sloped section of two parallel conducting rails at height  $H$ , with a resistor  $R$  at the top of the slope. The rails transition to a horizontal section. A region of magnetic field  $B_1$  (dots, pointing out of the page,  $+y$ -direction) covers part of the horizontal rails near the bottom of the slope. Farther along, a separate region of magnetic field  $B_2$  (arrows pointing into the page,  $+z$ -direction, shown as downward-slanting field lines with arrowheads) covers another part of the rails. Axes shown:  $+x$ ,  $+y$ ,  $+z$  in a right-handed orientation.]

[Figure, Top View (at time  $t_B$ ): Two horizontal parallel rails separated by distance  $d$ , viewed from above. A resistor  $R$  is at the left end (position  $x_0$ ). The bar is shown as a vertical rectangle. Positions marked along the  $x$ -axis:  $x_0$ ,  $x_1$ ,  $x_B$ ,  $x_2$ ,  $x_3$ ,  $x_4$ . The region between  $x_1$  and  $x_2$  is shaded with dots labeled  $B_1$  (field out of the page). The region between  $x_3$  and  $x_4$  has downward arrows labeled  $B_2$  (field into the page). The bar is shown at position  $x_B$ , between  $x_1$  and  $x_2$ .]

## Question 2

Two parallel conducting rails are separated by distance  $d = 0.30$  m. A resistor of resistance  $R = 0.20 \, \Omega$  connects the rails. A conducting bar is placed on a sloped section of the rails at height  $H$  above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time  $t = 0$ , the bar is released from rest from position  $x_0$  and slides down the sloped section of the rails, as shown in the Perspective View. - At time  $t_1$ , the bar reaches position  $x_1$  and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude  $B_1 = 0.40$  T that is directed in the  $+y$ -direction. - At time  $t_2$ , the bar reaches position  $x_2$  and enters a region with no magnetic field. - At time  $t_3$ , the bar reaches position  $x_3$  and enters a uniform magnetic field of magnitude  $B_2 = 0.60$  T that is directed in the  $+z$ -direction. - At time  $t_4$ , the bar reaches position  $x_4$  and enters a region with no magnetic field.

The bar is at position  $x_B$  (shown in Top View) at time  $t_B$  such that  $t_1 < t_B < t_2$ .



## Question 2

(c) On the following axes, sketch a graph of the speed  $v$  of the bar as a function of time  $t$  between  $t = 0$  and  $t_4$ .

[Graph: y-axis labeled " $v$ ", x-axis labeled " $t$ " with tick marks at  $t_1$ ,  $t_2$ ,  $t_3$ ,  $t_4$  (dashed vertical gridlines at each); axes otherwise unlabeled with numeric values, origin at  $O$ .]

## Question 2

2023 Set 2 ■ Q2

[Figure, Perspective View (at time  $t = 0$ ): A sloped section of two parallel conducting rails at height  $H$ , with a resistor  $R$  at the top of the slope. The rails transition to a horizontal section. A region of magnetic field  $B_1$  (dots, pointing out of the page,  $+y$ -direction) covers part of the horizontal rails near the bottom of the slope. Farther along, a separate region of magnetic field  $B_2$  (arrows pointing into the page,  $+z$ -direction, shown as downward-slanting field lines with arrowheads) covers another part of the rails. Axes shown:  $+x$ ,  $+y$ ,  $+z$  in a right-handed orientation.]

[Figure, Top View (at time  $t_B$ ): Two horizontal parallel rails separated by distance  $d$ , viewed from above. A resistor  $R$  is at the left end (position  $x_0$ ). The bar is shown as a vertical rectangle. Positions marked along the  $x$ -axis:  $x_0$ ,  $x_1$ ,  $x_B$ ,  $x_2$ ,  $x_3$ ,  $x_4$ . The region between  $x_1$  and  $x_2$  is shaded with dots labeled  $B_1$  (field out of the page). The region between  $x_3$  and  $x_4$  has downward arrows labeled  $B_2$  (field into the page). The bar is shown at position  $x_B$ , between  $x_1$  and  $x_2$ .]

## Question 2

Two parallel conducting rails are separated by distance  $d = 0.30$  m. A resistor of resistance  $R = 0.20 \, \Omega$  connects the rails. A conducting bar is placed on a sloped section of the rails at height  $H$  above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time  $t = 0$ , the bar is released from rest from position  $x_0$  and slides down the sloped section of the rails, as shown in the Perspective View. - At time  $t_1$ , the bar reaches position  $x_1$  and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude  $B_1 = 0.40$  T that is directed in the  $+y$ -direction. - At time  $t_2$ , the bar reaches position  $x_2$  and enters a region with no magnetic field. - At time  $t_3$ , the bar reaches position  $x_3$  and enters a uniform magnetic field of magnitude  $B_2 = 0.60$  T that is directed in the  $+z$ -direction. - At time  $t_4$ , the bar reaches position  $x_4$  and enters a region with no magnetic field.

The bar is at position  $x_B$  (shown in Top View) at time  $t_B$  such that  $t_1 < t_B < t_2$ .

## Question 2

**(d)(i)** The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance  $R = 0.20 \, \Omega$ . The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

Determine the total resistance of the closed circuit.

**(d)(ii)** In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is  $a_{\text{original}}$ . In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is  $a_{\text{new}}$ . Is  $a_{\text{new}}$  greater than, less than, or equal to  $a_{\text{original}}$ ? Justify your answer.

## Question 2

2023 Set 2 ■ Q2

[Figure, Perspective View (at time  $t = 0$ ): A sloped section of two parallel conducting rails at height  $H$ , with a resistor  $R$  at the top of the slope. The rails transition to a horizontal section. A region of magnetic field  $B_1$  (dots, pointing out of the page,  $+y$ -direction) covers part of the horizontal rails near the bottom of the slope. Farther along, a separate region of magnetic field  $B_2$  (arrows pointing into the page,  $+z$ -direction, shown as downward-slanting field lines with arrowheads) covers another part of the rails. Axes shown:  $+x$ ,  $+y$ ,  $+z$  in a right-handed orientation.]

[Figure, Top View (at time  $t_B$ ): Two horizontal parallel rails separated by distance  $d$ , viewed from above. A resistor  $R$  is at the left end (position  $x_0$ ). The bar is shown as a vertical rectangle. Positions marked along the  $x$ -axis:  $x_0$ ,  $x_1$ ,  $x_B$ ,  $x_2$ ,  $x_3$ ,  $x_4$ . The region between  $x_1$  and  $x_2$  is shaded with dots labeled  $B_1$  (field out of the page). The region between  $x_3$  and  $x_4$  has downward arrows labeled  $B_2$  (field into the page). The bar is shown at position  $x_B$ , between  $x_1$  and  $x_2$ .]

## Question 2

Two parallel conducting rails are separated by distance  $d = 0.30$  m. A resistor of resistance  $R = 0.20 \, \Omega$  connects the rails. A conducting bar is placed on a sloped section of the rails at height  $H$  above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time  $t = 0$ , the bar is released from rest from position  $x_0$  and slides down the sloped section of the rails, as shown in the Perspective View. - At time  $t_1$ , the bar reaches position  $x_1$  and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude  $B_1 = 0.40$  T that is directed in the  $+y$ -direction. - At time  $t_2$ , the bar reaches position  $x_2$  and enters a region with no magnetic field. - At time  $t_3$ , the bar reaches position  $x_3$  and enters a uniform magnetic field of magnitude  $B_2 = 0.60$  T that is directed in the  $+z$ -direction. - At time  $t_4$ , the bar reaches position  $x_4$  and enters a region with no magnetic field.

The bar is at position  $x_B$  (shown in Top View) at time  $t_B$  such that  $t_1 < t_B < t_2$ .

## Question 2

(e) Describe a modification to  $H$ ,  $B_1$ , or  $d$  that will result in a larger induced current in the new bar immediately after the bar enters the first magnetic field. Justify your answer.

## Solution 2

(a)

## Mark scheme

- On the Top View diagram of the bar, draw a single arrow pointing to the LEFT (with no extraneous arrows) representing the direction of the net force  $F_{net}$  on the bar at time  $t_B$ . This is the magnetic braking force from the induced current interacting with the field, which by Lenz's law opposes the bar's motion; on the horizontal rail section (no gravity component along the rails) it is the only horizontal force, so  $F_{net}$  points opposite the bar's velocity, i.e. to the left in the given view.



## Solution 2

**(b)(i)** Mark scheme

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- Calculate the magnitude of the current in the bar at  $t_B$ , where  $v = 2.5 \text{ m/s}$ . (1)  
 Use Faraday's law for the induced emf:  $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(B_1 dx)}{dt}$  (this point may be earned without the negative sign or a numerical answer). (2) Substitute  $v$  for  $\frac{dx}{dt}$ :  $\mathcal{E} = -B_1 d\frac{dx}{dt} = -B_1 dv$  (a student may also earn both of the first two points by starting directly from  $\mathcal{E} = B_1 dv$ ). (3) Substitute the correct resistance into Ohm's law to solve for current. Full solution:  

$$\mathcal{E} = -(0.40 \text{ T})(0.30 \text{ m})(2.5 \text{ m/s}) = -0.30 \text{ V}; I = \frac{|\mathcal{E}|}{R} = \frac{0.30 \text{ V}}{0.20 \Omega} = 1.5 \text{ A}.$$

## Solution 2

(b)(ii)

## Mark scheme

- Calculate the magnitude of the net force  $F_{net}$  on the bar at  $t_B$  by substituting the current (or an expression for it) from part (b)(i) into  $F = IdB_1$ :

$F = (1.5 \text{ A})(0.3 \text{ m})(0.4 \text{ T}) = 0.18 \text{ N}$ . (Equivalently, deriving symbolically:

$$F = \frac{B_1^2 d^2 v}{R} = \frac{(0.4)^2 (0.3)^2 (2.5)}{0.2} = 0.18 \text{ N}.)$$

## Solution 2

### (c) Mark scheme

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- Sketch  $v$  vs  $t$  from  $t = 0$  to  $t_4$  with four required features (1 point each): (1) a straight line with POSITIVE slope starting at the origin from  $t = 0$  to  $t_1$  (bar speeding up before/while entering the field region); (2) a curve that is DECREASING and CONCAVE UP from  $t_1$  to  $t_2$  (bar decelerating due to the speed-dependent magnetic braking force, asymptotically slowing its rate of decrease); (3) a NONZERO horizontal line from  $t_2$  to  $t_3$  (constant nonzero speed); (4) a NONZERO horizontal line from  $t_3$  to  $t_4$  (constant nonzero speed continues, possibly at a different level than segment 3). Net shape: rises linearly, then curves down concave-up leveling off, then two flat nonzero segments.

## Solution 2

(d)(i)

## Mark scheme

- Determine the total resistance of the closed circuit for the new bar ( $R = 0.20 \, \Omega$ ) in series with the fixed rail/other resistance of  $0.2 \, \Omega$ :  
 $R_s = \sum_i R_i = (0.2 \, \Omega) + (0.2 \, \Omega) = 0.4 \, \Omega$ . Correct answer with units ( $0.4 \, \Omega$ ) earns the point; it may be earned without supporting calculations.

## Solution 2

### (d)(ii) Mark scheme

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- Determine whether  $a_{new}$  is greater than, less than, or equal to  $a_{original}$ , and justify: the correct answer is  $a_{new} < a_{original}$ . Full credit requires a chain of correct statements (1 point each, and full credit can be earned with a justification consistent with the resistance calculated in part (d)(i)): (1) inverse relation between resistance and current — as resistance increases, current decreases; (2) direct relation between current and magnetic force — as current decreases, magnetic force  $F = IdB$  decreases; (3) direct relation between force and acceleration — by Newton's second law  $F = ma$ , as force decreases (same mass), acceleration decreases. Example justification: the new bar has greater total

## Solution 2

resistance, so it carries less current, so the magnetic force on it is less, so (same mass) its acceleration is less than the original.

## Solution 2

### (e) Mark scheme

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- Describe ONE modification to  $H$ ,  $B_1$ , or  $d$  that results in a LARGER induced current in the new bar immediately after entering the first magnetic field, and justify it. (1 point) Correctly identify one of: increasing  $H$ , increasing  $B_1$ , or increasing  $d$ , with an attempted relevant justification. (1 point) Correctly justify why that change increases current. Any one of: Increasing  $H$  — a higher release height gives greater potential energy, hence greater kinetic energy/speed entering the field, giving a greater rate of change of flux, so by Faraday's law a greater emf and thus greater current. OR Increasing  $B_1$  — a stronger field increases the flux through the circuit and its rate of change, giving a greater induced emf and thus greater current. OR Increasing  $d$  — a larger bar width increases the area enclosed

## Solution 2

by the circuit, increasing flux and its rate of change, giving a greater induced emf and thus greater current.



## Question 3

2019 Set 2 ■ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled  $\Delta V$ . To the left of and between the plates is a small filled circle labeled "mass =  $m$ , charge =  $e^-$ " with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of  $\times$  symbols labeled  $B$ , representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference  $V$  between them. A small sphere of mass  $m$  and charge  $-e$  is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude  $B$  directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of  $V$ ,  $m$ ,  $e$ ,  $B$ , and fundamental constants, as appropriate.

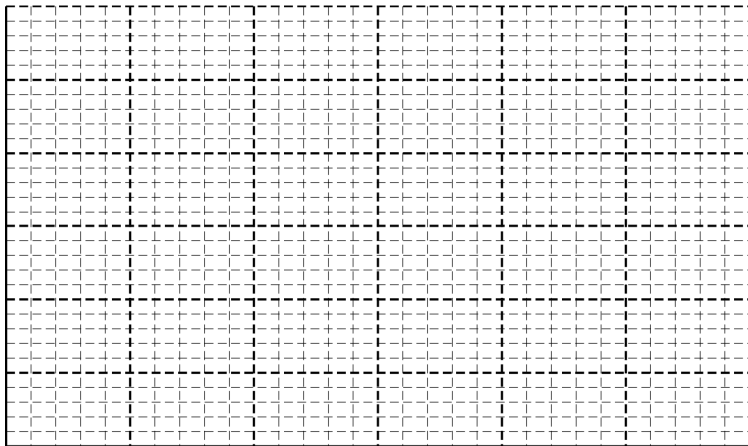
## Question 3

**(a)(i)** What is the initial direction of the force on the sphere as it enters the magnetic field?

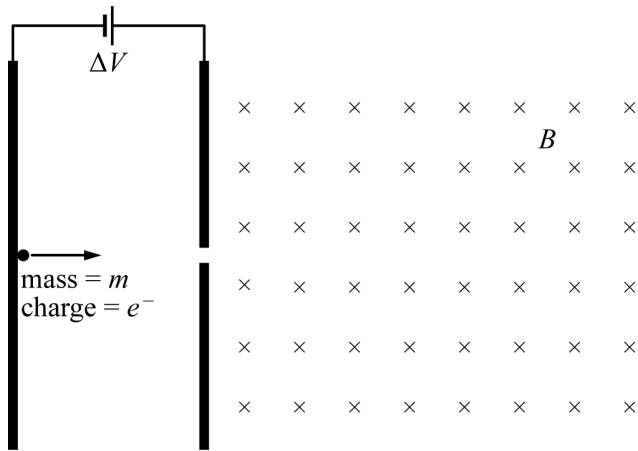
\_\_\_\_\_ Into the page \_\_\_\_\_ Out of the page \_\_\_\_\_ Toward the top of the page \_\_\_\_\_  
Toward the bottom of the page

**(a)(ii)** Describe the path taken by the sphere after it enters the magnetic field.

## Question 3



## Question 3



## Question 3

|                                       |      |      |      |      |      |      |
|---------------------------------------|------|------|------|------|------|------|
| Potential difference (V)              | 60   | 70   | 100  | 110  | 120  | 140  |
| Magnetic field ( $T \times 10^{-3}$ ) | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |
|                                       |      |      |      |      |      |      |
|                                       |      |      |      |      |      |      |

## Question 3

2019 Set 2 ■ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled  $\Delta V$ . To the left of and between the plates is a small filled circle labeled "mass =  $m$ , charge =  $e^-$ " with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of  $\times$  symbols labeled  $B$ , representing a uniform magnetic field directed into the page.]

Two plates are set up with a potential difference  $V$  between them. A small sphere of mass  $m$  and charge  $-e$  is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude  $B$  directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of  $V$ ,  $m$ ,  $e$ ,  $B$ , and fundamental constants, as appropriate.

**(b)** Derive an expression for the speed of the sphere as it passes through the small opening.

## Question 3

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[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled  $\Delta V$ . To the left of and between the plates is a small filled circle labeled "mass =  $m$ , charge =  $e^-$ " with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of  $\times$  symbols labeled  $B$ , representing a uniform magnetic field directed into the page.]

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**(c)** Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

## Question 3

2019 Set 2 ■ Q3

[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled  $\Delta V$ . To the left of and between the plates is a small filled circle labeled "mass =  $m$ , charge =  $e^-$ " with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of  $\times$  symbols labeled  $B$ , representing a uniform magnetic field directed into the page.]

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**(d)** An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through



## Question 3

the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

|                                              |      |      |      |      |      |      |   |   |   |   |   |   |  |
|----------------------------------------------|------|------|------|------|------|------|---|---|---|---|---|---|--|
| Potential difference (V)                     | 60   | 70   | 100  | 110  | 120  | 140  | — | — | — | — | — | — |  |
| Magnetic field ( $\text{T} \times 10^{-3}$ ) | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |   |   |   |   |   |   |  |

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:

Horizontal axis:

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Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

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[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled  $\Delta V$ . To the left of and between the plates is a small filled circle labeled "mass =  $m$ , charge =  $e^-$ " with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of  $\times$  symbols labeled  $B$ , representing a uniform magnetic field directed into the page.]

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## Question 3

**(e)** On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.

[Blank grid graph for plotting data.]

## Question 3

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[Figure: Two vertical parallel plates connected at the top to a battery symbol labeled  $\Delta V$ . To the left of and between the plates is a small filled circle labeled "mass =  $m$ , charge =  $e^-$ " with an arrow pointing right (toward the plates/opening). To the right of the plates is a grid of  $\times$  symbols labeled  $B$ , representing a uniform magnetic field directed into the page.]

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**(f)** Using the straight line from part (e), determine the mass-to-charge ratio of an electron.