# Past-paper walk-through

## Magnetic Fields ▪ 12.1

eXercise

# Questions in this set

- 2019 Set 1 Q3
- 2017 Q3
- 2016 Q3

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 3

2019 Set 1 ▪ Q3

Note: Figures not drawn to scale.
[Figure, left ("Solenoid"): A solenoid of inner radius $a$ and length $\ell$ with current $I$ entering, wound with wire, oriented along its axis.]
[Figure, right ("Cutaway View"): A cutaway view of the solenoid showing the xyz-coordinate system, with the x-axis along the axis of the solenoid, current $I$ shown entering at the near end, point P at the origin in the middle of the solenoid, and the y- and z-axes shown perpendicular to the x-axis at P.]
A solenoid is used to generate a magnetic field. The solenoid has an inner radius $a$, length $\ell$, and $N$ total turns of wire. A power supply, not shown, is connected to the solenoid and generates current $I$, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid, as shown in the cutaway view on the right above.

# Question 3

Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view. Assume $\ell \gg a$.

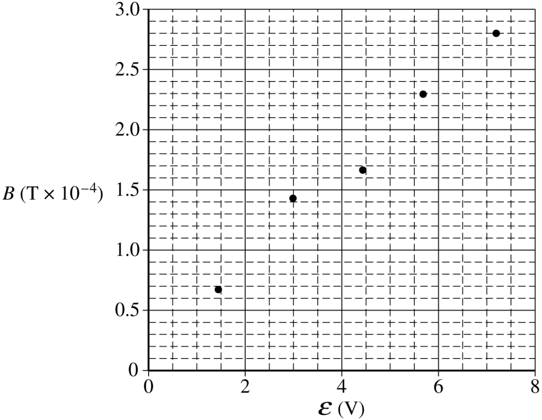
**(a)** Select the correct direction of the magnetic field at point P.

_____ +x-direction _____ +y-direction _____ +z-direction _____ -x-direction _____ -y-direction _____ -z-direction

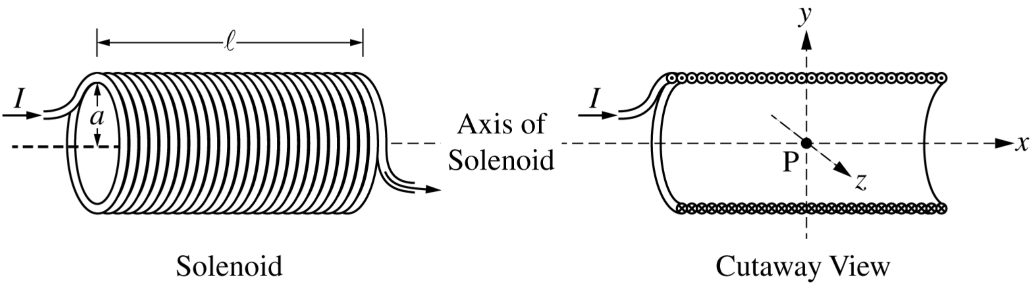
Justify your selection.

# Question 3

axis at the center of the solenoid. The plot of the magnetic field strength $B$ as a function of
power supply is shown below.



# Question 3

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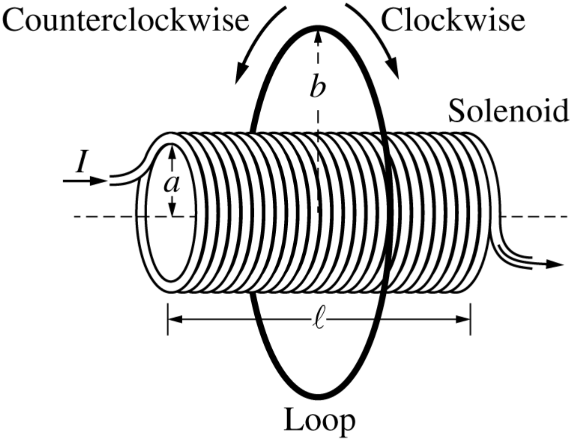
# Question 3



# Question 3

2019 Set 1 ▪ Q3

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A solenoid is used to generate a magnetic field. The solenoid has an inner radius $a$, length $\ell$, and $N$ total turns of wire. A power supply, not shown, is connected to the solenoid and generates current $I$, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid, as shown in the cutaway view on the right above. Point P is in the middle of the

# Question 3

solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view. Assume $\ell \gg a$.

**(b)(i)** On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.

[Figure: Cutaway view of the solenoid with current $I$ entering along the "Axis of Solenoid" dashed line, and point P marked at the center of the solenoid; blank for the student to draw an Amperian loop.]

**(b)(ii)** Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of $I$, $\ell$, $N$, $a$, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance $R_S$ of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor

# Question 3

is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength $B$ as a function of the emf $\mathcal{E}$ of the power supply is shown below.
[Graph: Scatter plot of $B$ (T $\times 10^{-4}$) on the vertical axis, ranging 0 to 3.0 in increments of 0.5, versus $\mathcal{E}$ (V) on the horizontal axis, ranging 0 to 8 in increments of 2. Data points approximately at: $(1.6, 0.65)$, $(3.0, 1.4)$, $(4.2, 1.55)$, $(5.4, 2.35)$, $(6.8, 2.85)$. Grid lines shown throughout; no line drawn yet — student draws a best-fit line.]

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2019 Set 1 ▪ Q3

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solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view. Assume $\ell \gg a$.

**(c)(i)** On the graph above, draw a best-fit line for the data.

**(c)(ii)** Use the straight line to determine the resistance $R_S$ of the solenoid used in the experiment.

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2019 Set 1 ▪ Q3

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# Question 3

solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view. Assume $\ell \gg a$.

**(d)(i)** One of the students notes that the horizontal component of the magnetic field of Earth is $2.5 \times 10^{-5}$ T. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for $B$?

Yes No

Justify your answer.

**(d)(ii)** Would the horizontal orientation of the solenoid affect the calculated value for $R_S$?

Yes No

Justify your answer.

# Question 3

[Figure: A solenoid with current $I$ entering, inner radius $a$, length $\ell$, labeled "Solenoid." A thin conducting loop of radius $b$ is shown concentric with and around the solenoid, with curved arrows labeled "Counterclockwise" and "Clockwise" indicating the two possible current directions in the loop, labeled "Loop."]

A thin conducting loop of radius $b$ and resistance $R_L$ is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from $I$ to zero over time $\Delta t$.

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# Question 3

solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view. Assume $\ell \gg a$.

**(e)(i)** Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

_____ Clockwise _____ Counterclockwise

Justify your answer.

**(e)(ii)** Derive an equation for the average induced current $i_{\mathrm{IND}}$ in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of $I$, $\ell$, $N$, $a$, $b$, $R_L$, $R_S$, $\Delta t$, and physical constants, as appropriate.

# Solution 3

## (a)

### Mark scheme

- $+$x-direction (1 pt for choosing this with a justification attempt). Justification (1 pt): using the right-hand rule for a solenoid – curl the fingers of the right hand in the direction of current flow around the loops – the thumb points in the $+x$-direction, so the magnetic field at the center of the solenoid (point P) points in the $+x$-direction. (Equivalently: using the right-hand rule for the current on the near/left side of the solenoid, the fingers curl into the loop and point in the $+x$ direction at the axis.)

# Solution 3

## (b)(i)

**Mark scheme**

- Draw a rectangular Amperian loop with one long side lying along the central axis of the solenoid, passing through point P (the side where $B$ is to be found), and the opposite long side lying outside the solenoid (where $B \approx 0$); the two short sides are perpendicular to the axis. Both long sides must stay within the solenoid's length $\ell$ (must not extend beyond the ends of the solenoid). 1 point.

# Solution 3

## (b)(ii) Mark scheme

- Apply Ampere's law around the rectangular loop (4 segments): $\oint B \cdot d\ell = \mu_0 I_{enc} \;\therefore\; (\int B\, d\ell)_1 + (\int B\, d\ell)_2 + (\int B\, d\ell)_3 + (\int B\, d\ell)_4 = \mu_0 I_{enc}$. The two perpendicular segments and the outside leg contribute zero (either $B \perp d\ell$, or $B \approx 0$ outside), leaving $(\int Bd\ell)_1 + 0 + \big(-(\int Bd\ell)_1\big) + Bh = \mu_0 \dfrac{N}{\ell} hI$, where $h$ is the length of the on-axis segment and $\mu_0 \frac{N}{\ell} hI$ is the enclosed current (the fraction $h/\ell$ of the $N$ turns, each carrying current $I$) – this simplifies to $Bh = \mu_0 \dfrac{N}{\ell} hI$ (1 pt for correctly applying Ampere's law this way). Solving: $B = \dfrac{\mu_0 NI}{\ell}$ (1 pt for the correct final answer).

# Solution 3

## (c)(i)

### Mark scheme

- Draw a straight best-fit line through the plotted $(\mathcal{E}, B)$ data points, positioned so that at least one data point lies above the line and at least one lies below it, following the overall positive-slope trend of the data (roughly rising from about $(1\ \text{V}, 0.3 \times 10^{-4}\ \text{T})$ to $(7\ \text{V}, 2.6 \times 10^{-4}\ \text{T})$). 1 point.

# Solution 3

## (c)(ii) Mark scheme

- Compute the slope from the best-fit line, not raw data points (1 pt): slope $= \dfrac{\Delta B}{\Delta \mathcal{E}} = \dfrac{(2.5 - 0.9) \times 10^{-4}\ \mathrm{T}}{(6.4 - 2.0)\ \mathrm{V}} = 0.36 \times 10^{-4} = 3.6 \times 10^{-5}\ \mathrm{T/V}$. Relate the slope to $R_S$ (1 pt): since $B = \dfrac{\mu_0 N I}{\ell}$ and (treating the solenoid as the circuit's resistance) $I = \mathcal{E}/R_S$, we get $B = \dfrac{\mu_0 N \mathcal{E}}{\ell R_S}$, so slope $= \dfrac{\mu_0 N}{\ell R_S} \Rightarrow R_S = \dfrac{\mu_0 N}{\ell \times \text{slope}} = \dfrac{(4\pi \times 10^{-7}\ \mathrm{T\ m/A})(100)}{(0.40\ \mathrm{m})(3.6 \times 10^{-5}\ \mathrm{T/V})} = 8.7\ \Omega$.

# Solution 3

## (d)(i) Mark scheme

- Yes – the best-fit line does not pass through the origin (there is a nonzero $B$-intercept of roughly $0.3 \times 10^{-4}$ T at $\mathcal{E} = 0$). 1 point for a correct justification, e.g.: the horizontal component of Earth's magnetic field adds to or subtracts from the solenoid's own field depending on how the solenoid is oriented, which offsets the measured $B$ values and is evidence that orientation affects them. (If a student's line does pass through the origin, 'No' with the matching justification – the line passing through the origin meaning $B = 0$ when $\mathcal{E} = 0$, so Earth's field is not affecting the values – also earns full credit.)

# Solution 3

## (d)(ii)

**Mark scheme**

- No. 1 point for a correct justification: Earth's horizontal field component is a constant additive offset to $B$ – it shifts the line's intercept but does not change its slope as $\mathcal{E}$ (and hence $I$) is varied. Since $R_S$ is calculated from the slope of the line (not its intercept), the horizontal orientation of the solenoid does not affect the calculated value of $R_S$.

# Solution 3

## (e)(i) Mark scheme

- Clockwise. 1 point for a justification indicating that the magnetic field inside the solenoid (and hence through the loop) will decrease as the solenoid's current decreases. 1 point for using Lenz's law to relate this decreasing field to the induced current's direction: since the solenoid's field points in $+x$ and is decreasing, the loop's induced current opposes the decrease by creating its own field also in the $+x$ direction inside the loop; by the right-hand rule, producing a $+x$-directed field means the current in the loop must flow clockwise (as viewed per the figure's labeled Clockwise/Counterclockwise arrows).

# Solution 3

## (e)(ii) Mark scheme

- Faraday's law (1 pt): $\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \frac{d(BA)}{dt} = A\frac{\Delta B}{\Delta t}$. Ohm's law for the loop (1 pt): $I_{IND} = \frac{V}{R} = \frac{A\,\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L\Delta t}$. Using the correct area – the solenoid's cross-sectional area $A = \pi a^2$ (radius $a$, NOT the larger loop radius $b$, since only the flux through the solenoid's cross-section threads the loop) – and $\Delta B = \frac{\mu_0 NI}{\ell}$ (the field drops from $\frac{\mu_0 NI}{\ell}$ to 0) (1 pt): $I_{IND} = \frac{\pi a^2\, \mu_0 NI/\ell}{R_L\Delta t} = \frac{\pi a^2 \mu_0 NI}{R_L\ell\,\Delta t}$.

# Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length $\ell$ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space $\mu_0$. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length $\ell$ with $N$ turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current $I$ through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

# Question 3

**(a)(i)** On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.
**(a)(ii)** Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?
_____ Toward the top of the page _____ To the left _____ Out of the page _____ Toward the bottom of the page _____ To the right _____ Into the page
The rectangle shown below represents the solenoid (the loops of wire are not shown). Points $A$, $B$, and $C$ are along the central axis of the solenoid with point $B$ at the middle of the solenoid. Point $D$ is directly above point $B$.
[Diagram: a rectangle representing the solenoid, with dot $D$ above the rectangle at its midpoint, and inside the rectangle three dots along the central axis labeled $\bullet A$, $\bullet B$

# Question 3

(both toward the left-middle), and $\bullet C$ (toward the right edge, outside the main cluster of $A$ and $B$, near the right end).]

**(a)(iii)** From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space $\mu_0$.

$^{A}{}_{B}{}_{C}{}_{D}$

Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.

# Question 3

[Two diagrams of coiled solenoids: "Solenoid 1" shown as a long tightly-wound coil, and "Solenoid 2" shown as a short coil with few turns. Note: Figures not drawn to scale.]

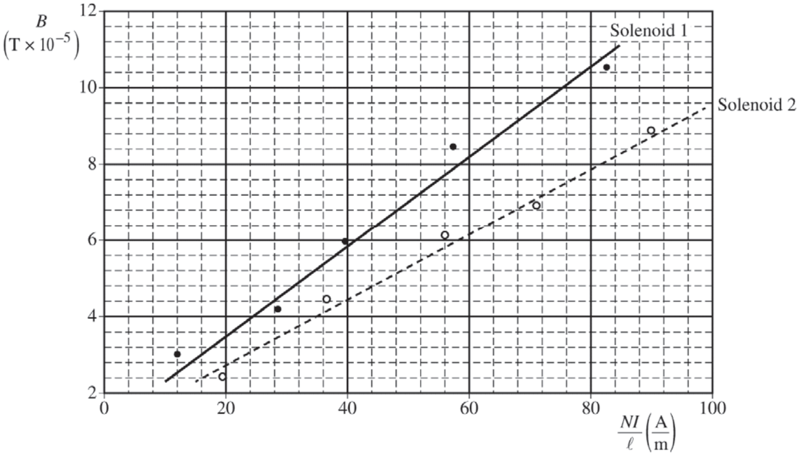
A graph of the magnitude of the magnetic field $B$ as a function of $NI/\ell$ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.

[Graph: vertical axis $B$ ($\mathrm{T} \times 10^{-5}$) ranging 0 to 12 in increments of 2; horizontal axis $\frac{NI}{\ell}\ \left(\frac{\mathrm{A}}{\mathrm{m}}\right)$ ranging 0 to 100 in increments of 20. Solid line "Solenoid 1" is a best-fit line through filled data points, rising from about $(12, 2.3)$ to about $(95, 11)$, roughly linear with a positive $y$-intercept near 0.5–1. Dashed line "Solenoid 2" is a best-fit line through open-circle data points, rising from about $(15, 1.7)$ to about $(85, 9)$, also

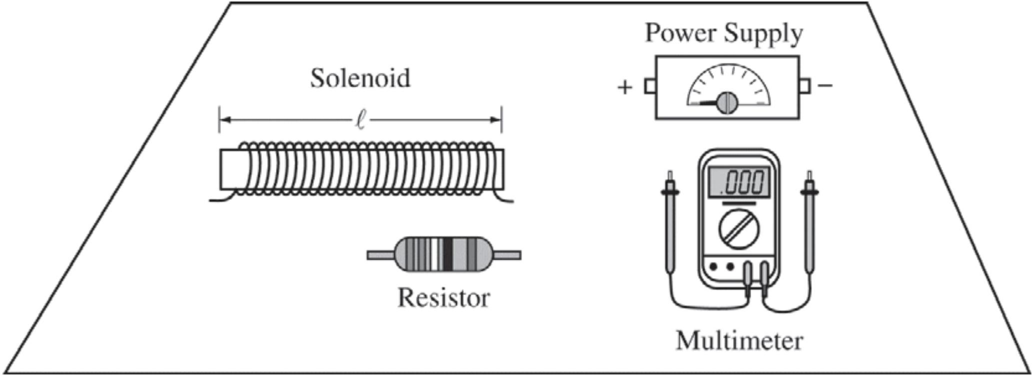
## Question 3

roughly linear, lying slightly below Solenoid 1's line at low $NI/\ell$ and crossing to be below it at high $NI/\ell$ as well, with more scatter in its data points.]

# Question 3



# Question 3



# Question 3

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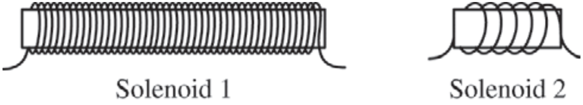


Note: Figures not drawn to scale.

# Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length $\ell$ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space $\mu_0$. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length $\ell$ with $N$ turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current $I$ through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

**(b)** Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space $\mu_0$?

# Question 3

_____ Solenoid 1 _____ Solenoid 2

Justify your answer.

# Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length $\ell$ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

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**(c)(i)** Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space $\mu_0$.

# Question 3

**(c)(ii)** Calculate the percent error for the experimental value of the magnetic permeability of free space $\mu_0$ determined in part (c)i.

# Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length $\ell$ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

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**(d)(i)** What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

# Question 3

**(d)(ii)** Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

# Solution 3

## (a)(i)

**Mark scheme**

- Draw wire connections that form a working circuit: the power supply, solenoid, and resistor connected in series with each other (1 point); the multimeter connected in series with the solenoid/resistor combination so it reads the current through the solenoid (1 point).

# Solution 3

## (a)(ii)

**Mark scheme**

- 1 point, awarded for a direction of the magnetic field inside the solenoid (toward top of page / left / out of page / toward bottom / right / into page) that is logically consistent (via the right-hand rule) with the current direction implied by the student's own wiring/connections chosen in part (a)i — there is no single fixed correct choice independent of that answer.

# Solution 3

## (a)(iii) Mark scheme

- Correct selection: point B (the point at the middle of the solenoid, on its central axis). Justification: the formula for the magnetic field of a solenoid ($B = \mu_0 nI$) is derived for an ideal (infinitely long) solenoid and does not hold outside the solenoid — this rules out points C and D, which lie outside (1 point, for any indication that the field outside the solenoid is less than inside). Near the ends/edges of the solenoid the ideal-solenoid approximation becomes less accurate due to edge effects — this rules out point A (1 point, for any indication that being closer to the center gives a better approximation because it avoids edge effects). Point B, at the solenoid's center, therefore gives the best approximation of an ideal solenoid (1 point, for selecting B).

# Solution 3

## (b)

### Mark scheme

- Correct selection: Solenoid 1. Justification: Solenoid 1 has more turns per unit length than Solenoid 2 (1 point); Solenoid 1 is also longer, and so is more like an ideal (infinite) solenoid (1 point). Since $B = \mu_0 nI$ assumes an ideal solenoid, Solenoid 1's greater turn density and length make it a better approximation, so its best-fit-line slope gives a more accurate experimental value for $\mu_0$.

# Solution 3

## (c)(i) Mark scheme

- Calculate the slope for the chosen solenoid (Solenoid 1) using the best-fit line (not individual data points):
  slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(10-4)(\mathsf{T} \times 10^{-5})}{(75-24)(\mathsf{A/m})} = 1.17 \times 10^{-6}$ T · m/A (1 point, for correctly calculating the slope from the best-fit line, not the data points). Relate the permeability of free space to the slope: $\mu_0 =$ slope (1 point). Correct units: $\mu_0 = 1.17 \times 10^{-6}$ T · m/A (1 point). [For reference, Solenoid 2's slope, not used here since Solenoid 1 was chosen in part (b):
  slope $= \dfrac{(8-4)(\mathsf{T} \times 10^{-5})}{(82-35)(\mathsf{A/m})} = 8.51 \times 10^{-7}$ T · m/A.]

# Solution 3

## (c)(ii) Mark scheme

- Use the correct percent-error formula, substituting the accepted value ($\mu_0 = 1.26 \times 10^{-6}$ T · m/A) and the experimental value from part (c)i:
  $$\%\,\text{error} = \frac{|acc - exp|}{acc} \times 100\% = \frac{|(1.26 \times 10^{-6}) - (1.18 \times 10^{-6})|}{1.26 \times 10^{-6}} \times 100\% \approx 6.35\%$$
  (1 point) [as transcribed in the official guideline for Solenoid 1; for Solenoid 2 the analogous calculation gives $\frac{|(1.26 \times 10^{-6}) - (8.51 \times 10^{-7})|}{1.26 \times 10^{-6}} \times 100\% \approx 32.4\%$].

# Solution 3

## (d)(i)

### Mark scheme

- 1 point for a correct explanation involving any extraneous (stray) magnetic field, e.g.: a component of Earth's magnetic field lies along the direction of the solenoid's (inductor's) own magnetic field, shifting the best-fit line so it does not pass through the origin.

# Solution 3

## (d)(ii)

### Mark scheme

- State that either the solenoid (inductor) or the circuit has near-zero resistance (1 point); state that, as a result of this near-zero resistance, the current becomes very high/near-infinite (1 point). Example: the inductor has near-zero resistance, so without the resistor in series with it the inductor draws a near-infinite current, which the multimeter cannot handle — causing the multimeter to stop functioning.

# Question 3

2016 ▪ Q3

[Diagram: Two long vertical conducting rails, separated by horizontal distance, both drawn as vertical lines, oriented into a uniform magnetic field directed into the page (shown by an array of $\times$ symbols filling the region between and around the rails). A resistor of resistance $R$ connects the tops of the two rails (zig-zag resistor symbol labeled $R$). A horizontal conducting bar of mass $M$ and length $L$ (labeled "$M, L$") lies across the rails below the resistor, shown shaded. Point $C$ is marked just above the bar, between the rails. Point $D$ is marked further down between the rails, below the bar's initial position. The left rail is labeled $B$ near its lower end (labeling the magnetic field region). A downward arrow to the right of the figure is labeled "Vertically down," indicating the bar's direction of motion.]

A conducting bar of mass $M$, length $L$, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a

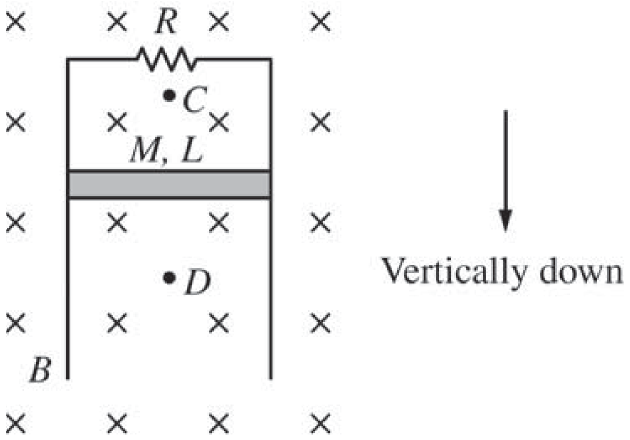
# Question 3

resistor of resistance $R$ at the top. The entire apparatus is located in a magnetic field of magnitude $B$ directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

**(a)** What is the direction of the current in the resistor?

______ Left ______ Right

# Question 3



# Question 3

2016 ▪ Q3

[Diagram: Two long vertical conducting rails, separated by horizontal distance, both drawn as vertical lines, oriented into a uniform magnetic field directed into the page (shown by an array of $\times$ symbols filling the region between and around the rails). A resistor of resistance $R$ connects the tops of the two rails (zig-zag resistor symbol labeled $R$). A horizontal conducting bar of mass $M$ and length $L$ (labeled "$M, L$") lies across the rails below the resistor, shown shaded. Point $C$ is marked just above the bar, between the rails. Point $D$ is marked further down between the rails, below the bar's initial position. The left rail is labeled $B$ near its lower end (labeling the magnetic field region). A downward arrow to the right of the figure is labeled "Vertically down," indicating the bar's direction of motion.]

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# Question 3

directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

**(b)(i)** Is the magnitude of the net magnetic field above the bar at point $C$ greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

**(b)(ii)** While the bar is above point $D$, is the magnitude of the net magnetic field at point $D$ greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

# Question 3

Express your answers to parts (c) and (d) in terms of $M$, $L$, $R$, $B$, and physical constants, as appropriate.

# Question 3

2016 ▪ Q3

[Diagram: Two long vertical conducting rails, separated by horizontal distance, both drawn as vertical lines, oriented into a uniform magnetic field directed into the page (shown by an array of $\times$ symbols filling the region between and around the rails). A resistor of resistance $R$ connects the tops of the two rails (zig-zag resistor symbol labeled $R$). A horizontal conducting bar of mass $M$ and length $L$ (labeled "$M, L$") lies across the rails below the resistor, shown shaded. Point $C$ is marked just above the bar, between the rails. Point $D$ is marked further down between the rails, below the bar's initial position. The left rail is labeled $B$ near its lower end (labeling the magnetic field region). A downward arrow to the right of the figure is labeled "Vertically down," indicating the bar's direction of motion.]

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# Question 3

directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

**(c)** Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time $t$.

# Question 3

2016 ▪ Q3

[Diagram: Two long vertical conducting rails, separated by horizontal distance, both drawn as vertical lines, oriented into a uniform magnetic field directed into the page (shown by an array of $\times$ symbols filling the region between and around the rails). A resistor of resistance $R$ connects the tops of the two rails (zig-zag resistor symbol labeled $R$). A horizontal conducting bar of mass $M$ and length $L$ (labeled "$M, L$") lies across the rails below the resistor, shown shaded. Point $C$ is marked just above the bar, between the rails. Point $D$ is marked further down between the rails, below the bar's initial position. The left rail is labeled $B$ near its lower end (labeling the magnetic field region). A downward arrow to the right of the figure is labeled "Vertically down," indicating the bar's direction of motion.]

A conducting bar of mass $M$, length $L$, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance $R$ at the top. The entire apparatus is located in a magnetic field of magnitude $B$

# Question 3

directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

**(d)** Determine an expression for the terminal velocity $v_T$ of the bar.

Express your answers to parts (e) and (f) in terms of $v_T$, $M$, $L$, $R$, $B$, and physical constants, as appropriate.

# Question 3

2016 ▪ Q3

[Diagram: Two long vertical conducting rails, separated by horizontal distance, both drawn as vertical lines, oriented into a uniform magnetic field directed into the page (shown by an array of $\times$ symbols filling the region between and around the rails). A resistor of resistance $R$ connects the tops of the two rails (zig-zag resistor symbol labeled $R$). A horizontal conducting bar of mass $M$ and length $L$ (labeled "$M, L$") lies across the rails below the resistor, shown shaded. Point $C$ is marked just above the bar, between the rails. Point $D$ is marked further down between the rails, below the bar's initial position. The left rail is labeled $B$ near its lower end (labeling the magnetic field region). A downward arrow to the right of the figure is labeled "Vertically down," indicating the bar's direction of motion.]

A conducting bar of mass $M$, length $L$, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance $R$ at the top. The entire apparatus is located in a magnetic field of magnitude $B$

# Question 3

directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

**(e)** Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

# Question 3

2016 ▪ Q3

[Diagram: Two long vertical conducting rails, separated by horizontal distance, both drawn as vertical lines, oriented into a uniform magnetic field directed into the page (shown by an array of $\times$ symbols filling the region between and around the rails). A resistor of resistance $R$ connects the tops of the two rails (zig-zag resistor symbol labeled $R$). A horizontal conducting bar of mass $M$ and length $L$ (labeled "$M, L$") lies across the rails below the resistor, shown shaded. Point $C$ is marked just above the bar, between the rails. Point $D$ is marked further down between the rails, below the bar's initial position. The left rail is labeled $B$ near its lower end (labeling the magnetic field region). A downward arrow to the right of the figure is labeled "Vertically down," indicating the bar's direction of motion.]

A conducting bar of mass $M$, length $L$, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance $R$ at the top. The entire apparatus is located in a magnetic field of magnitude $B$

# Question 3

directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

**(f)** Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time $t$.

# Solution 3

## (a)

**Mark scheme**

- Correct choice: Left — the induced current flows to the left through the resistor. Points: 1 for correctly selecting 'Left'.

# Solution 3

## (b)(i)

**Mark scheme**

- Correct choice: 'Less than.' Justification: as the bar falls, the magnetic flux at point C (above the bar) is increasing, so by Lenz's law the induced current creates a magnetic field that opposes this increase — i.e. a field that decreases the net field at C, making it less than the original (before-release) magnitude. Points: 1 for selecting 'Less than'; 1 for a correct justification. No points are earned if the wrong answer is selected.

# Solution 3

## (b)(ii)

**Mark scheme**

- Correct choice: 'Greater than.' Justification: the induced field at point C (above the bar) is less than the original field, and point D (below the bar) is on the opposite side of the bar from C, so the induced field's direction at D is opposite to its direction at C — i.e. it adds to the original field there. Hence the net magnetic field at D while the bar is falling is greater than the original (before-release) magnitude. Points: 1 for selecting 'Greater than'; 1 for a correct justification. No points are earned if the wrong answer is selected.

# Solution 3

## (c) Mark scheme

- Derive (do not solve) a differential equation for $v(t)$. Newton's second law on the falling bar: $F_{net} = Mg - F_M = Mg - BIL$, with $I = \mathcal{E}/R$. Faraday's law gives the emf induced in the bar: $\mathcal{E} = \frac{d\Phi}{dt} = BL\frac{dx}{dt}$, so the current is $I = \frac{BL(dx/dt)}{R} = \frac{BLv}{R}$. Substituting: $Ma = Mg - B\left(\frac{BLv}{R}\right)L = Mg - \frac{B^2L^2v}{R}$, so $a = g - \frac{B^2L^2v}{MR}$. Writing $a = dv/dt$ gives the final differential equation: $\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$. Points: 1 for correctly applying Newton's second law to the bar's motion; 1 for attempting to use Faraday's law to obtain an expression for the

# Solution 3

emf in the bar; 1 for correctly using the emf expression to obtain an expression for the current and substituting into Newton's second law; 1 for writing the acceleration as $dv/dt$ to state the final differential equation.

# Solution 3

## (d) Mark scheme

- At terminal velocity the net force is zero: $0 = Mg - \dfrac{B^2L^2v_T}{R}$, so $\dfrac{B^2L^2v_T}{R} = Mg$, giving $v_T = \dfrac{MgR}{B^2L^2}$. Points: 1 for setting the net force at terminal velocity equal to zero; 1 for an answer consistent with part (c), $v_T = \dfrac{MgR}{B^2L^2}$.

# Solution 3

## (e) Mark scheme

- Using $P = I^2 R$ with $I = \dfrac{BLv_T}{R}$: $P = \dfrac{B^2 L^2 v_T^2}{R}$ (equivalently, substituting $v_T = \dfrac{MgR}{B^2 L^2}$ gives $P = \dfrac{M^2 g^2 R}{B^2 L^2}$, the same result reached via the alternate route $Mg = BIL \Rightarrow I = \dfrac{Mg}{BL} \Rightarrow P = I^2 R = \dfrac{M^2 g^2 R}{B^2 L^2}$). Points: 1 for a correct or consistent substitution into an appropriate power equation ($P = I^2 R$ or $P = VI$), by either solution route.

# Solution 3

## (f) Mark scheme

- Starting from part (c)'s equation $\frac{dv}{dt} = g - \frac{B^2L^2v}{MR} = -\frac{B^2L^2}{MR}\left(v - \frac{MRg}{B^2L^2}\right)$, separate variables: $\frac{dv}{v - \frac{MRg}{B^2L^2}} = -\frac{B^2L^2}{MR}dt$. Integrating from $v' = 0$ to $v' = v(t)$ and $t' = 0$ to $t' = t$: $\ln\left[\frac{v(t) - \frac{MRg}{B^2L^2}}{-\frac{MRg}{B^2L^2}}\right] = -\frac{B^2L^2}{MR}t$. Solving for $v(t)$ gives the final answer: $v(t) = \frac{MgR}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)$. Using a trial solution in the differential equation and verifying its correctness is also acceptable. Points: 1 for attempting

# Solution 3

separation of variables; 1 for attempting to integrate with the correct limits or the correct constant of integration; 1 for the correct final answer,
$$v(t) = \frac{MgR}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right).$$