Past-paper walk-through

Electric Power ▪ 11.4

eXercise

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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2025 ▪ Q2

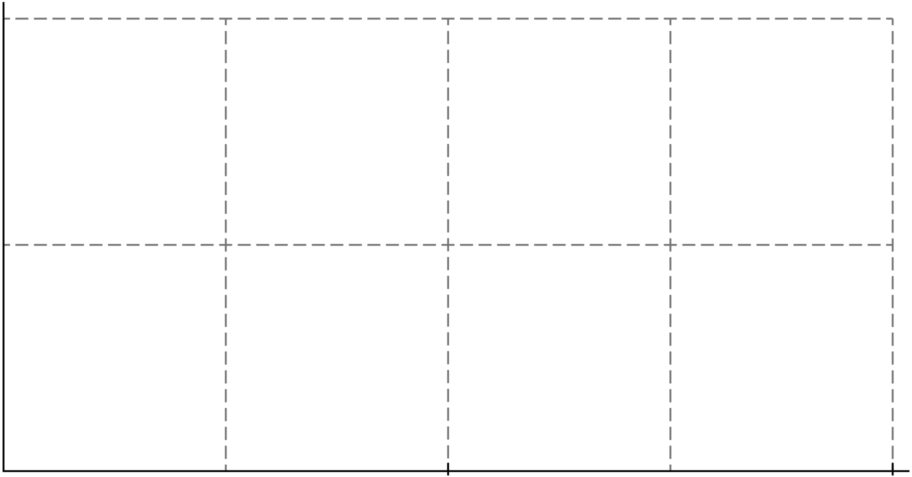
A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.
[Figure 1: A circular loop shown face-on with uniform magnetic field B (into the page, shown by an array of $\times$ symbols) filling the region around and through the loop. A dashed vertical line along a diameter of the loop is the rotation axis, with angular velocity ω indicated by a curved arrow at the top. A small 3D axis to the right shows $+y$ up, $+x$ to the right, and $+z$ out of the page (circle with a dot).]

Question 2

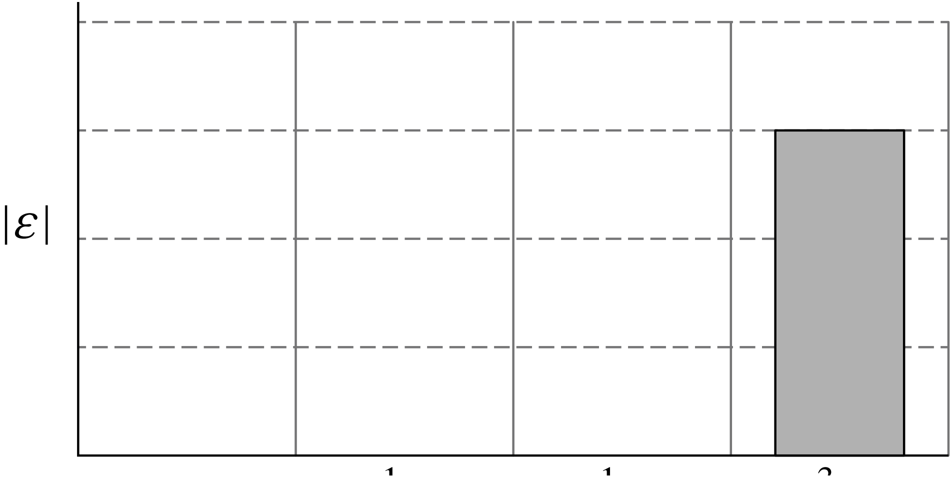
(a) The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \dfrac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\dfrac{1}{4}T$, and $\dfrac{1}{2}T$ relative to $|\varepsilon|$ shown at $\dfrac{3}{4}T$. If $|\varepsilon| = 0$, **write** a "0" in that column.

[Figure 2: A bar chart with vertical axis $|\varepsilon|$ and horizontal axis showing four columns labeled $t = 0$, $\frac{1}{4}T$, $\frac{1}{2}T$, $\frac{3}{4}T$. Only the $\frac{3}{4}T$ column already has a bar drawn, reaching to a height roughly midway up the chart; the other three columns are blank for the student to fill in.]

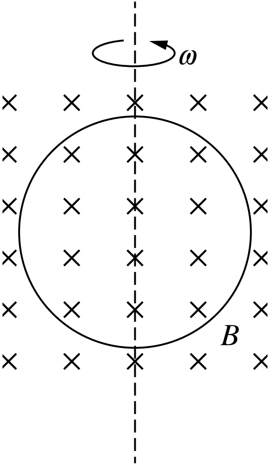
Question 2



Question 2



Question 2



Question 2

2025 ▪ Q2

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[Figure 1: A circular loop shown face-on with uniform magnetic field B (into the page, shown by an array of $\times$ symbols) filling the region around and through the loop. A dashed vertical line along a diameter of the loop is the rotation axis, with angular velocity ω indicated by a curved arrow at the top. A small 3D axis to the right shows $+y$ up, $+x$ to the right, and $+z$ out of the page (circle with a dot).]

Question 2

(b) Derive an expression for the maximum induced current in the loop in terms of A, R, B, ω, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2025 ▪ Q2

A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

[Figure 1: A circular loop shown face-on with uniform magnetic field B (into the page, shown by an array of $\times$ symbols) filling the region around and through the loop. A dashed vertical line along a diameter of the loop is the rotation axis, with angular velocity ω indicated by a curved arrow at the top. A small 3D axis to the right shows $+y$ up, $+x$ to the right, and $+z$ out of the page (circle with a dot).]

Question 2

(c) On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \leq t \leq T$.
[Figure 3: A blank set of axes with vertical axis labeled P and horizontal axis labeled t, origin O. Two vertical dashed reference lines are marked on the t-axis at positions labeled $\frac{1}{2}T$ and T.]

Question 2

2025 ▪ Q2

A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

[Figure 1: A circular loop shown face-on with uniform magnetic field B (into the page, shown by an array of $\times$ symbols) filling the region around and through the loop. A dashed vertical line along a diameter of the loop is the rotation axis, with angular velocity ω indicated by a curved arrow at the top. A small 3D axis to the right shows $+y$ up, $+x$ to the right, and $+z$ out of the page (circle with a dot).]

Question 2

(d) Indicate whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\varepsilon|$.

Solution 2

(a)

Mark scheme

- The bar chart shows $|\varepsilon| att = 3/4Talreadydrawn(height3units).Draw: abaratt = 1/4Twithheight3units(equaltothegiven3/4Tbar), andwrite'0'att = 0andatt = 1/2T(|\varepsilon|iszeroatthosetimes, when \cos(\omega t)isatanextremumsoitsderivative - -theemf- -iszero).Points: (1)abarat1/4T, (2)thatbarhasheight3units(matchingthe3/4Tbar$
$0att = 0andt = 1/2T.$

Solution 2

(b) Mark scheme

- Start from Faraday's law: $\varepsilon = -\frac{d\Phi_B}{dt} (signnotrequired). With \Phi_B = BA\cos(\omega t), \varepsilon = -\frac{d}{dt}[BA\cos(\omega t)] = BA\omega\sin(\omega t), so \varepsilon_{max} = BA\omega. Using Ohm's law, I = \varepsilon/R (or I = \Delta V/R with \Delta V = \varepsilon), so I_{max} = \varepsilon_{max}/R = \frac{BA\omega}{R}. Points: (1) writing \varepsilon = -d\Phi_B/dt, (2) correct expression for induced emf \varepsilon = BA\omega\sin(\omega t), (3) using Ohm's law I = \varepsilon/R, (4) correct expression for the maximum induced current I_{max} = BA\omega/R.$

Solution 2

(c) Mark scheme

- Sketch P vs t (power dissipated, P $\propto \varepsilon^2 \propto \sin^2(\omega t)$) :
$anapproximatelysinusoidalcurve(allhumpsabovetheaxis, sincepowerisnevernegative)thatcomplet$
$0 andt = T(since \sin^2 hashalftheperiodof \sin), startingattheorigin(P = 0 att =$
$0) withequalmaximumvaluesandequalminimumvalues(minimaatP = 0, occurringatt =$
$0, T/2, T; maximamidwaybetween). AcurvethatstartsandendsatP =$
$0 withasinglemaximumcanalsoearnthethirdpoint. Points :$
$(1) approximatelysinusoidalcurve, (2) exactlytwocyclesshownover 0 to T, (3) startsattheoriginwithe$

Solution 2

(d)

Mark scheme

- Yes, the graph in part C is consistent with the bars drawn in part A. Because P is proportional to $\varepsilon^2, Pismaximumwhenever|\varepsilon|ismaximum(att = 1/4Tand3/4T)andPiszero(minimum)whenever\varepsilon iszero(att = 0and1/2T) - -i.e., themaxima/minimaofthepart - CgraphalignwiththebarsofpartA.Points : (1)correctlystatingtherepresentationsareconsistent, (2)correctjustificationreferencingthatPisp$

Question 3

2024 Set 1 ▪ Q3

[Figure 1: An x-z coordinate system indicator (with $+z$ up, $+x$ to the right) is shown at top right. A rigid rectangular loop of width L and height $2L$ lies in the plane of the page, with corners forming a rectangle; the loop's leading (right) vertical edge is labeled with an arrow pointing right labeled v, indicating the loop moves in the $+x$-direction with constant speed v. Inside the loop near its left edge, a resistor R (zigzag symbol) is shown in the left vertical side, and point S is marked at the midpoint of the loop's right (leading) edge. To the right of the loop, the page is divided into two vertical field regions along the x-axis: "Region 1" spans from $x = L$ to $x = 2L$, shown with dots (indicating field out of the page, $+z$-direction); "Region 2" spans from $x = 2.5L$ to $x = 3.5L$, shown with alternating dot/cross symbols (two external uniform magnetic fields of equal magnitude B directed in opposite z-directions). The horizontal axis below is marked with tick labels $0, L, 2L, 3L, 4L$.]

Question 3

A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop first enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, but directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the loop's leading edge B that is directed at the horizontal boundary in Region 2 that separates the two magnetic fields.

(a) [Figure: A blank graph with vertical axis Φ (unlabeled scale) and horizontal axis x marked with gridlines and tick labels $0, L, 2L, 3L, 4L, 4.5L$ (position of Point S from $x = 0$ to $x = 4.5L$).]

Question 3

On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.

Question 3

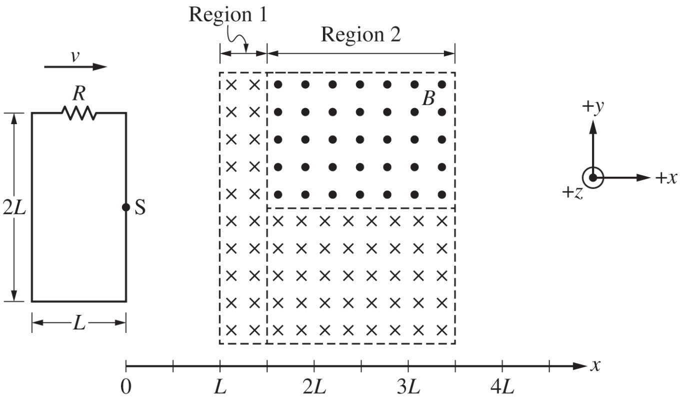


Figure 2

Question 3

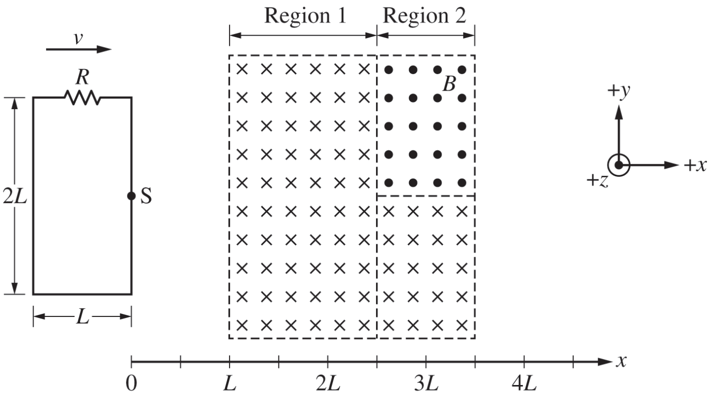
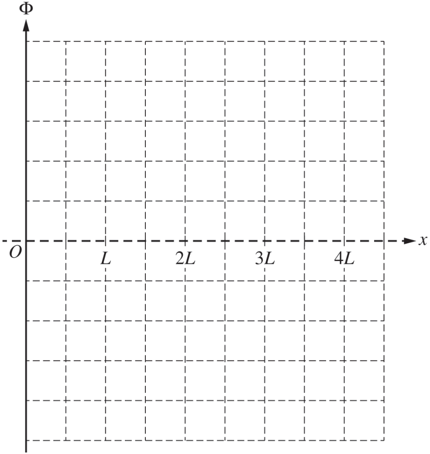


Figure 1

Question 3



Question 3

2024 Set 1 ▪ Q3

[Figure 1: An x-z coordinate system indicator (with $+z$ up, $+x$ to the right) is shown at top right. A rigid rectangular loop of width L and height $2L$ lies in the plane of the page, with corners forming a rectangle; the loop's leading (right) vertical edge is labeled with an arrow pointing right labeled v, indicating the loop moves in the $+x$-direction with constant speed v. Inside the loop near its left edge, a resistor R (zigzag symbol) is shown in the left vertical side, and point S is marked at the midpoint of the loop's right (leading) edge. To the right of the loop, the page is divided into two vertical field regions along the x-axis: "Region 1" spans from $x = L$ to $x = 2L$, shown with dots (indicating field out of the page, $+z$-direction); "Region 2" spans from $x = 2.5L$ to $x = 3.5L$, shown with alternating dot/cross symbols (two external uniform magnetic fields of equal magnitude B directed in opposite z-directions). The horizontal axis below is marked with tick labels $0, L, 2L, 3L, 4L$.]

A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with

Question 3

constant speed v in the $+x$-direction, as shown in Figure 1. The loop first enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, but directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the loop's leading edge B that is directed at the horizontal boundary in Region 2 that separates the two magnetic fields.

(b)(i) Consider the instant when Point S reaches $x = 1.5L$.

Indicate whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

Question 3

(b)(ii) Derive an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

(b)(iii) Derive an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: An x-z coordinate system indicator (with $+z$ up, $+x$ to the right) is shown at top right. A rigid rectangular loop of width L and height $2L$ lies in the plane of the page, with corners forming a rectangle; the loop's leading (right) vertical edge is labeled with an arrow pointing right labeled v, indicating the loop moves in the $+x$-direction with constant speed v. Inside the loop near its left edge, a resistor R (zigzag symbol) is shown in the left vertical side, and point S is marked at the midpoint of the loop's right (leading) edge. To the right of the loop, the page is divided into two vertical field regions along the x-axis: "Region 1" spans from $x = L$ to $x = 2L$, shown with dots (indicating field out of the page, $+z$-direction); "Region 2" spans from $x = 2.5L$ to $x = 3.5L$, shown with alternating dot/cross symbols (two external uniform magnetic fields of equal magnitude B directed in opposite z-directions). The horizontal axis below is marked with tick labels $0, L, 2L, 3L, 4L$.]

A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with

Question 3

constant speed v in the $+x$-direction, as shown in Figure 1. The loop first enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, but directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the loop's leading edge B that is directed at the horizontal boundary in Region 2 that separates the two magnetic fields.

(c) The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

Question 3

[Figure 2: Same setup as Figure 1, but Region 1 (dots, $+z$ field out of page) now spans from $x = L$ to $x = 1.5L$, and Region 2 (alternating dot/cross field) spans from $x = 1.5L$ to $x = 2.5L$. The loop, resistor R, and point S are shown in the same configuration as before, moving in $+x$ with speed v. Axis tick labels: $0, L, 2L, 3L, 4L$.]
Indicate whether E_{new} is greater than, less than, or equal to E_{original}.
______$E_{\text{new}} > E_{\text{original}}$ ______$E_{\text{new}} < E_{\text{original}}$ ______$E_{\text{new}} = E_{\text{original}}$
Briefly **justify** your answer.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: An x-z coordinate system indicator (with $+z$ up, $+x$ to the right) is shown at top right. A rigid rectangular loop of width L and height $2L$ lies in the plane of the page, with corners forming a rectangle; the loop's leading (right) vertical edge is labeled with an arrow pointing right labeled v, indicating the loop moves in the $+x$-direction with constant speed v. Inside the loop near its left edge, a resistor R (zigzag symbol) is shown in the left vertical side, and point S is marked at the midpoint of the loop's right (leading) edge. To the right of the loop, the page is divided into two vertical field regions along the x-axis: "Region 1" spans from $x = L$ to $x = 2L$, shown with dots (indicating field out of the page, $+z$-direction); "Region 2" spans from $x = 2.5L$ to $x = 3.5L$, shown with alternating dot/cross symbols (two external uniform magnetic fields of equal magnitude B directed in opposite z-directions). The horizontal axis below is marked with tick labels $0, L, 2L, 3L, 4L$.]

A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with

Question 3

constant speed v in the $+x$-direction, as shown in Figure 1. The loop first enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, but directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the loop's leading edge B that is directed at the horizontal boundary in Region 2 that separates the two magnetic fields.

(d) [Figure 3: A right-triangular loop with base L and height $2L$ is shown, with a resistor R on its vertical left side and point S at the lower-right corner (lower leading corner) of the loop. To the right, the region $L < x < 3.5L$ contains a uniform magnetic field of magnitude B shown with cross symbols (directed in the $-z$-direction). The loop moves in the $+x$-direction with speed v. Axis tick labels: $0, L, 2L, 3L, 4L$.]

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

Question 3

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.
[Figure: A blank graph with vertical axis I_S (unlabeled scale) and horizontal axis x marked with gridlines and tick labels $0, L, 2L, 3L, 4L$.]
On the following axes, **sketch** a graph of the induced current I_S in the triangular loop as Point S moves from $x = L$ to $x = 3L$.

Solution 3

(a) Mark scheme

- Sketch magnetic flux Φ through the rectangular loop versus position x of Point S, from $x = 0$ to $x = 4.5L$, 4 points total (1 pt each region): (1) flux is zero for $0 \leq x \leq L$ and for $x > 3.5L$; (2) the absolute value of the flux increases with a constant slope for $L < x < 2L$; (3) the flux is constant and nonzero for $2L < x < 2.5L$; (4) the absolute value of the flux decreases from its nonzero value at $x = 2.5L$ to zero at $x = 3.5L$. (A sketch reflected across the horizontal axis also earns full credit; the magnitudes of the slopes in the ramp regions are not scored.) Shape: flat at zero from 0 to L, ramps to a (negative, since $+z$ is $+\Phi$ and the field here is into the page) plateau between roughly 2L-2.5L, ramps back to zero by 3.5L, flat afterward to 4.5L.

Solution 3

(b)(i)

Mark scheme

- Select Counterclockwise, with an attempt at justification (1 pt). The justification must indicate either that the magnetic flux through the loop is increasing in the $-z$-direction, OR that the magnetic field due to the induced current will be directed in the $+z$-direction (1 pt). Reasoning: by Lenz's law the induced current opposes the increasing $-z$ flux by creating a field in $+z$ inside the loop, which (by the right-hand rule) requires a counterclockwise current.

Solution 3

(b)(ii) Mark scheme

- Derive I_R using Faraday's law, 3 points: (1) a multistep derivation that includes Faraday's law, $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA)$ (1 pt; the point is earned even if the negative sign is omitted); (2) indicate that $\frac{dA}{dt} = 2Lv$ (1 pt; sign not required) since the loop's covered area within the field grows at rate $2L \cdot v$ as Point S advances; (3) apply Ohm's law to obtain an expression for I_R consistent with the derived emf (1 pt; sign not required). Result: $\mathcal{E} = B\frac{dA}{dt} = 2BLv$, so $I_R = \frac{2BLv}{R}$.

Solution 3

(b)(iii) Mark scheme

- Derive the dissipated power P: (1) use a correct general expression for power, $P = I^2 R$ or $P = \frac{(\Delta V)^2}{R}$ or $P = I\Delta V$ (1 pt); (2) substitute the I_R expression from part (b)(ii) to write P in terms of the given quantities R, L, v, B only, consistently with that earlier response (1 pt). Result: $P = \frac{(2BLv)^2}{R} = \frac{4B^2L^2v^2}{R}$.

Solution 3

(c) Mark scheme

- Select $E_{\text{new}} < E_{\text{original}}$, with an attempt at a relevant justification (1 pt). The justification must indicate one of: the change in magnetic flux is greater for the original scenario, the induced current occurs for a greater amount of time in the original scenario, or the induced emf occurs for a greater amount of time in the original scenario (1 pt). Reasoning: shifting the field-region boundary to $x = 1.5L$ reduces the flux change and/or the duration over which emf/current is induced compared to the original setup, so less total energy is dissipated in the resistor: $E_{\text{new}} < E_{\text{original}}$.

Solution 3

(d)

Mark scheme

- Sketch the magnitude of the induced current I_T versus x for the triangular-loop scenario with the field region now $L < x < 3.5L$: (1) the sketch's absolute value only increases (e.g., roughly linearly, reflecting the growing chord length of the triangle) from $x = L$ to $x = 2L$ (1 pt); (2) the sketch is zero from $x = 2L$ to $x = 3L$ (the loop is fully inside the field region with constant flux) (1 pt). (A response reflected across the horizontal axis earns both points; any portion of the graph before $x = L$ or after $x = 3L$ is not scored.)

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A diagram on the left shows a square loop of side length D approaching a rectangular field region from the left, with $+x$ to the right and $+y$ upward indicated by a small axis. The loop has a resistor R on its left side and width D, moving right with velocity v in the $+x$ direction. To the right is a number line marked $0, D, 2D, 3D, 4D$. Above the number line, two adjacent rectangular field regions are shown: "Region 1" spans from $x = D$ to $x = 2D$, containing an array of small circles with dots (field out of the page, magnitude B); "Region 2" spans from $x = 2D$ to $x = 3D$, containing an array of "$\times$" symbols (field into the page, magnitude $2B$). Point S is the leading (right) edge of the loop.]

A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop enters Region 1

Question 3

of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $-z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.

[Blank graph: vertical axis labeled Φ, horizontal axis labeled x with tick marks at D, $2D$, $3D$, $4D$, gridlines shown, origin labeled O. No curve plotted; axes are empty for the student to sketch.]

Question 3

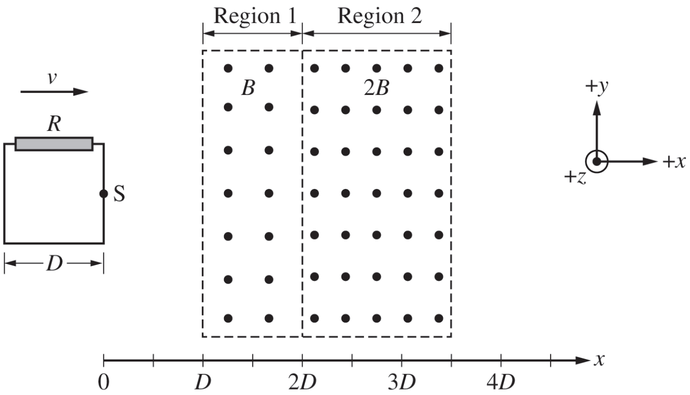


Figure 1

Question 3

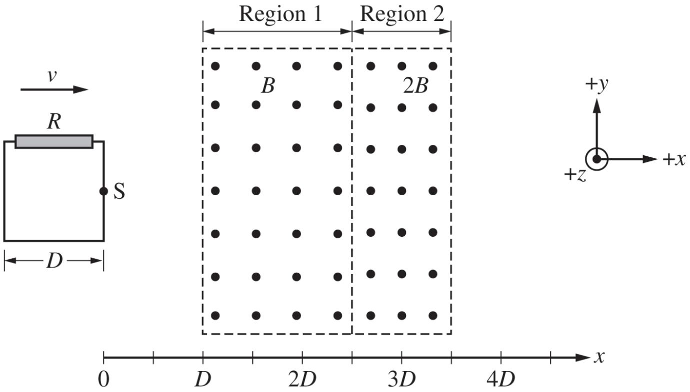
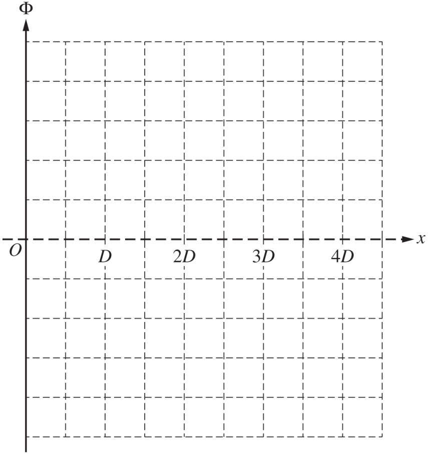


Figure 2

Question 3



Question 3

2024 Set 2 ▪ Q3

[Figure 1: A diagram on the left shows a square loop of side length D approaching a rectangular field region from the left, with $+x$ to the right and $+y$ upward indicated by a small axis. The loop has a resistor R on its left side and width D, moving right with velocity v in the $+x$ direction. To the right is a number line marked $0, D, 2D, 3D, 4D$. Above the number line, two adjacent rectangular field regions are shown: "Region 1" spans from $x = D$ to $x = 2D$, containing an array of small circles with dots (field out of the page, magnitude B); "Region 2" spans from $x = 2D$ to $x = 3D$, containing an array of "$\times$" symbols (field into the page, magnitude $2B$). Point S is the leading (right) edge of the loop.]

A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$

Question 3

that is directed in the $-z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(b)(i) Consider the instant when Point S reaches $x = 1.5D$.

Indicate whether the current I_0 that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_______ Clockwise _______ Counterclockwise _______ Zero

Briefly **justify** your answer.

(b)(ii) Derive an expression for I_0 when Point S reaches $x = 1.5D$. If $I_0 = 0$, indicate how the derived expression shows that $I_0 = 0$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

(b)(iii) Derive an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A diagram on the left shows a square loop of side length D approaching a rectangular field region from the left, with $+x$ to the right and $+y$ upward indicated by a small axis. The loop has a resistor R on its left side and width D, moving right with velocity v in the $+x$ direction. To the right is a number line marked $0, D, 2D, 3D, 4D$. Above the number line, two adjacent rectangular field regions are shown: "Region 1" spans from $x = D$ to $x = 2D$, containing an array of small circles with dots (field out of the page, magnitude B); "Region 2" spans from $x = 2D$ to $x = 3D$, containing an array of "$\times$" symbols (field into the page, magnitude $2B$). Point S is the leading (right) edge of the loop.]

A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$

Question 3

that is directed in the $-z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(c) The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted, so the boundary is at $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new}.

[Figure 2: Same layout as Figure 1, but the boundary between Region 1 (dots, field out of page, magnitude B) and Region 2 ($\times$'s, field into page, magnitude $2B$) is now at $x = 2.5D$; Region 1 spans $x = D$ to $x = 2.5D$, Region 2 spans $x = 2.5D$ to $x = 3.5D$ (approximately). Number line marked $0, D, 2D, 3D, 4D$. Square loop with resistor R and side length D shown at the left, moving in $+x$ direction with speed v.]

Question 3

Indicate whether E_{new} is greater than, less than, or equal to E_{original}.
$\text{E}_{\text{new}} > E_{\text{original}}$ $\text{E}_{\text{new}} < E_{\text{original}}$ $\text{E}_{\text{new}} = E_{\text{original}}$
Briefly **justify** your answer.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A diagram on the left shows a square loop of side length D approaching a rectangular field region from the left, with $+x$ to the right and $+y$ upward indicated by a small axis. The loop has a resistor R on its left side and width D, moving right with velocity v in the $+x$ direction. To the right is a number line marked $0, D, 2D, 3D, 4D$. Above the number line, two adjacent rectangular field regions are shown: "Region 1" spans from $x = D$ to $x = 2D$, containing an array of small circles with dots (field out of the page, magnitude B); "Region 2" spans from $x = 2D$ to $x = 3D$, containing an array of "$\times$" symbols (field into the page, magnitude $2B$). Point S is the leading (right) edge of the loop.]

A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$

Question 3

that is directed in the $-z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

(d) The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field B that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D, as shown in Figure 3. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the upper-left corner of the loop.

[Figure 3: A right triangular loop with a resistor R on its vertical left side of height D, base length D along the bottom, and hypotenuse sloping from the top-left down to the bottom-right, with Point S marked at the top-left corner. The triangle moves in the $+x$ direction with speed v. To the right, a single rectangular field region spans from

Question 3

$x = D$ to $x = 3.5D$, filled with dots (field out of the page, magnitude B). Number line marked $0, D, 2D, 3D, 4D$, with small axis indicator showing $+x$ and $+z$ directions.]
On the following axes, **sketch** a graph of the induced current I_2 in the loop as Point S moves from $x = D$ to $x = 3D$.
[Blank graph: vertical axis labeled I_2, horizontal axis with tick marks at D, $2D$, $3D$, $4D$; shaded vertical bands appear at the far left (near 0–D) and in the middle region (around $2D$–$3D$) as reference/shading, with the rest of the axes empty for the student to sketch a curve.]

Solution 3

(a) Mark scheme

- Sketch of magnetic flux Φ vs. position x (from $x = 0$ to $x = 4.5D$): flux is zero until $x = D$; from $x = D$ to $x = 2D$, $|\Phi|$ increases with a constant slope; the curve continues smoothly (continuous slope, no kink) at $x = 2D$ and keeps increasing with that same slope until $x = 3D$; Φ is then constant and nonzero for $3D < x < 3.5D$; finally $|\Phi|$ decreases from that nonzero value at $x = 3.5D$ down to zero at $x = 4.5D$. (A graph reflected across the horizontal axis also earns full credit; only the magnitudes/shape of the slopes over the full regions $D < x < 3D$ and $3.5D < x < 4.5D$ matter, not their exact values.) 1 point each for: the initial linear rise from $x = D$ to $2D$; continuity with the same slope from $2D$ to $3D$; the

Solution 3

constant nonzero plateau from $3D$ to $3.5D$; the decrease from $3.5D$ to zero at $4.5D$.

Solution 3

(b)(i)

Mark scheme

- Clockwise. Justification: as Point S reaches $x = 1.5D$, the magnetic flux through the loop is increasing in the $+z$-direction. By Lenz's law, the induced current opposes this increase, so it creates a magnetic field in the $-z$-direction inside the loop; by the right-hand rule this requires the induced current to flow clockwise. 1 point for selecting Clockwise with an attempted relevant justification, 1 point for indicating the flux is increasing in $+z$ (or, alternately, that the induced current's own field points in $-z$).

Solution 3

(b)(ii) Mark scheme

- Apply Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA)$. Since the loop has side length D and moves at speed v, the area swept per unit time is $\frac{dA}{dt} = Dv$, giving $\mathcal{E} = -B\frac{dA}{dt} = -BDv$. By Ohm's law, $I_S = \mathcal{E}/R$, so $I_0 = I_S = -\frac{BDv}{R}$ (magnitude BDv/R). 1 point for a multistep derivation that includes Faraday's law, 1 point for indicating $dA/dt = Dv$, 1 point for using Ohm's law to get an expression for I_S consistent with the derived emf (negative sign not required for any of the three points).

Solution 3

(b)(iii)

Mark scheme

- Use a correct general power expression, e.g. $P = I^2 R$, and substitute the current found in part (b)(ii), $I_S = -BDv/R$: $P = \left(-\frac{BDv}{R}\right)^2 R = \frac{B^2 D^2 v^2}{R}$. 1 point for using a correct general expression for P ($P = I^2 R$, $P = (\Delta V)^2/R$, or $P = I\Delta V$), 1 point for an expression for P consistent with the part (b)(ii) result, expressed only in terms of R, D, v, B (and constants).

Solution 3

(c) Mark scheme

- $E_{\text{new}} = E_{\text{original}}$. Justification: the total change in magnetic flux through the loop over the full transit (from $x = 0$ to $x = 4.5D$) is the same in both scenarios — shifting the region boundary only changes when the flux changes, not how much it changes overall — so the induced emf and current profile produce the same total energy dissipated by the resistor in both cases. 1 point for selecting $E_{\text{new}} = E_{\text{original}}$ with an attempted justification, 1 point for indicating one of: the change in magnetic flux is the same for both scenarios / the induced current occurs for the same total time in both scenarios / the induced emf occurs for the same total time in both scenarios.

Solution 3

(d) Mark scheme

- Sketch of induced current magnitude I_T in the new triangular loop as it moves through the field region $D < x < 3.5D$ (only the region $x = D$ to $x = 3D$ is scored): $|I_T|$ starts at a nonzero value at $x = D$ and only decreases (no local increase) until it reaches zero at $x = 2D$ (as the triangle's widening base continues sweeping into the uniform field, the rate of flux change — and hence the induced current — decreases as the loop approaches being fully inside the field). I_T is then zero from $x = 2D$ to $x = 3D$, since the loop is entirely within the uniform field there and the flux through it is constant. (A graph reflected across the horizontal axis also earns full credit; portions before $x = D$ or after $x = 3D$ are not scored.) 1

Solution 3

point for a sketch whose absolute value only decreases from $x = D$ to $x = 2D$, 1 point for a sketch that is zero from $x = 2D$ to $x = 3D$.

Question 3

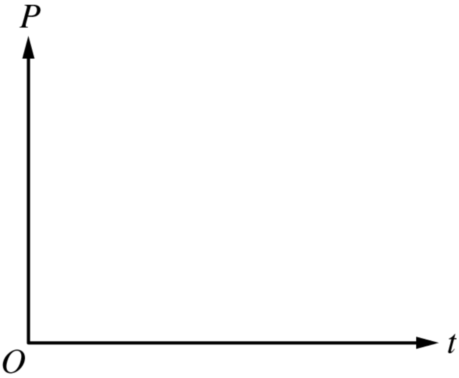
2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is

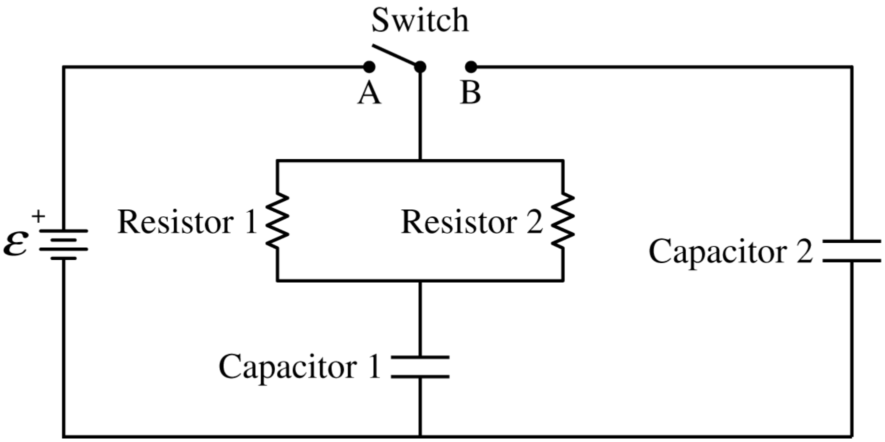
Question 3

closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

Question 3



Question 3


Switch
A
B
+
$\mathcal{E}$
Resistor 1
Resistor 2
Capacitor 2
Capacitor 1

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.

Question 3

[Graph 1: Blank axes, vertical axis σ, horizontal axis t, origin O, for the student to sketch surface charge density vs. time.]
[Graph 2: Blank axes, vertical axis P, horizontal axis t, origin O, for the student to sketch total power dissipated vs. time.]
A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(c)(i) At time t_1, the switch is closed to Position B.

Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

Question 3

(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
(c)(iii) Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(d) With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

Question 3

Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(e)(i) With the capacitors still charged as in part (d), the switch is now closed to Position A.

Question 3

Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.
(e)(ii) With the capacitors still charged as in part (d), the switch is now closed to Position A.
Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

Solution 3

(a) Mark scheme

- Loop rule around the circuit with Capacitor 1 and the two parallel resistors (each of resistance R, combining to an equivalent resistance $R/2$):
  $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$ (1 pt, for a loop-rule expression that includes terms for the equivalent resistance $R/2$ and the potential difference across the battery).
  Writing the capacitor voltage as $\Delta V_C = Q/C$ and the resistor-pair voltage as $\Delta V_{R,eq} = \dfrac{R}{2}\dfrac{dQ}{dt}$ (1 pt, for an expression that includes charge Q in the capacitor-voltage term and dQ/dt in the resistor-voltage term) gives the final differential equation: $\mathcal{E} - \dfrac{Q}{C} - \dfrac{R}{2}\dfrac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Two sketches vs. time t, from $t = 0$ until steady state is nearly reached: the surface charge density σ on the positive plate of Capacitor 1 increases and is concave down (rises quickly, then levels off toward a maximum) (1 pt); the total power P dissipated by the resistors decreases and is concave up (falls quickly, then levels off toward zero) (1 pt); both curves approach a slope of zero as time increases, i.e. both flatten out as steady state is approached (1 pt — this point can be earned even if the first two points are not).

Solution 3

(c)(i) Mark scheme

- The current in the switch immediately after t_1 is directed toward the right. Justification: while the switch was closed to Position A, Capacitor 1 charged so that its top plate accumulated positive charge (and the bottom plate negative), making the top plate's electric potential higher than the bottom plate's; immediately after moving the switch to Position B, current flows from high to low potential, i.e., away from the top plate through the switch toward the right (into the second loop with Capacitor 2), so conventional current is directed toward the right. 1 point for this indication with justification along the lines of the accumulated plate charge or the relative potentials of the plates.

Solution 3

(c)(ii) Mark scheme

- The total charge on the positive plate of Capacitor 2 a long time after t_1 is $\frac{2}{3}Q_0$. Reasoning: once connected in the second loop, Capacitor 1 and Capacitor 2 reach the same potential difference across them; since Capacitor 2 has twice the capacitance of Capacitor 1, it stores twice as much charge as Capacitor 1 in that final state. By conservation of the original charge Q_0 (all of which was on Capacitor 1 before t_1), splitting it in a 1:2 ratio between Capacitor 1 and Capacitor 2 gives Capacitor 2 a final charge of $\frac{2}{3}Q_0$ (and Capacitor 1 a final charge of $\frac{1}{3}Q_0$). 1 point; can be earned without supporting calculations.

Solution 3

(c)(iii) Mark scheme

- The energy dissipated by resistors 1 and 2 equals the decrease in total capacitor stored energy: $E_R = U_{0C} - U_C$, i.e., $\Delta E_R = U_C - U_{0C}$ with $E_R = -\Delta E_R$ (1 pt, for indicating the total energy dissipated is the difference between the initial electric PE stored at $t = t_1$ and the final electric PE stored once the new steady state is reached). Before t_1, only Capacitor 1 stores nonzero energy, $U_{0C} = U_{01} = \dfrac{1}{2}\dfrac{Q_0^2}{C}$; after the new steady state, both capacitors store nonzero energy, $U_C = U_1 + U_2$ (1 pt, for this indication, or an alternative consistent with part (c)(ii)). Substituting the steady-state charges from part (c)(ii), $Q_1 = Q_0/3$ on Capacitor 1 (capacitance C) and $Q_2 = 2Q_0/3$ on Capacitor 2 (capacitance $2C$) (1 pt, for

Solution 3

correct substitutions consistent with (c)(ii)): $\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\frac{1}{2C}\left(\frac{2Q_0}{3}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \frac{Q_0^2}{6C} - \frac{1}{2}\frac{Q_0^2}{C} = -\frac{Q_0^2}{3C}$. So the resistors dissipate a total energy of $\frac{Q_0^2}{3C}$.

Solution 3

(d) Mark scheme

- $\frac{U_2}{U_1} = 1$. Justification (each part 1 pt): after the new steady state is reached, the potential difference across each capacitor is the same (equivalently, the charge stored on each capacitor is the same), $\Delta V_1 = \Delta V_2$ or $Q_1 = Q_2$ (1 pt, indicating equal potential difference or equal charge across the two capacitors in steady state); and since capacitance is inversely related to plate separation, doubling Capacitor 2's plate separation halves its capacitance, making it equal to Capacitor 1's capacitance in this new configuration (1 pt, recognizing the capacitances are

Solution 3

now equal). Then, since $U_C = \dfrac{1}{2}C(\Delta V)^2$ with equal C and equal ΔV for both capacitors, $\dfrac{U_2}{U_1} = 1$.

Solution 3

(e)(i) Mark scheme

- With the switch closed to Position A again (capacitors 1 and 2, each already charged as in part (d), now effectively act together across the battery loop with the pair of resistors), the loop rule is $\mathcal{E} - \Delta V_R - \Delta V_C = 0$ (1 pt, for a loop rule including terms for the battery emf, the potential difference across the pair of resistors, and the potential difference across Capacitor 1 / the capacitor combination). Using $\mathcal{E} = Q_0/C$, $\Delta V_R = I(R/2)$, and $\Delta V_C = Q_0/(2C)$ (the steady-state, equal charge/capacitor voltage from part (d)): $\dfrac{Q_0}{C} - I\dfrac{R}{2} - \dfrac{Q_0}{2C} = 0 \Rightarrow \dfrac{Q_0}{2C} = I\dfrac{R}{2} \Rightarrow \dfrac{Q_0}{C} = IR \Rightarrow I_0 = \dfrac{Q_0}{RC}$ (1 pt, for the correct answer $I_0 = Q_0/(RC)$).

Solution 3

(e)(ii)

Mark scheme

- A long time after the switch is closed to Position A, the capacitors reach a new steady state (fully charged) and, in DC steady state, no current flows into or out of a fully-charged capacitor branch. Therefore $I_\infty = 0$. 1 point for indicating the current is zero.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

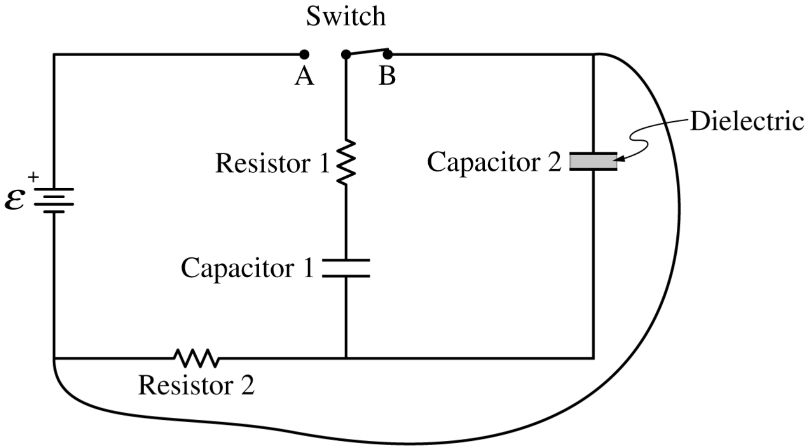
The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

Question 3

At time $t = 0$, the switch is closed to Position A.

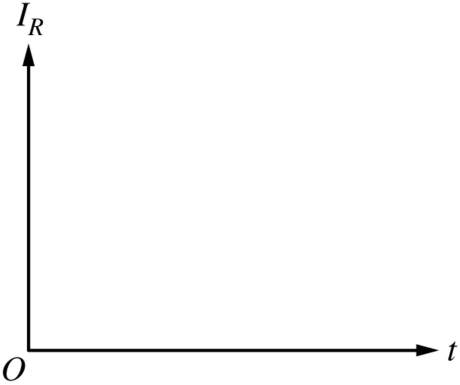
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

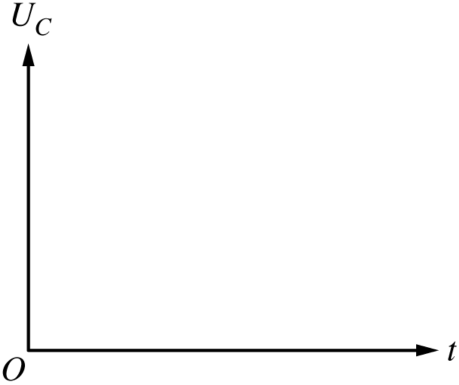
Question 3



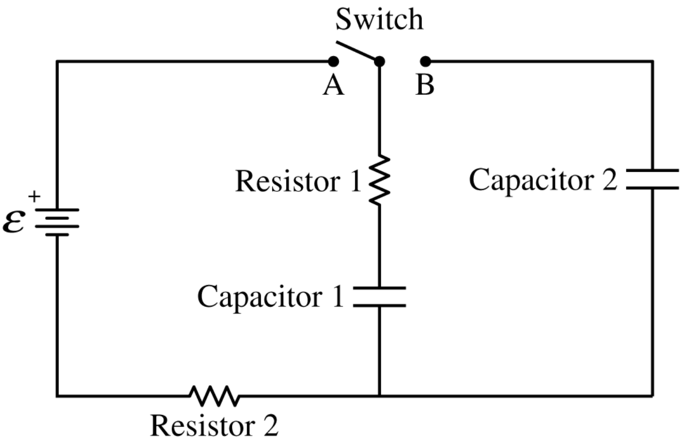
Switch
A
B
Dielectric
Resistor 1
Capacitor 2
$\mathcal{E}$
+
Capacitor 1
Resistor 2

Question 3





Question 3



Switch
A
B
Resistor 1
Capacitor 2
+
$\mathcal{E}$
Capacitor 1
Resistor 2

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.
[Graph 1: y-axis labeled "I_R", x-axis labeled "t", origin at O, no numeric scale shown.]
[Graph 2: y-axis labeled "U_C", x-axis labeled "t", origin at O, no numeric scale shown.]
A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.
At time t_1, the switch is closed to Position B.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]
The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(c)(i) Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.
(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
(c)(iii) Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.
With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
[Figure: The same circuit as before (battery $\mathcal{E}$, switch positions A/B, Resistor 1, Capacitor 1, Resistor 2, Capacitor 2), now with a dielectric slab labeled "Dielectric" shown inserted between the plates of Capacitor 2. An additional wire of negligible resistance is shown connected between two corners of the circuit (from the node between the switch and Resistor 1/Capacitor 1, curving around to the node below Capacitor 2/Resistor 2), as described below.]
With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(e)(i) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.
(e)(ii) Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

Solution 3

(a) Mark scheme

- Write, but do NOT solve, the differential equation for charge Q on the positive plate of Capacitor 1 vs time t after the switch closes to Position A, in terms of $\mathcal{E}, R, C, Q, t$. (1 point) Loop-rule expression that includes terms for the equivalent resistance $2R$ (the pair of resistors) and the potential difference across the battery. (1 point) An expression that includes charge Q in the term for the potential difference across the capacitor, and charge per unit time $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Result: $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0 \Rightarrow \mathcal{E} - \frac{Q}{C} - 2R\frac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 vs time t from $t = 0$ until steady state is nearly reached (RC charging curves). (1 point) I_R vs t is a CONCAVE UP, DECREASING curve starting at its maximum value at $t = 0$. (1 point) U_C vs t is a CONCAVE DOWN, INCREASING curve starting at zero. (1 point, can be earned even if the first two are not) Both curves approach a slope of zero (level off / asymptote) as time increases, consistent with exponential charging behavior.

Solution 3

(c)(i) Mark scheme

- Immediately after t_1 (switch moved to Position B, connecting the charged Capacitor 1 through Resistor 1 to the initially uncharged Capacitor 2), the current in Resistor 1 is directed UP. Justification (either form acceptable): positive charge has accumulated on the top plate of Capacitor 1 (and/or negative charge on its bottom plate) from being charged while the switch was at Position A, so at t_1 the higher potential top plate drives current up through Resistor 1 and down through Capacitor 2 to charge it. Equivalently: the top plate of Capacitor 1 is at higher electric potential than the bottom plate, so current flows from high to low potential, i.e. up through Resistor 1, around the loop through Capacitor 2.

Solution 3

(c)(ii) Mark scheme

- Determine the total charge on the positive plate of Capacitor 2 a long time after t_1: the answer is $\frac{Q_0}{2}$ (this point can be earned without supporting calculations). Justification: at new steady state, the potential difference across Capacitor 1 equals that across Capacitor 2 (parallel connection via Resistor 1 with no current, so no voltage drop across R1). Since Capacitor 2 has the same capacitance as Capacitor 1, it stores the same charge. By conservation of the total charge Q_0 originally on Capacitor 1, the two capacitors split it evenly, so Capacitor 2 stores $Q_0/2$.

Solution 3

(c)(iii) Mark scheme

- Derive an expression for the total energy dissipated by Resistor 1 from immediately after t_1 until the new steady state, in terms of C, Q_0. (1 point) Indicate that the energy dissipated equals the difference between the initial electric potential energy (at $t = t_1$) and the final electric potential energy (after new steady state): $\Delta E_R = U_C - U_{0C}$. (1 point) Indicate that only Capacitor 1 stores nonzero PE initially, and BOTH capacitors store PE after the new steady state (or an alternate response consistent with part (c)(ii)): $U_{0C} = U_{01}$ AND $U_C = U_1 + U_2$. (1 point) Correct substitution of the charges from part (c)(ii) ($Q_0/2$ on each capacitor). Full solution: $\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \dfrac{Q_0^2}{4C} - \dfrac{Q_0^2}{2C} = -\dfrac{Q_0^2}{4C}$. The

Solution 3

magnitude of energy dissipated by Resistor 1 is $\dfrac{Q_0^2}{4C}$ (the negative sign reflects the decrease in stored electric PE, which is dissipated as heat).

Solution 3

(d) Mark scheme

- Determine the charge on the positive plate of Capacitor 2 a long time after a dielectric ($\kappa = 2$) is inserted between its plates, with the switch still at Position B: $Q_2 = \dfrac{2Q_0}{3}$ (this point can be earned without supporting calculations). Derivation: with $C_1 = C$ and $C_2 = \kappa C = 2C$, the plates are still connected so $\Delta V_{C1} = \Delta V_{C2} \Rightarrow \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_1 = \frac{1}{2}Q_2$. Total charge is conserved: $Q_1 + Q_2 = Q_0 \Rightarrow \frac{1}{2}Q_2 + Q_2 = Q_0 \Rightarrow Q_2 = \dfrac{2}{3}Q_0$.

Solution 3

(e)(i) Mark scheme

- Derive an expression for the current in Resistor 2 immediately after an additional (negligible-resistance) wire is connected into the circuit, in terms of R, C, Q_0. (1 point) Loop-rule equation with terms for the potential difference across Resistor 2 and the potential difference across (dielectric-filled) Capacitor 2: $\Delta V_{R,2} - \Delta V_{C,2} = 0$. (1 point) Correct substitution of IR for the potential difference across Resistor 2: $\Delta V_{R,2} = IR$. (1 point) Correct substitution of $2C$ for Capacitor 2's new (dielectric) capacitance and of the charge from part (d) $(2Q_0/3)$ into the expression for $\Delta V_{C,2} = \dfrac{Q_2}{C_2} = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right)$. Full solution: $IR = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right) = \dfrac{Q_0}{3C} \Rightarrow I = \dfrac{Q_0}{3RC}$.

Solution 3

Solution 3

(e)(ii)

Mark scheme

- Determine the current in Resistor 2 a long time after the wire is connected to the circuit: the current is ZERO. At the new steady state, Capacitor 2 is fully charged (no more charge flow needed to maintain the loop's voltage balance), so no current flows through Resistor 2.

Question 3

2022 Set 2 ▪ Q3

[Side View: a solenoid drawn as a coil of wire with current $I(t)$ entering at the left, labeled "Solenoid"; a single circular "Loop of Wire" is shown positioned partway along/around the solenoid's axis, oriented with its plane perpendicular to the solenoid's axis.] [End View: the solenoid seen end-on as a circle, with a smaller concentric circle labeled "Loop of Wire" carrying current $I(t)$ shown with a circular arrow indicating direction.] [Note: Figures not drawn to scale.]
A single loop of wire with resistance $3.0\ \Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

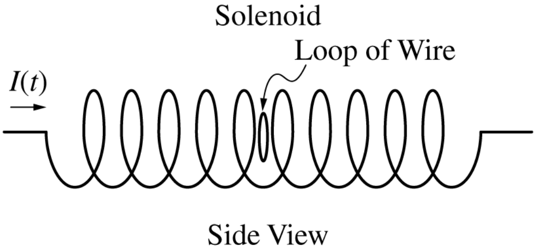
Question 3

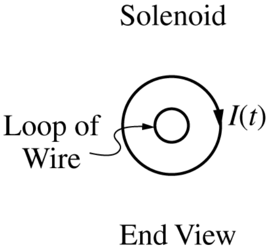
(a) At time $t = 2.0$ s, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

$c^{lockwise}{}_{c^{ounterclockwise}{}_{z^{ero}}}$

Justify your answer.

Question 3





Note: Figures not drawn to scale.

Question 3

2022 Set 2 ▪ Q3

[Side View: a solenoid drawn as a coil of wire with current $I(t)$ entering at the left, labeled "Solenoid"; a single circular "Loop of Wire" is shown positioned partway along/around the solenoid's axis, oriented with its plane perpendicular to the solenoid's axis.] [End View: the solenoid seen end-on as a circle, with a smaller concentric circle labeled "Loop of Wire" carrying current $I(t)$ shown with a circular arrow indicating direction.] [Note: Figures not drawn to scale.]

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Question 3

(b) Calculate the current in the loop of wire at time $t = 2.0$ s.

Question 3

2022 Set 2 ▪ Q3

[Side View: a solenoid drawn as a coil of wire with current $I(t)$ entering at the left, labeled "Solenoid"; a single circular "Loop of Wire" is shown positioned partway along/around the solenoid's axis, oriented with its plane perpendicular to the solenoid's axis.] [End View: the solenoid seen end-on as a circle, with a smaller concentric circle labeled "Loop of Wire" carrying current $I(t)$ shown with a circular arrow indicating direction.] [Note: Figures not drawn to scale.]

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Question 3

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0$ s.

Question 3

2022 Set 2 ▪ Q3

[Side View: a solenoid drawn as a coil of wire with current $I(t)$ entering at the left, labeled "Solenoid"; a single circular "Loop of Wire" is shown positioned partway along/around the solenoid's axis, oriented with its plane perpendicular to the solenoid's axis.] [End View: the solenoid seen end-on as a circle, with a smaller concentric circle labeled "Loop of Wire" carrying current $I(t)$ shown with a circular arrow indicating direction.] [Note: Figures not drawn to scale.]

A single loop of wire with resistance $3.0\ \Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

Question 3

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0$ s is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

TheexperimentdidnotaccountforEarth′smagneticfield. Theplaneoftheloopisnotperpendiculartotheaxisofthesolenoid. Thecenteroftheloopisnotontheaxisofthesolenoid.

m.

Justify your answer.