Past-paper walk-through

Kirchhoff's Junction Rule ▪ 11.7

eXercise

Questions in this set

- 2019 Set 1 Q2
- 2014 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)(i) Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

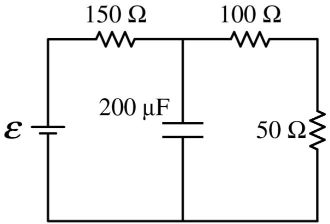
(a)(ii) Calculate the current in the $200\ \Omega$ resistor.

Question 2

(a)(iii) Calculate the power dissipated by the $200\ \Omega$ resistor.
[Figure 2: A circuit with a battery of emf $\mathcal{E}$, a $150\ \Omega$ resistor and $100\ \Omega$ resistor along the top in series from the battery's positive terminal, a $200\ \Omega$ resistor in the middle branch, a voltmeter V connected across the $200\ \Omega$ resistor, and a $50\ \Omega$ resistor in the right branch below the $100\ \Omega$ resistor.]
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

Question 2

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



Question 2

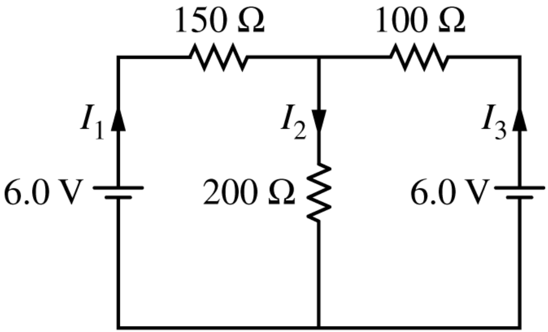


Figure 1

Question 2

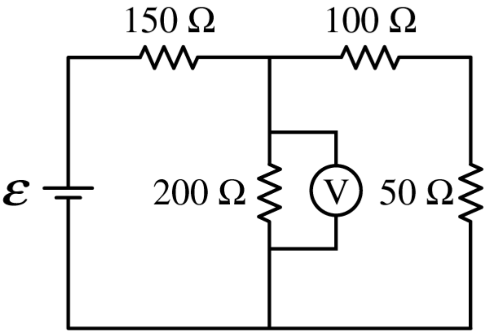


Figure 2

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(b) Calculate the current through the $50\ \Omega$ resistor.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(d)(i) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with a $200\ \mu$F capacitor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

Question 2

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu$F capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.
(d)(ii) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with an ideal 50 mH inductor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]
The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?

$G^{reaterthan}{}_{L}{}^{essthan}{}_{E}{}^{qualto}$

Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Using Kirchhoff's rules (do not solve): Junction rule at the node joining all three branches: $I_1 - I_2 + I_3 = 0$ (1 pt). Loop rule, left loop (through the left 6.0 V battery, the 150 ohm resistor, and the 200 ohm resistor): $6 - 150I_1 - 200I_2 = 0$ (1 pt). Loop rule, right loop (through the right 6.0 V battery, the 100 ohm resistor, and the 200 ohm resistor): $6 - 100I_3 - 200I_2 = 0$ (1 pt). (Equivalently, the outer loop $6 - 150I_1 + 100I_3 - 6 = 0$ may replace one of these; full credit is also earned for two correct loop equations written using loop currents.)

Solution 2

(a)(ii)

Mark scheme

- Combine the three equations from part (a)(i) (1 pt for combining them / using a calculator's system-solver, e.g. $I_1 - I_2 + I_3 = 0$, $-I_1 - 1.33I_2 = -0.04$, $-I_3 - 2I_2 = -0.06 \Rightarrow -4.33I_2 = -0.10$). Solving: $I_2 = \dfrac{-0.10}{-4.33} = 0.023$ A (1 pt for the correct answer with correct units).

Solution 2

(a)(iii)

Mark scheme

- $P = I_2^2 R = (0.023\ \mathrm{A})^2 (200\ \Omega) = 0.107$ W. 1 point for using a correct power equation and computing this value.

Solution 2

(b) Mark scheme

- In the modified circuit (Figure 2), the 100 ohm and 50 ohm resistors are in series in the right-hand branch, so their combined resistance is $R = 100\ \Omega + 50\ \Omega = 150\ \Omega$ (1 pt). This branch is in parallel with the 200 ohm resistor, across which the voltmeter reads 4.4 V, so that same 4.4 V is the potential difference across the 150 ohm series combination: $I = \frac{V}{R} = \frac{4.4\ \text{V}}{100\ \Omega + 50\ \Omega} = 0.029$ A (1 pt for a correct answer using the correct potential difference).

Solution 2

(c) Mark scheme

- Using $I_1 = I_2 + I_3$ (1 pt for the correct equation determining current through the 150 ohm resistor) and substituting $I_3 = 0.029$ A from part (b) with $I_2 = \dfrac{4.4\text{ V}}{200\ \Omega}$ (1 pt for correct substitution): $I_1 = \dfrac{4.4\text{ V}}{200\ \Omega} + 0.029\text{ A} = 0.051\text{ A}$. Then apply the loop equation through the battery/150-ohm branch (1 pt): $\mathcal{E} = I_1 R_1 + V_{200\Omega\text{ branch}} = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$. (Equivalent alternate method: find the circuit's total equivalent resistance $R_T = 150\ \Omega + \left(\frac{1}{200} + \frac{1}{150}\right)^{-1} \Omega = 236\ \Omega$, giving $\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) \approx 12.0\text{-}12.1\text{ V}$ – also full credit.)

Solution 2

(d)(i) Mark scheme

- At steady state, the fully-charged 200 uF capacitor carries no current, so the 150 ohm, 100 ohm, and 50 ohm resistors form a single series loop carrying the same current (1 pt for correctly finding the total series resistance $R_{tot} = 150 + 100 + 50 = 300\ \Omega$). Using the battery voltage found in part (c), $\mathcal{E} = 12.1$ V (1 pt for substituting this consistent voltage into Ohm's law): $I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{12.1 \text{ V}}{300\ \Omega} = 40.3$ mA.

Solution 2

(d)(ii)

Mark scheme

- Less than (1 pt for correctly selecting 'Less than' with an attempt at a relevant justification). Justification (1 pt): at steady state an ideal inductor acts as a short circuit (zero resistance / zero back-emf), so essentially all the current from the source flows through the (now zero-resistance) inductor branch and none flows through the 100-ohm + 50-ohm branch – so the current through the 50 ohm resistor is less than the 0.029 A found in part (b) (in fact it approaches zero).

Question 1

2014 ▪ Q1

[Circuit diagram: A variable DC power supply connects to an ammeter (A) in series, which connects to a node. From that node, resistor $R_2 = 50\ \Omega$ runs across the top back to the power supply, an unknown resistor R_1 runs down the middle in parallel, and resistor $R_3 = 50\ \Omega$ runs across the bottom, also back to the power supply. So R_1, R_2, and R_3 are all in parallel with each other, and this parallel combination is in series with the ammeter and the DC power supply.]

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

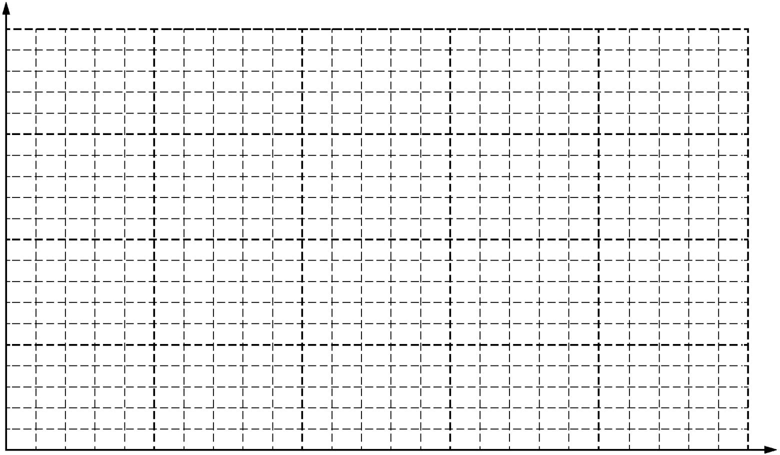
Question 1

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

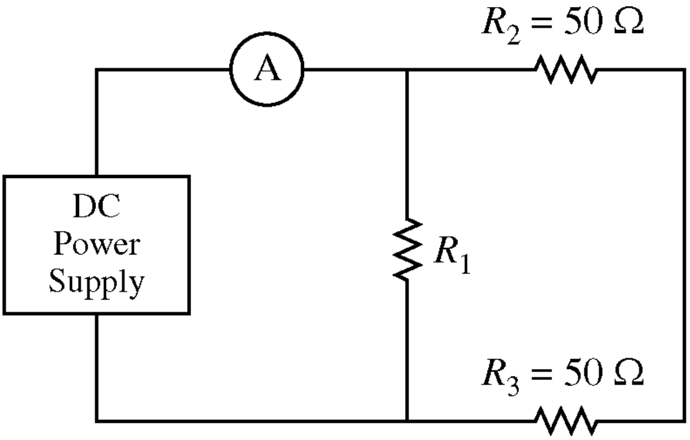
(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.

[Grid: a blank dashed-gridline graph with a vertical axis (arrow pointing up, unlabeled) and a horizontal axis (arrow pointing right, unlabeled), for the student to plot Current (mA) vs. Potential Difference (V) and draw a best-fit line.]

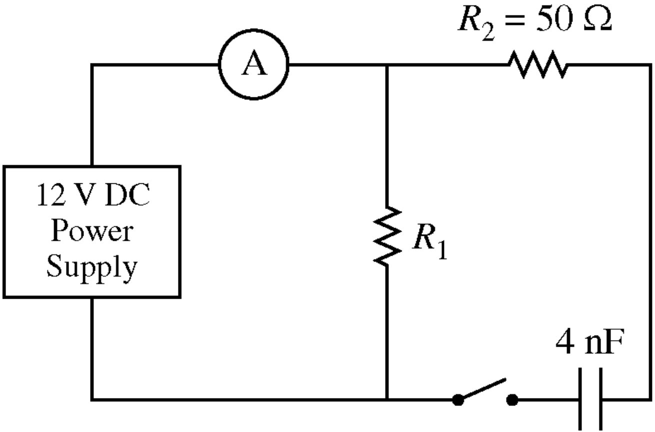
Question 1



Question 1



Question 1



Question 1

2014 ▪ Q1

[Circuit diagram: A variable DC power supply connects to an ammeter (A) in series, which connects to a node. From that node, resistor $R_2 = 50\ \Omega$ runs across the top back to the power supply, an unknown resistor R_1 runs down the middle in parallel, and resistor $R_3 = 50\ \Omega$ runs across the bottom, also back to the power supply. So R_1, R_2, and R_3 are all in parallel with each other, and this parallel combination is in series with the ammeter and the DC power supply.]

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

Question 1

(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

Question 1

2014 ▪ Q1

[Circuit diagram: A variable DC power supply connects to an ammeter (A) in series, which connects to a node. From that node, resistor $R_2 = 50\ \Omega$ runs across the top back to the power supply, an unknown resistor R_1 runs down the middle in parallel, and resistor $R_3 = 50\ \Omega$ runs across the bottom, also back to the power supply. So R_1, R_2, and R_3 are all in parallel with each other, and this parallel combination is in series with the ammeter and the DC power supply.]

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

| Potential Difference (V) | 2 | 4 | 6 | 8 | 10 | |—|—|—|—|—|—| | Current (mA) | 40 | 55 | 97 | 138 | 155 |

Question 1

(c) Calculate the value of R_1.
The power supply is now fixed at 12 V.

Question 1

2014 ▪ Q1

[Circuit diagram: A variable DC power supply connects to an ammeter (A) in series, which connects to a node. From that node, resistor $R_2 = 50\ \Omega$ runs across the top back to the power supply, an unknown resistor R_1 runs down the middle in parallel, and resistor $R_3 = 50\ \Omega$ runs across the bottom, also back to the power supply. So R_1, R_2, and R_3 are all in parallel with each other, and this parallel combination is in series with the ammeter and the DC power supply.]

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

Question 1

(d) Calculate the current through R_2.

Question 1

2014 ▪ Q1

[Circuit diagram: A variable DC power supply connects to an ammeter (A) in series, which connects to a node. From that node, resistor $R_2 = 50\ \Omega$ runs across the top back to the power supply, an unknown resistor R_1 runs down the middle in parallel, and resistor $R_3 = 50\ \Omega$ runs across the bottom, also back to the power supply. So R_1, R_2, and R_3 are all in parallel with each other, and this parallel combination is in series with the ammeter and the DC power supply.]

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

Question 1

(e)(i) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V. [Circuit diagram: A 12 V DC power supply connects through an ammeter (A) to a node; from that node $R_2 = 50\ \Omega$ runs across the top back to the supply, R_1 runs down the middle in parallel, and a branch containing an open switch in series with a 4 nF capacitor runs across the bottom back to the supply.]

i. Calculate the current in R_2 immediately after the switch is closed.

(e)(ii) ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d)?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

Question 1

2014 ▪ Q1

[Circuit diagram: A variable DC power supply connects to an ammeter (A) in series, which connects to a node. From that node, resistor $R_2 = 50\ \Omega$ runs across the top back to the power supply, an unknown resistor R_1 runs down the middle in parallel, and resistor $R_3 = 50\ \Omega$ runs across the bottom, also back to the power supply. So R_1, R_2, and R_3 are all in parallel with each other, and this parallel combination is in series with the ammeter and the DC power supply.]

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

Question 1

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i?

___ Greater than ___ Less than ___ Equal to

Justify your answer.