Past-paper walk-through

Electric Circuits ▪ 11.2

eXercise

Questions in this set

- 2024 Set 1 Q2
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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

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[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a

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function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)(i) On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that <u>decreases</u> with time.

[Voltmeter Symbol: a circle containing the letter "V", with two connection leads.]

(a)(ii) **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that <u>decreases</u> with time. Provide enough detail so that another student could replicate the experiment.

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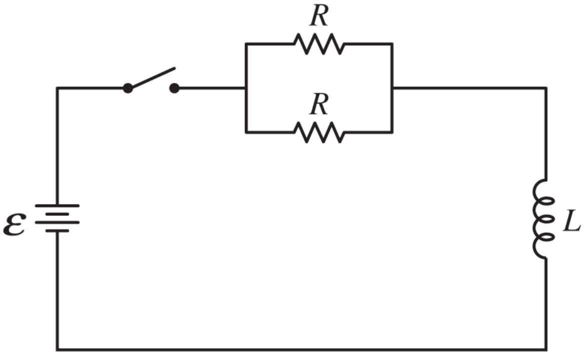

Figure 1

Question 2

Figure 2

Question 2

Voltmeter Symbol

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

Question 2

(b)(i) [Figure 2: A blank graph with a vertical axis (unlabeled) and a horizontal axis (unlabeled, shown as a dashed line), both with arrowheads, meeting at the origin.]
On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

(b)(ii) Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

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[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

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(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

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[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

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(d) After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.
Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.
_____$|\Delta V_2| > |\Delta V_1|$ _____$|\Delta V_2| < |\Delta V_1|$ _____$|\Delta V_2| = |\Delta V_1|$
Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Draw the voltmeter symbol connected in parallel across the two terminals of the inductor L, so that it measures the potential difference across the inductor — the quantity that decreases with time as the circuit approaches steady state after the switch is closed.

Solution 2

(a)(ii) Mark scheme

- Describe a data-collection procedure: after placing the voltmeter across the inductor, close the switch and use the voltmeter to record the potential difference across the inductor as a function of time (1 pt, for a procedure using the voltmeter to measure the potential difference at least once). The measurements must span from immediately after the switch is closed until steady-state conditions are established (or otherwise over a time interval that would allow the time constant to be determined) (1 pt). Example: 'Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.'

Solution 2

(b)(i) Mark scheme

- Sketch the voltmeter reading ΔV (potential difference across the inductor) versus time t, 4 points total (1 pt each): (1) correctly label the vertical axis as potential difference and the horizontal axis as time; (2) draw a concave-up, decreasing curve; (3) the curve should asymptotically approach zero (or start at the origin and approach a horizontal asymptote), consistent with the voltmeter placement across the inductor from part (a)(i); (4) include a vertical intercept (or horizontal asymptote) correctly labeled as $\mathcal{E}$ — the curve begins at $\Delta V = \mathcal{E}$ when $t = 0$ and decays smoothly toward the time axis.

Solution 2

(b)(ii) Mark scheme

- (1) Indicate that the graph should be fit with an exponential function, $V_L = \mathcal{E}\, e^{-tR/2L}$, or correctly relate the area under the curve to the current in the circuit (1 pt). (2) Indicate that the coefficient in front of t in that exponential fit equals $\frac{R}{2L}$, or equivalently correctly relate R to L and the current — i.e. that the time constant equals $\frac{2L}{R}$ (1 pt). Since $\mathcal{E}$ and L are known, this coefficient/time-constant lets R be calculated. Alternate: identify the time on the graph where $\Delta V \approx 0.37\mathcal{E}$ as the time constant $2L/R$, from which R follows because L is known.

Solution 2

(c) Mark scheme

- Derive the differential equation for the inductor current $I(t)$ using Kirchhoff's loop rule, 3 points: (1) a multi-step derivation that begins with an attempt at the loop rule, e.g. $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ (1 pt); (2) indicate that the potential difference across the parallel combination of the two resistors R is $I\left(\frac{R}{2}\right)$ (1 pt); (3) indicate that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$ (1 pt). Combining: $\mathcal{E} - I\left(\frac{R}{2}\right) - L\frac{dI}{dt} = 0$, which rearranges to $-\frac{R}{L}\left(\frac{I}{2} - \frac{\mathcal{E}}{R}\right) = \frac{dI}{dt}$.

Solution 2

(d)

Mark scheme

- Select $|\Delta V_2| > |\Delta V_1|$, with an attempt at a relevant justification (1 pt). The justification must indicate that the potential difference across the original, ideal inductor is zero once steady-state conditions are established (1 pt), and that the potential difference across the new inductor (which has nonnegligible resistance) is nonzero at steady state — equal to the product of the steady-state current and the inductor's resistance (1 pt).

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[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

Question 2

(a)(i) On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.
[Voltmeter Symbol: a circle containing the letter “V”, with a wire lead extending from each side, labeled “Voltmeter Symbol”.]
(a)(ii) **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

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long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

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Figure 2

Question 2

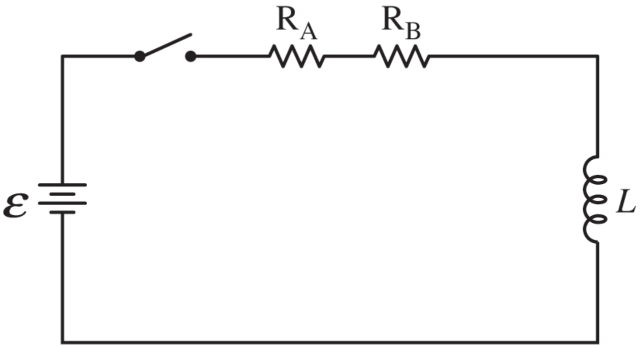

Figure 1

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

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(b)(i) On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

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- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

[Figure 2: A blank graph with a vertical axis (upward arrow, unlabeled) and a horizontal axis (rightward arrow, unlabeled, drawn dashed). No scale or curve shown; axes are empty for the student to label and sketch.]

(b)(ii) Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

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2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

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(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the

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inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

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[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(d) After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has

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nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.
Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.
$|\Delta V_2| > |\Delta V_1|$ $|\Delta V_2| < |\Delta V_1|$ $|\Delta V_2| = |\Delta V_1|$
Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Place the voltmeter in parallel with Resistor A, Resistor B, or the series combination of Resistors A and B (not across the inductor) — this location's potential difference increases with time as current builds up after the switch closes, unlike the inductor voltage which decreases with time. 1 point for correctly placing the voltmeter across one of these three valid options.

Solution 2

(a)(ii)

Mark scheme

- Procedure: Close the switch, then use the voltmeter to record the potential difference across the chosen resistor(s) as a function of time, from immediately after the switch is closed until steady-state conditions are established (or until enough time has elapsed that the time constant can be determined from the data). 1 point for a procedure indicating the voltmeter is used to measure the potential difference for at least one time, 1 point for measuring from immediately after the switch closes through steady state (or until the time constant is determinable).

Solution 2

(b)(i) Mark scheme

- Sketch potential difference (vertical axis) vs. time (horizontal axis): a curve that starts at the origin $(0,0)$, is concave-down and increasing, and asymptotically approaches a nonzero potential-difference value. The labeled horizontal asymptote must be consistent with the voltmeter placement chosen in part (a)(i) — e.g. $\varepsilon/2$ if measuring across one resistor (A or B alone), or ε if measuring across the series combination of A and B. 1 point for correctly labeling potential difference vertically and time horizontally, 1 point for a concave-down increasing curve, 1 point for a curve starting at the origin and asymptotically approaching a nonzero value, 1 point for correctly labeling the horizontal asymptote consistent with part (a)(i).

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(b)(ii) Mark scheme

- Fit the data to an exponential function of the form $V_R = \varepsilon\left(1 - e^{-t2R/L}\right)$ (for the series-combination case; for a single resistor the analogous form applies). Since ε and L are known, the coefficient multiplying t in the fit equals $2R/L$, from which R can be calculated. Alternate: read off the time at which the potential difference reaches $\approx 0.63\varepsilon/2$ (one resistor) or $\approx 0.63\varepsilon$ (combination); this time equals the time constant $L/2R$, and since L is known, R can be calculated. 1 point for indicating the curve fit is an exponential function (or, alternately, indicating the 0.63-value time), 1 point for indicating the fitted coefficient of t equals $2R/L$ (or, alternately, that the time constant equals $L/2R$).

Solution 2

(c) Mark scheme

- Apply Kirchhoff's loop rule: $\varepsilon - \Delta V_R - \Delta V_L = 0$. The potential difference across the series combination of the two resistors is $I(2R)$, and the magnitude of the potential difference across the inductor is $L\, dI/dt$. Substituting: $\varepsilon - I(2R) - L\dfrac{dI}{dt} = 0$, which rearranges to $\dfrac{dI}{dt} = \dfrac{\varepsilon}{L} - \dfrac{2IR}{L} = -\dfrac{R}{L}\left(2I - \dfrac{\varepsilon}{R}\right)$ (do not solve further). 1 point for a multi-step derivation beginning with an attempt at Kirchhoff's loop rule, 1 point for indicating the resistor-combination potential difference is $I(2R)$, 1 point for indicating the inductor's potential-difference magnitude is $L\, dI/dt$.

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(d) Mark scheme

- $|\Delta V_2| < |\Delta V_1|$. Justification: because the new inductor has nonnegligible resistance, the total resistance of the new circuit is greater than in the original circuit, so at steady state the current is smaller than before, and the potential difference across R_A (which equals IR_A) decreases. (Equivalently: the increased total resistance means more of the source emf is dropped across the inductor's internal resistance at steady state, increasing the potential difference across the inductor and thereby decreasing the potential difference across R_A.) 1 point for selecting $|\Delta V_2| < |\Delta V_1|$ with an attempted justification, 1 point for indicating the total resistance has increased, 1 point for indicating the current decreases due

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to the increased resistance (or, alternately, that the inductor's potential difference increases so R_A's must decrease).

Question 2

2023 Set 1 ▪ Q2

Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.
- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring. - At time t_1, the bar loses contact with the spring and slides to the right. - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown. - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page. - At time t_4, the bar enters a region with no magnetic field.

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Consider time t_B such that $t_2 < t_B < t_3$.
[Diagram: Top view at time t_B. Two horizontal parallel rails separated by distance L, connected on the left by a resistor R and a coiled spring, with the Conducting Bar shown between two field regions. The left field region (between the spring/bar and the bar's current position) is shaded with $\times$ symbols labeled B, indicating a uniform magnetic field into the page. The right region is marked with $\cdot$ (dot) symbols labeled B, indicating a uniform magnetic field out of the page.]
(a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.
[Diagram: A vertical rectangle representing the conducting bar, for the student to draw a force arrow on.]

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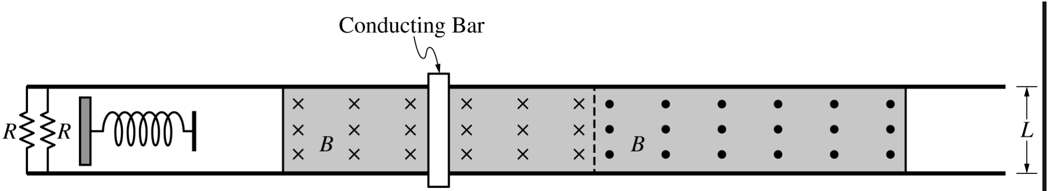

Top View (at time t_B)

) The scenario is repeated but an additional resistor of resistance $R = 0.20\ \Omega$ is connected, as shown

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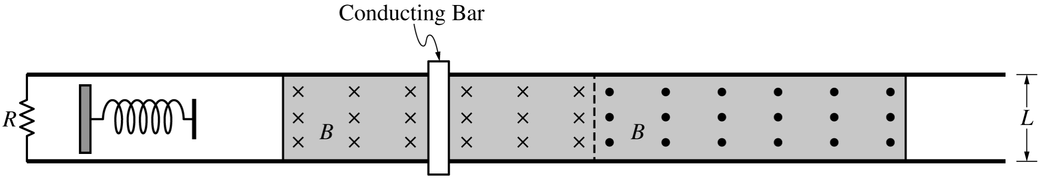

Top View (at time t_B)

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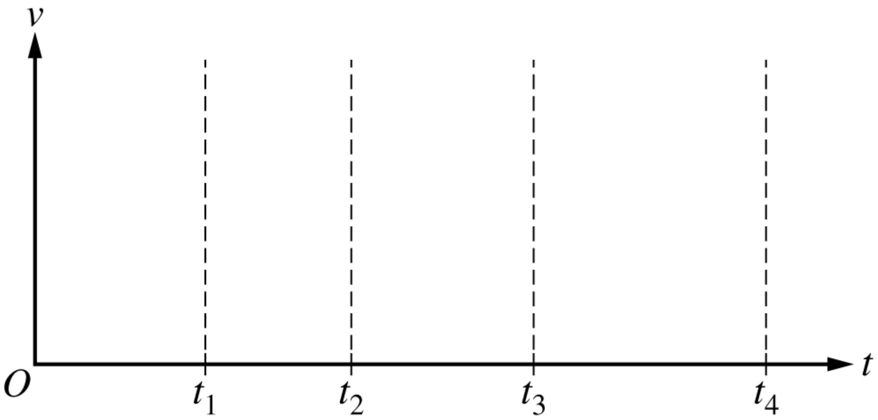

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2023 Set 1 ▪ Q2

Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.
- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring. - At time t_1, the bar loses contact with the spring and slides to the right. - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown. - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page. - At time t_4, the bar enters a region with no magnetic field.
Consider time t_B such that $t_2 < t_B < t_3$.

Question 2

[Diagram: Top view at time t_B. Two horizontal parallel rails separated by distance L, connected on the left by a resistor R and a coiled spring, with the Conducting Bar shown between two field regions. The left field region (between the spring/bar and the bar's current position) is shaded with $\times$ symbols labeled B, indicating a uniform magnetic field into the page. The right region is marked with $\cdot$ (dot) symbols labeled B, indicating a uniform magnetic field out of the page.]

(b)(i) At time t_B, the speed of the bar is $v = 2.5\,\mathrm{m/s}$.

Calculate the magnitude of the current in the bar at time t_B.

(b)(ii) Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

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2023 Set 1 ▪ Q2

Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\mathrm{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\mathrm{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.
- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring. - At time t_1, the bar loses contact with the spring and slides to the right. - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\mathrm{T}$ that is directed into the page, as shown. - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\mathrm{T}$ but is directed out of the page. - At time t_4, the bar enters a region with no magnetic field.
Consider time t_B such that $t_2 < t_B < t_3$.

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[Diagram: Top view at time t_B. Two horizontal parallel rails separated by distance L, connected on the left by a resistor R and a coiled spring, with the Conducting Bar shown between two field regions. The left field region (between the spring/bar and the bar's current position) is shaded with $\times$ symbols labeled B, indicating a uniform magnetic field into the page. The right region is marked with $\cdot$ (dot) symbols labeled B, indicating a uniform magnetic field out of the page.]

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.

[Graph: Blank axes with vertical axis v and horizontal axis t, origin labeled O. Four vertical dashed reference lines are marked on the horizontal axis at t_1, t_2, t_3, and t_4, for the student to sketch the speed-time curve.]

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Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\text{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\text{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.
- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring. - At time t_1, the bar loses contact with the spring and slides to the right. - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\text{T}$ that is directed into the page, as shown. - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\text{T}$ but is directed out of the page. - At time t_4, the bar enters a region with no magnetic field.
Consider time t_B such that $t_2 < t_B < t_3$.

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[Diagram: Top view at time t_B. Two horizontal parallel rails separated by distance L, connected on the left by a resistor R and a coiled spring, with the Conducting Bar shown between two field regions. The left field region (between the spring/bar and the bar's current position) is shaded with $\times$ symbols labeled B, indicating a uniform magnetic field into the page. The right region is marked with $\cdot$ (dot) symbols labeled B, indicating a uniform magnetic field out of the page.]

(d)(i) The scenario is repeated but an additional resistor of resistance $R = 0.30\,\Omega$ is connected, as shown.

[Diagram: Top view at time t_B, same rail/bar/field setup as before, but now with a second resistor R connected in the circuit next to the first resistor R and the spring, both on the left end of the rails.]

Determine the total resistance R_{total} of the closed circuit for the new scenario.

(d)(ii) In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new

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scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}.
Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

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2023 Set 1 ▪ Q2

Two horizontal, parallel, conducting rails are separated by distance $L = 0.40\,\mathrm{m}$. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23\,\mathrm{kg}$ is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.
- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring. - At time t_1, the bar loses contact with the spring and slides to the right. - At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50\,\mathrm{T}$ that is directed into the page, as shown. - At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50\,\mathrm{T}$ but is directed out of the page. - At time t_4, the bar enters a region with no magnetic field.
Consider time t_B such that $t_2 < t_B < t_3$.

Question 2

[Diagram: Top view at time t_B. Two horizontal parallel rails separated by distance L, connected on the left by a resistor R and a coiled spring, with the Conducting Bar shown between two field regions. The left field region (between the spring/bar and the bar's current position) is shaded with $\times$ symbols labeled B, indicating a uniform magnetic field into the page. The right region is marked with $\cdot$ (dot) symbols labeled B, indicating a uniform magnetic field out of the page.]

(e) Describe a modification to m, B, or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

Solution 2

(a)

Mark scheme

- At time t_B the induced current in the bar produces a magnetic (retarding) force that, by Lenz's law, opposes the bar's motion into the field; draw a single arrow on the bar pointing to the left, with no other/extraneous force arrows included. (If a student instead correctly argues the net force is zero at t_B, write $F_{net} = 0$ for full credit.) 1 point total for a correct single left-pointing arrow with no extraneous arrows.

Solution 2

(b)(i) Mark scheme

- Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(BLx)}{dt}$ (1 pt, using Faraday's law to set up the induced emf; may be earned without the negative sign or a numerical answer). Substituting $v = \frac{dx}{dt}$: $\mathcal{E} = -BL\frac{dx}{dt} = -BLv$ (1 pt, correct substitution of v for dx/dt; points 1 and 2 may also be earned by starting directly from $\mathcal{E} = BLv$). Numerically, $\mathcal{E} = -(0.50\,\mathrm{T})(0.40\,\mathrm{m})(2.5\,\mathrm{m/s}) = -0.50\,\mathrm{V}$. Using Ohm's law with the correct resistance (1 pt, substituting the correct resistance into Ohm's law to solve for current): $I = \frac{|\mathcal{E}|}{R} = \frac{|-0.50\,\mathrm{V}|}{0.30\,\Omega} = 1.7\,\mathrm{A}$.

Solution 2

(b)(ii) Mark scheme

- Substituting the current from part (b)(i) into the magnetic force equation: $\vec{F} = \int I\, d\vec{\ell} \times \vec{B} \Rightarrow F = ILB = (1.7\,\mathrm{A})(0.4\,\mathrm{m})(0.5\,\mathrm{T}) = 0.33\,\mathrm{N}$ (equivalently, using $F = \dfrac{B^2 L^2 v}{R} = \dfrac{(0.5)^2(0.4)^2(2.5)}{0.3} = 0.33\,\mathrm{N}$). 1 point for substituting the current (or an expression for it) into an appropriate equation for the magnetic force on the bar and obtaining $F = 0.33\,\mathrm{N}$.

Solution 2

(c)

Mark scheme

- Sketch of speed v vs. time t from $t = 0$ to t_4: the curve starts at the origin, is increasing and concave down from $t = 0$ to t_1 (1 pt); is a horizontal line (constant speed) from t_1 to t_2 (1 pt); is decreasing and concave up from t_2 to t_4 (1 pt); and is differentiable (smooth, no sharp corner) at t_3 with a nonzero slope there (1 pt) — i.e. the curve rises smoothly to a plateau between t_1 and t_2, then decays smoothly and without a kink through t_3 on its way down to t_4.

Solution 2

(d)(i) Mark scheme

- The two $0.30\,\Omega$ resistors are now in parallel: $\frac{1}{R_p} = \frac{1}{0.3\,\Omega} + \frac{1}{0.3\,\Omega} = \frac{2}{0.3\,\Omega} \Rightarrow R_p = 0.15\,\Omega$. 1 point for the correct numeric answer with units ($0.15\,\Omega$); this point can be earned without supporting calculations shown.

Solution 2

(d)(ii) Mark scheme

- a_{new} is greater than $a_{original}$. Chain of reasoning (each statement worth 1 point, full credit earnable with a justification consistent with the resistance calculated in part (d)(i)): since the new total resistance is smaller ($0.15\,\Omega < 0.30\,\Omega$), the current increases (inverse relationship between resistance and current) (1 pt); since the current increases, the magnetic force on the bar increases (direct relationship between current and force, via $F = BIL$) (1 pt); since the force increases, the magnitude of the bar's acceleration increases (direct relationship between force and acceleration, via $F = ma$) (1 pt). Example: "Since there is less resistance in the new circuit, there will be more current in the new circuit, so a

Solution 2

larger force on the bar. Thus, since the force on the bar is larger, the new acceleration is greater than the original acceleration."

Solution 2

(e) Mark scheme

- Any ONE of the following modifications, with an attempt at justification (1 pt): decreasing B, decreasing L, or increasing m. A second point (1 pt) is earned for correctly justifying why the chosen modification results in a smaller induced potential difference across the original resistor. Since $\mathcal{E} = -BLv$: decreasing B directly decreases $\mathcal{E}$ (proportional relationship); decreasing L directly decreases $\mathcal{E}$ (proportional relationship, L is the rail separation); increasing m causes the bar to enter the field with a smaller speed v (e.g., if launched by the same spring/energy source, larger mass gives smaller speed), and since $\mathcal{E} \propto v$, a smaller v gives a smaller induced $\mathcal{E}$.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

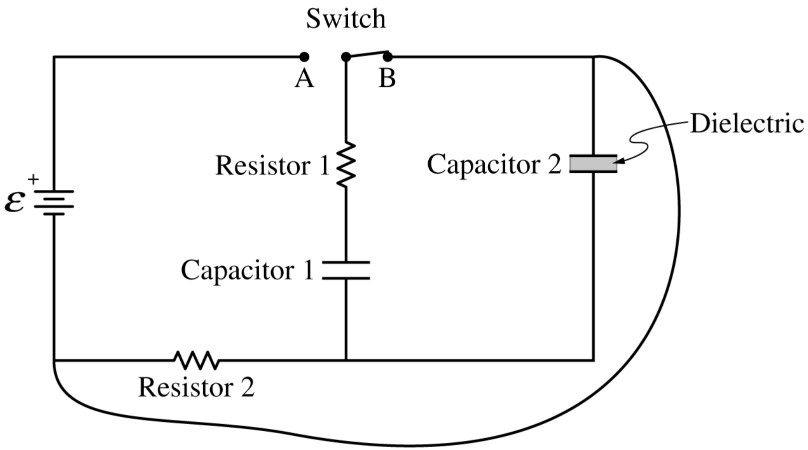
The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

Question 3

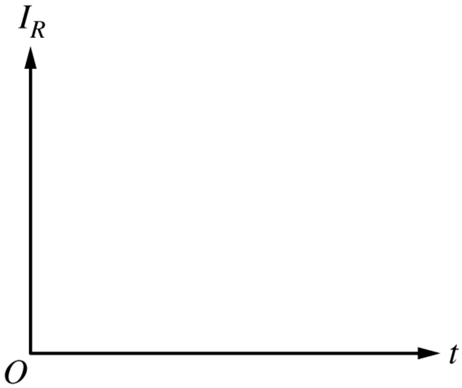
At time $t = 0$, the switch is closed to Position A.

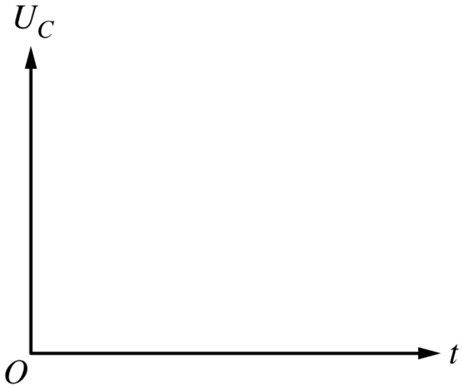
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

Question 3

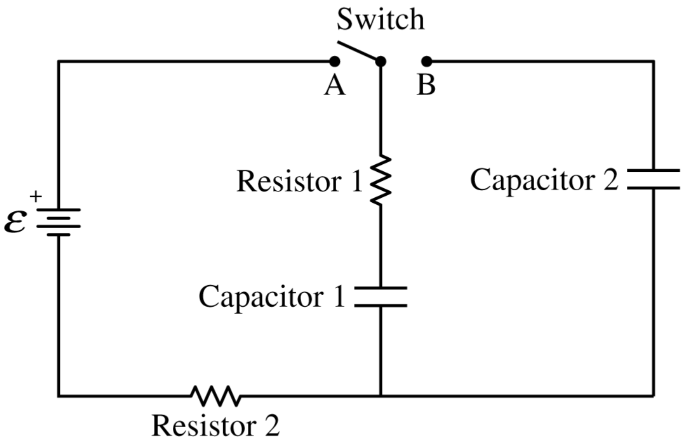

Switch
A
B
Resistor 1
Capacitor 2
Dielectric
ε
+
Capacitor 1
Resistor 2

Question 3





Question 3



Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.
[Graph 1: y-axis labeled "I_R", x-axis labeled "t", origin at O, no numeric scale shown.]
[Graph 2: y-axis labeled "U_C", x-axis labeled "t", origin at O, no numeric scale shown.]
A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.
At time t_1, the switch is closed to Position B.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(c)(i) Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.
(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
(c)(iii) Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.
With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
[Figure: The same circuit as before (battery $\mathcal{E}$, switch positions A/B, Resistor 1, Capacitor 1, Resistor 2, Capacitor 2), now with a dielectric slab labeled "Dielectric" shown inserted between the plates of Capacitor 2. An additional wire of negligible resistance is shown connected between two corners of the circuit (from the node between the switch and Resistor 1/Capacitor 1, curving around to the node below Capacitor 2/Resistor 2), as described below.]
With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(e)(i) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.
(e)(ii) Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

Solution 3

(a) Mark scheme

- Write, but do NOT solve, the differential equation for charge Q on the positive plate of Capacitor 1 vs time t after the switch closes to Position A, in terms of $\mathcal{E}, R, C, Q, t$. (1 point) Loop-rule expression that includes terms for the equivalent resistance $2R$ (the pair of resistors) and the potential difference across the battery. (1 point) An expression that includes charge Q in the term for the potential difference across the capacitor, and charge per unit time $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Result: $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0 \Rightarrow \mathcal{E} - \frac{Q}{C} - 2R\frac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 vs time t from $t = 0$ until steady state is nearly reached (RC charging curves). (1 point) I_R vs t is a CONCAVE UP, DECREASING curve starting at its maximum value at $t = 0$. (1 point) U_C vs t is a CONCAVE DOWN, INCREASING curve starting at zero. (1 point, can be earned even if the first two are not) Both curves approach a slope of zero (level off / asymptote) as time increases, consistent with exponential charging behavior.

Solution 3

(c)(i) Mark scheme

- Immediately after t_1 (switch moved to Position B, connecting the charged Capacitor 1 through Resistor 1 to the initially uncharged Capacitor 2), the current in Resistor 1 is directed UP. Justification (either form acceptable): positive charge has accumulated on the top plate of Capacitor 1 (and/or negative charge on its bottom plate) from being charged while the switch was at Position A, so at t_1 the higher potential top plate drives current up through Resistor 1 and down through Capacitor 2 to charge it. Equivalently: the top plate of Capacitor 1 is at higher electric potential than the bottom plate, so current flows from high to low potential, i.e. up through Resistor 1, around the loop through Capacitor 2.

Solution 3

(c)(ii) Mark scheme

- Determine the total charge on the positive plate of Capacitor 2 a long time after t_1: the answer is $\frac{Q_0}{2}$ (this point can be earned without supporting calculations). Justification: at new steady state, the potential difference across Capacitor 1 equals that across Capacitor 2 (parallel connection via Resistor 1 with no current, so no voltage drop across R1). Since Capacitor 2 has the same capacitance as Capacitor 1, it stores the same charge. By conservation of the total charge Q_0 originally on Capacitor 1, the two capacitors split it evenly, so Capacitor 2 stores $Q_0/2$.

Solution 3

(c)(iii) Mark scheme

- Derive an expression for the total energy dissipated by Resistor 1 from immediately after t_1 until the new steady state, in terms of C, Q_0. (1 point) Indicate that the energy dissipated equals the difference between the initial electric potential energy (at $t = t_1$) and the final electric potential energy (after new steady state): $\Delta E_R = U_C - U_{0C}$. (1 point) Indicate that only Capacitor 1 stores nonzero PE initially, and BOTH capacitors store PE after the new steady state (or an alternate response consistent with part (c)(ii)): $U_{0C} = U_{01}$ AND $U_C = U_1 + U_2$. (1 point) Correct substitution of the charges from part (c)(ii) ($Q_0/2$ on each capacitor). Full solution: $\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \dfrac{Q_0^2}{4C} - \dfrac{Q_0^2}{2C} = -\dfrac{Q_0^2}{4C}$. The

Solution 3

magnitude of energy dissipated by Resistor 1 is $\dfrac{Q_0^2}{4C}$ (the negative sign reflects the decrease in stored electric PE, which is dissipated as heat).

Solution 3

(d) Mark scheme

- Determine the charge on the positive plate of Capacitor 2 a long time after a dielectric ($\kappa = 2$) is inserted between its plates, with the switch still at Position B: $Q_2 = \dfrac{2Q_0}{3}$ (this point can be earned without supporting calculations). Derivation: with $C_1 = C$ and $C_2 = \kappa C = 2C$, the plates are still connected so $\Delta V_{C1} = \Delta V_{C2} \Rightarrow \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_1 = \frac{1}{2}Q_2$. Total charge is conserved: $Q_1 + Q_2 = Q_0 \Rightarrow \frac{1}{2}Q_2 + Q_2 = Q_0 \Rightarrow Q_2 = \dfrac{2}{3}Q_0$.

Solution 3

(e)(i) Mark scheme

- Derive an expression for the current in Resistor 2 immediately after an additional (negligible-resistance) wire is connected into the circuit, in terms of R, C, Q_0. (1 point) Loop-rule equation with terms for the potential difference across Resistor 2 and the potential difference across (dielectric-filled) Capacitor 2: $\Delta V_{R,2} - \Delta V_{C,2} = 0$. (1 point) Correct substitution of IR for the potential difference across Resistor 2: $\Delta V_{R,2} = IR$. (1 point) Correct substitution of $2C$ for Capacitor 2's new (dielectric) capacitance and of the charge from part (d) $(2Q_0/3)$ into the expression for $\Delta V_{C,2} = \frac{Q_2}{C_2} = \left(\frac{2Q_0}{3}\right)\left(\frac{1}{2C}\right)$. Full solution: $IR = \left(\frac{2Q_0}{3}\right)\left(\frac{1}{2C}\right) = \frac{Q_0}{3C} \Rightarrow I = \frac{Q_0}{3RC}$.

Solution 3

Solution 3

(e)(ii)

Mark scheme

- Determine the current in Resistor 2 a long time after the wire is connected to the circuit: the current is ZERO. At the new steady state, Capacitor 2 is fully charged (no more charge flow needed to maintain the loop's voltage balance), so no current flows through Resistor 2.

Question 2

2022 Set 1 ▪ Q2

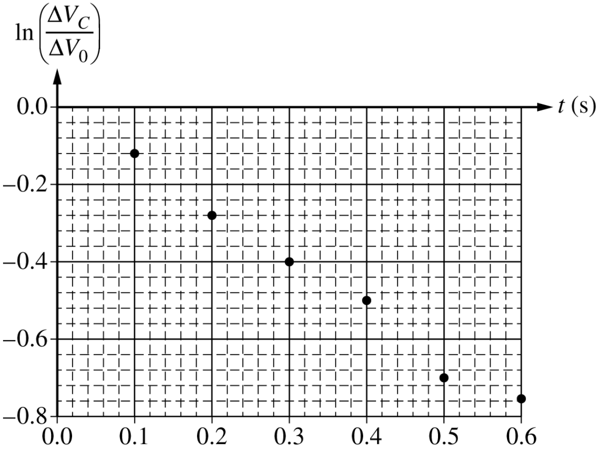
The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

Question 2

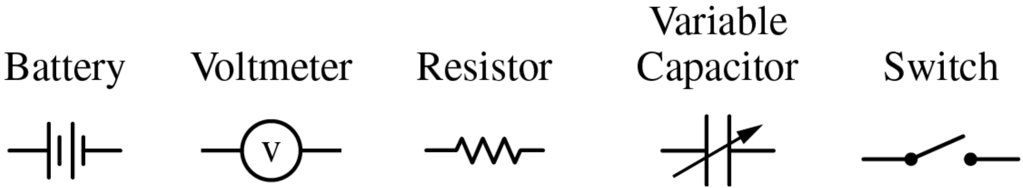
(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

Question 2



$\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$
0.0
–0.2
–0.4
–0.6
–0.8
t (s)
0.0
0.1
0.2
0.3
0.4
0.5
0.6

Question 2



Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(b) The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

Question 2

Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(c)(i) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\dfrac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.

Question 2

[Scatter plot on graph paper. Vertical axis labeled $\ln\left(\dfrac{\Delta V_C}{\Delta V_0}\right)$, ranging from 0.0 to -0.8 in increments of 0.2. Horizontal axis labeled t (s), ranging from 0.0 to 0.6 in increments of 0.1. Data points approximately at: $(0.1, -0.10)$, $(0.2, -0.27)$, $(0.3, -0.38)$, $(0.4, -0.45)$, $(0.5, -0.63)$, $(0.6, -0.72)$.]
Draw the best-fit line for the data.
(c)(ii) Using the best-fit line, calculate a value for the unknown capacitance C.

Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second

Question 2

experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

____ More steep ____ Less steep ____ Unchanged

Briefly justify your answer.

Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(e)(i) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

Question 2

Would the slope of the graph in this final experiment change compared to the graph in part (c)?

____ Yes ____ No

Briefly justify your answer.

(e)(ii) Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

____ Yes ____ No

Briefly justify your answer.

Solution 2

(a) Mark scheme

- Any schematic diagram satisfying all four criteria earns full credit: (1 pt) the capacitor in series with the resistor OR the resistor on a separate parallel path; (1 pt) the voltmeter connected in parallel with the capacitor; (1 pt) a switch used to connect the battery to the capacitor (to charge it); (1 pt) a switch that allows the capacitor to discharge through the resistor (bypassing the battery). Example circuit: a battery in series with switch S_1 and the resistor R, leading to capacitor C; the voltmeter V is wired in parallel across the capacitor; a second switch S_2 provides a path so the capacitor can discharge through R once S_1 is opened.

Solution 2

(b) Mark scheme

- Loop equation around the discharging RC loop: $V_C - V_R = 0$ (1 pt: appropriate loop equation). Substituting $\dfrac{q}{C}$ for V_C and IR for V_R: $\dfrac{q}{C} = IR$ (1 pt: correct substitution consistent with the loop equation). Using $I = -\dfrac{dq}{dt}$ for the discharging capacitor: $\dfrac{q}{C} = -\dfrac{dq}{dt}R \Rightarrow -\dfrac{1}{RC}dt = \dfrac{1}{q}dq$ (1 pt: differential expression consistent with the loop rule that includes the $I = -dq/dt$ substitution; sign is not graded). Integrating, $-\displaystyle\int_0^t \frac{1}{RC}dt' = \int_{q_0}^{q} \frac{1}{q'}dq'$ gives $-\dfrac{t}{RC} = \ln\left(\dfrac{q(t)}{q_0}\right)$, so $q(t) = q_0 e^{-t/RC}$, and dividing by C: $\Delta V_C(t) = \Delta V_0\, e^{-t/RC}$, as required.

Solution 2

(c)(i)

Mark scheme

- Draw a straight best-fit line through the plotted points of $\ln(\Delta V_C/\Delta V_0)$ vs. t — e.g. a line starting near $(0.0 \text{ s}, 0.0)$ and sloping down to about $(0.6 \text{ s}, -0.75)$, passing close to the scattered data points. Full credit for any reasonable straight best-fit line through the data.

Solution 2

(c)(ii) Mark scheme

- Start from $V(t) = V_0 e^{-t/RC} \Rightarrow \ln\left(\frac{V(t)}{V_0}\right) = -\frac{t}{RC}$ (1 pt: appropriate equation relating the unknown C to the data). Using two points on the best-fit line, e.g. $(0.30\ \text{s}, -0.40)$ and $(0.54\ \text{s}, -0.70)$: slope $= \frac{-0.70-(-0.40)}{0.54\ \text{s} - 0.30\ \text{s}} = -1.25\ \text{s}^{-1}$ (1 pt: correctly calculating the slope from two points on the best-fit line; the associated unit is not graded). Since slope $= -\frac{1}{RC}$:
$C = \frac{-1}{\text{slope} \times R} = \frac{-1}{(-1.25\ \text{s}^{-1})(150\ \text{k}\Omega)} = 5\ \mu\text{F}$ (1 pt: correctly relating the slope to the unknown capacitance).

Solution 2

(d)

Mark scheme

- Answer: Less steep (1 pt, for the correct selection with an attempt at relevant justification). Increasing the plate area increases the capacitance C of the capacitor; since the slope of the graph is $-\dfrac{1}{RC}$, a larger C makes the magnitude of the slope smaller, so the best-fit line is less steep (1 pt: correctly relating the change in capacitance to the slope of the graph). (Scored consistently with the response to part (c).)

Solution 2

(e)(i)

Mark scheme

- Answer: No (1 pt, for the correct selection AND correctly describing the effect on the slope at $t = 0$). The capacitor still discharges only through resistor R — the battery and its internal resistance are disconnected from the loop during discharge — so the time constant RC, and hence the slope of the graph, is unchanged.

Solution 2

(e)(ii)

Mark scheme

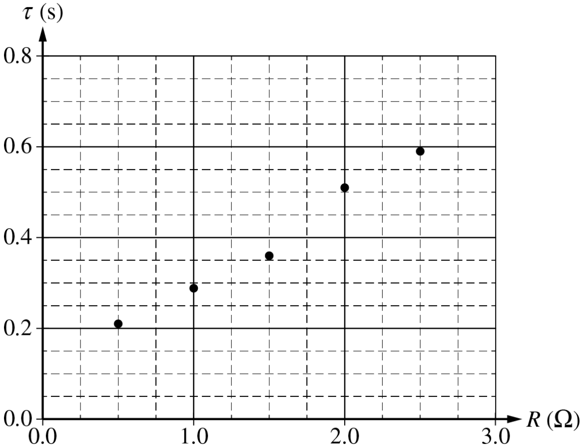
- Answer: No (1 pt, for the correct selection AND correctly describing the effect on the vertical intercept/initial value). The best-fit line's intercept does not change, because the internal resistance of the battery does not affect the final potential difference across the capacitor once it is fully charged (no current flows through the internal resistance at that point, so there is no voltage drop across it). (Scored with consistency to the circuit drawn in part (a)/(b).)

Question 2

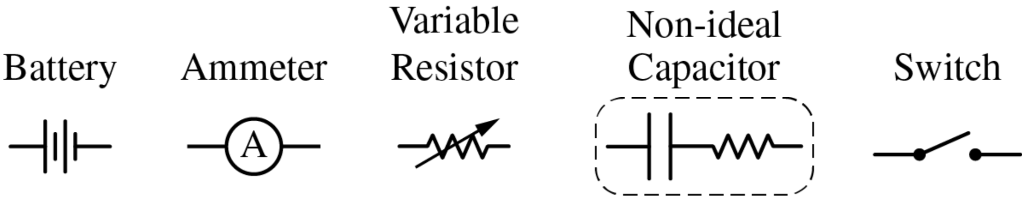
2022 Set 2 ▪ Q2

A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed. [Circuit symbol key shown: Battery (given potential difference ΔV_0), Ammeter (A), Variable Resistor (resistance R), Non-ideal Capacitor (capacitance C, drawn as a dashed box containing an ideal capacitor and a small resistor), Switch.]
(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

Question 2



Question 2


Battery
Ammeter
A
Variable Resistor
Non-ideal Capacitor
Switch

Question 2

2022 Set 2 ▪ Q2

A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.

[Circuit symbol key shown: Battery (given potential difference ΔV_0), Ammeter (A), Variable Resistor (resistance R), Non-ideal Capacitor (capacitance C, drawn as a dashed box containing an ideal capacitor and a small resistor), Switch.]

(b) The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

Question 2

Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

Question 2

2022 Set 2 ▪ Q2

A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.

[Circuit symbol key shown: Battery (given potential difference ΔV_0), Ammeter (A), Variable Resistor (resistance R), Non-ideal Capacitor (capacitance C, drawn as a dashed box containing an ideal capacitor and a small resistor), Switch.]

(c)(i) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.

Question 2

[Graph: vertical axis τ (s) from 0.0 to 0.8 in increments of 0.2 (gridlines every 0.1); horizontal axis R (Ω) from 0.0 to 3.0 in increments of 1.0 (gridlines every 0.25). Plotted data points approximately at: (0.5, 0.20), (1.0, 0.28), (1.5, 0.36), (2.0, 0.51), (2.5, 0.60).]

Draw the best-fit line for the data.

(c)(ii) Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

Question 2

2022 Set 2 ▪ Q2

A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.

[Circuit symbol key shown: Battery (given potential difference ΔV_0), Ammeter (A), Variable Resistor (resistance R), Non-ideal Capacitor (capacitance C, drawn as a dashed box containing an ideal capacitor and a small resistor), Switch.]

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

Question 2

____ Greater than ____ Less than ____ Equal to
Briefly justify your answer using features of the graph in part (c).

Question 2

2022 Set 2 ▪ Q2

A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.

[Circuit symbol key shown: Battery (given potential difference ΔV_0), Ammeter (A), Variable Resistor (resistance R), Non-ideal Capacitor (capacitance C, drawn as a dashed box containing an ideal capacitor and a small resistor), Switch.]

(e) The values of the variable resistor in the original experiment ranged from $0.5\ \Omega$ to $2.5\ \Omega$. The experiment is repeated with values ranging from $3.0\ \Omega$ to $6.0\ \Omega$. Would

Question 2

the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

$M^{oresteep}{}_{L}{}^{esssteep}{}_{R}{}^{emainunchanged}$

Briefly justify your answer.

Solution 2

(a) Mark scheme

- A working circuit: a battery in series with a switch used to charge the capacitor; the capacitor (drawn with its internal resistance r_C, e.g. a resistor bundled with it) in series with the variable resistor R and an ammeter; and a second switch arranged so the charged capacitor can instead discharge through R (bypassing the battery). Points (1 each, 4 total): capacitor in series with the resistor (the variable resistor need not be included for this point); ammeter in series with the capacitor and resistor; a switch that connects the battery to charge the capacitor; a switch that lets the capacitor discharge through the resistor.

Solution 2

(b) Mark scheme

- Loop equation with $V_C - V_R = 0$: $\dfrac{q}{C} - Ir_C = IR \Rightarrow \dfrac{q}{C} = I(R + r_C)$, i.e. total resistance $R_{total} = R + r_C$ (accept expressions without R if the variable resistor is omitted). Differentiate to get a differential equation for $I(t)$, using $I = -dq/dt$ (the charge on the plate decreases with time):
  $\dfrac{dq}{dt}\dfrac{1}{C} = (R + r_C)\dfrac{dI}{dt} \Rightarrow -I\dfrac{1}{C} = (R + r_C)\dfrac{dI}{dt}$. Separate and integrate:
  $\dfrac{1}{I}\dfrac{dI}{dt} = \dfrac{-1}{C(R + r_C)} \Rightarrow \ln\dfrac{I}{I_0} = \dfrac{-t}{C(R + r_C)} \Rightarrow I = I_0 e^{-t/[C(R+r_C)]}$, as required.
  Points: (1) loop equation substituting q/C, Ir_C, and IR; (2) substituting $R + r_C$ as

Solution 2

the total resistance; (3) a correct differential equation for $I(t)$ consistent with point 1.

Solution 2

(c)(i)

Mark scheme

- Draw a straight best-fit line through the τ vs R data points (running roughly from about $(0,\ 0.10\text{ s})$ up through $(2.5\ \Omega,\ 0.60\text{ s})$, e.g. passing near $(0.2, 0.20)$ and $(1.0, 0.28)$ — a line of the form $\tau \approx 0.2R + 0.1$). 1 point for drawing an appropriate straight best-fit line through the data (not connecting the dots).

Solution 2

(c)(ii) Mark scheme

- From the best-fit line: slope $= \dfrac{(0.24 - 0.08)\ \text{s}}{(1.0 - 0.2)\ \Omega} = 0.2$ F. Since $\tau = RC$ becomes $\tau = (R + r_C)C = CR + r_C C$, the SLOPE of τ vs R equals C, so $C = 0.2$ F. The vertical intercept equals $\tau_0 = r_C C = 0.08$ s, so $r_C = \dfrac{\text{vertical intercept}}{C} = \dfrac{0.08\ \text{s}}{0.2\ \text{F}} = 0.4\ \Omega$. Points: (1) correctly determining the slope of the line; (2) using $\tau = RC$ to identify that the slope must be C; (3) correctly relating the vertical intercept to r_C.

Solution 2

(d)

Mark scheme

- Answer: LESS THAN. Justification: the ammeter's own internal resistance is in series with the capacitor's internal resistance, so the experiment actually measures the EQUIVALENT resistance of the ammeter and the capacitor together (larger than r_C alone). Thus the true internal resistance of the capacitor is smaller than the experimentally determined value from part (c). Points: (1) selecting 'Less than' with an attempt at justification; (2) a correct justification identifying the measured resistance as the ammeter+capacitor equivalent resistance.

Solution 2

(e)

Mark scheme

- Answer: REMAIN UNCHANGED. Justification: the slope of the τ vs R best-fit line equals the capacitance C (from $\tau = CR + r_C C$), a property of the capacitor that does not depend on the range of R values used, so repeating the experiment with a different R range would not change the slope. (Scored for consistency with the circuit drawn in part a.) Points: (1) selecting 'Remain unchanged' with an attempt at justification; (2) a correct justification that the slope equals the capacitance, independent of resistance.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)(i) Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

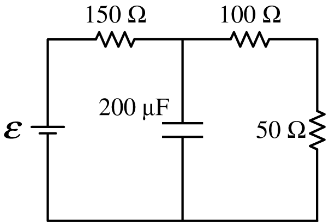
(a)(ii) Calculate the current in the $200\ \Omega$ resistor.

Question 2

(a)(iii) Calculate the power dissipated by the $200\ \Omega$ resistor.
[Figure 2: A circuit with a battery of emf $\mathcal{E}$, a $150\ \Omega$ resistor and $100\ \Omega$ resistor along the top in series from the battery's positive terminal, a $200\ \Omega$ resistor in the middle branch, a voltmeter V connected across the $200\ \Omega$ resistor, and a $50\ \Omega$ resistor in the right branch below the $100\ \Omega$ resistor.]
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

Question 2

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



Question 2

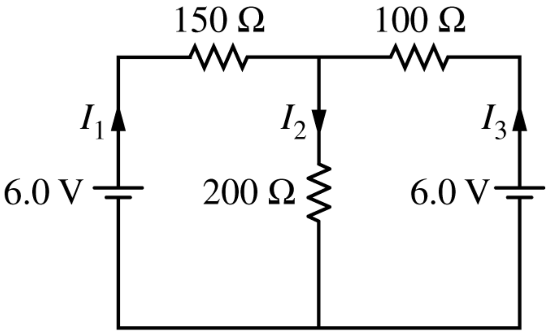


Figure 1

Question 2

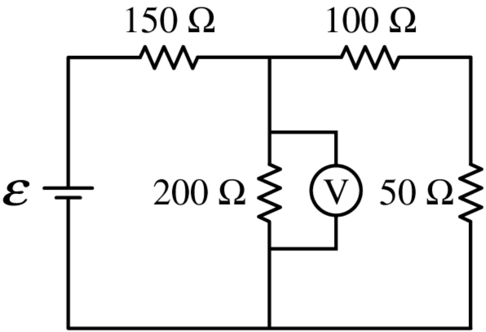


Figure 2

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(b) Calculate the current through the $50\ \Omega$ resistor.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(d)(i) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with a $200\ \mu$F capacitor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

Question 2

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu$F capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

(d)(ii) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with an ideal 50 mH inductor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?

$G^{reaterthan}{}_{L}{}^{essthan}{}_{E}{}^{qualto}$

Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Using Kirchhoff's rules (do not solve): Junction rule at the node joining all three branches: $I_1 - I_2 + I_3 = 0$ (1 pt). Loop rule, left loop (through the left 6.0 V battery, the 150 ohm resistor, and the 200 ohm resistor): $6 - 150I_1 - 200I_2 = 0$ (1 pt). Loop rule, right loop (through the right 6.0 V battery, the 100 ohm resistor, and the 200 ohm resistor): $6 - 100I_3 - 200I_2 = 0$ (1 pt). (Equivalently, the outer loop $6 - 150I_1 + 100I_3 - 6 = 0$ may replace one of these; full credit is also earned for two correct loop equations written using loop currents.)

Solution 2

(a)(ii)

Mark scheme

- Combine the three equations from part (a)(i) (1 pt for combining them / using a calculator's system-solver, e.g. $I_1 - I_2 + I_3 = 0$, $-I_1 - 1.33 I_2 = -0.04$, $-I_3 - 2 I_2 = -0.06 \Rightarrow -4.33 I_2 = -0.10$). Solving: $I_2 = \dfrac{-0.10}{-4.33} = 0.023$ A (1 pt for the correct answer with correct units).

Solution 2

(a)(iii)

Mark scheme

- $P = I_2^2 R = (0.023\text{ A})^2(200\ \Omega) = 0.107$ W. 1 point for using a correct power equation and computing this value.

Solution 2

(b) Mark scheme

- In the modified circuit (Figure 2), the 100 ohm and 50 ohm resistors are in series in the right-hand branch, so their combined resistance is $R = 100\ \Omega + 50\ \Omega = 150\ \Omega$ (1 pt). This branch is in parallel with the 200 ohm resistor, across which the voltmeter reads 4.4 V, so that same 4.4 V is the potential difference across the 150 ohm series combination: $I = \frac{V}{R} = \frac{4.4\ \text{V}}{100\ \Omega + 50\ \Omega} = 0.029$ A (1 pt for a correct answer using the correct potential difference).

Solution 2

(c) Mark scheme

- Using $I_1 = I_2 + I_3$ (1 pt for the correct equation determining current through the 150 ohm resistor) and substituting $I_3 = 0.029$ A from part (b) with $I_2 = \dfrac{4.4\text{ V}}{200\ \Omega}$ (1 pt for correct substitution): $I_1 = \dfrac{4.4\text{ V}}{200\ \Omega} + 0.029\text{ A} = 0.051\text{ A}$. Then apply the loop equation through the battery/150-ohm branch (1 pt): $\mathcal{E} = I_1 R_1 + V_{200\Omega\text{ branch}} = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$. (Equivalent alternate method: find the circuit's total equivalent resistance $R_T = 150\ \Omega + \left(\frac{1}{200} + \frac{1}{150}\right)^{-1}\Omega = 236\ \Omega$, giving $\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) \approx 12.0\text{-}12.1\text{ V}$ – also full credit.)

Solution 2

(d)(i) Mark scheme

- At steady state, the fully-charged 200 uF capacitor carries no current, so the 150 ohm, 100 ohm, and 50 ohm resistors form a single series loop carrying the same current (1 pt for correctly finding the total series resistance $R_{tot} = 150 + 100 + 50 = 300\ \Omega$). Using the battery voltage found in part (c), $\mathcal{E} = 12.1$ V (1 pt for substituting this consistent voltage into Ohm's law): $I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{12.1\text{ V}}{300\ \Omega} = 40.3$ mA.

Solution 2

(d)(ii)

Mark scheme

- Less than (1 pt for correctly selecting 'Less than' with an attempt at a relevant justification). Justification (1 pt): at steady state an ideal inductor acts as a short circuit (zero resistance / zero back-emf), so essentially all the current from the source flows through the (now zero-resistance) inductor branch and none flows through the 100-ohm + 50-ohm branch – so the current through the 50 ohm resistor is less than the 0.029 A found in part (b) (in fact it approaches zero).

Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length ℓ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

Question 3

(a)(i) On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.

(a)(ii) Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

_____ Toward the top of the page _____ To the left _____ Out of the page _____ Toward the bottom of the page _____ To the right _____ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A, B, and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B.

[Diagram: a rectangle representing the solenoid, with dot D above the rectangle at its midpoint, and inside the rectangle three dots along the central axis labeled $\bullet A$, $\bullet B$

Question 3

(both toward the left-middle), and •C (toward the right edge, outside the main cluster of A and B, near the right end).]

(a)(iii) From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0.

A B C D

Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.

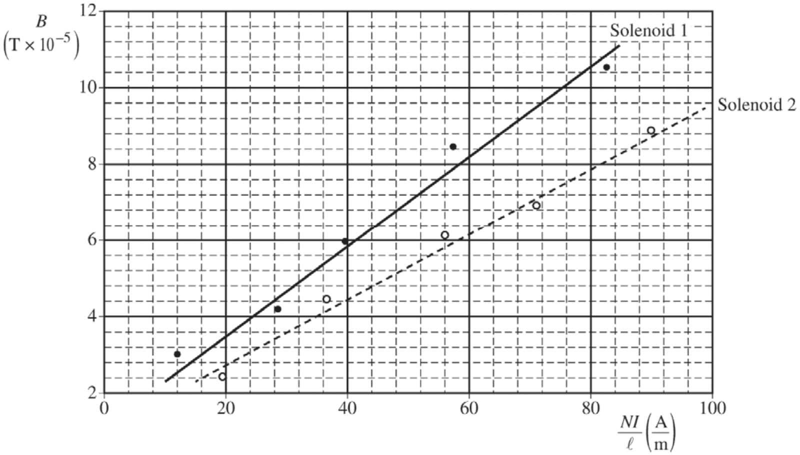
Question 3

[Two diagrams of coiled solenoids: "Solenoid 1" shown as a long tightly-wound coil, and "Solenoid 2" shown as a short coil with few turns. Note: Figures not drawn to scale.]
A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.
[Graph: vertical axis B $(\mathrm{T} \times 10^{-5})$ ranging 0 to 12 in increments of 2; horizontal axis $\frac{NI}{\ell}$ $\left(\frac{\mathrm{A}}{\mathrm{m}}\right)$ ranging 0 to 100 in increments of 20. Solid line "Solenoid 1" is a best-fit line through filled data points, rising from about $(12, 2.3)$ to about $(95, 11)$, roughly linear with a positive y-intercept near 0.5–1. Dashed line "Solenoid 2" is a best-fit line through open-circle data points, rising from about $(15, 1.7)$ to about $(85, 9)$, also

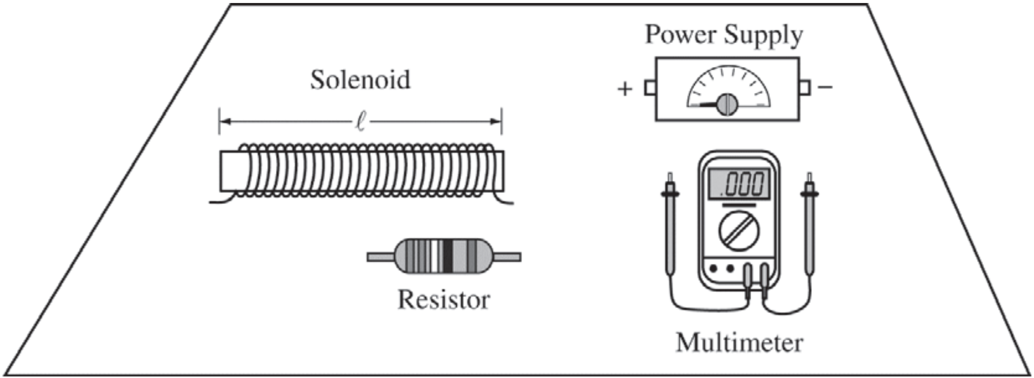
Question 3

roughly linear, lying slightly below Solenoid 1's line at low NI/ℓ and crossing to be below it at high NI/ℓ as well, with more scatter in its data points.]

Question 3

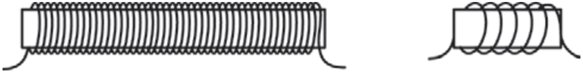


Question 3



Question 3

The figures below show two different solenoids that will be connected in the circuit above. Sole
a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Solenoid 1 Solenoid 2

<u>Note:</u> Figures not drawn to scale.

Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length ℓ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0?

Question 3

_____ Solenoid 1 _____ Solenoid 2

Justify your answer.

Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length ℓ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(c)(i) Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0.

Question 3

(c)(ii) Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

Question 3

2017 ▪ Q3

[Figure: a trapezoidal table setup showing a solenoid of length ℓ (a coiled wire wound on a hollow tube) connected in series with a resistor, and a power supply (with a dial/gauge, terminals $+$ and $-$) and a multimeter (with a digital display and a dial selector) sitting on the table nearby, not yet wired together.]

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(d)(i) What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

Question 3

(d)(ii) Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

Solution 3

(a)(i)

Mark scheme

- Draw wire connections that form a working circuit: the power supply, solenoid, and resistor connected in series with each other (1 point); the multimeter connected in series with the solenoid/resistor combination so it reads the current through the solenoid (1 point).

Solution 3

(a)(ii)

Mark scheme

- 1 point, awarded for a direction of the magnetic field inside the solenoid (toward top of page / left / out of page / toward bottom / right / into page) that is logically consistent (via the right-hand rule) with the current direction implied by the student's own wiring/connections chosen in part (a)i — there is no single fixed correct choice independent of that answer.

Solution 3

(a)(iii) Mark scheme

- Correct selection: point B (the point at the middle of the solenoid, on its central axis). Justification: the formula for the magnetic field of a solenoid ($B = \mu_0 nI$) is derived for an ideal (infinitely long) solenoid and does not hold outside the solenoid — this rules out points C and D, which lie outside (1 point, for any indication that the field outside the solenoid is less than inside). Near the ends/edges of the solenoid the ideal-solenoid approximation becomes less accurate due to edge effects — this rules out point A (1 point, for any indication that being closer to the center gives a better approximation because it avoids edge effects). Point B, at the solenoid's center, therefore gives the best approximation of an ideal solenoid (1 point, for selecting B).

Solution 3

(b)

Mark scheme

- Correct selection: Solenoid 1. Justification: Solenoid 1 has more turns per unit length than Solenoid 2 (1 point); Solenoid 1 is also longer, and so is more like an ideal (infinite) solenoid (1 point). Since $B = \mu_0 nI$ assumes an ideal solenoid, Solenoid 1's greater turn density and length make it a better approximation, so its best-fit-line slope gives a more accurate experimental value for μ_0.

Solution 3

(c)(i) Mark scheme

- Calculate the slope for the chosen solenoid (Solenoid 1) using the best-fit line (not individual data points):
  slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(10-4)(\mathsf{T} \times 10^{-5})}{(75-24)(\mathsf{A/m})} = 1.17 \times 10^{-6}$ T · m/A (1 point, for correctly calculating the slope from the best-fit line, not the data points). Relate the permeability of free space to the slope: $\mu_0 =$ slope (1 point). Correct units: $\mu_0 = 1.17 \times 10^{-6}$ T · m/A (1 point). [For reference, Solenoid 2's slope, not used here since Solenoid 1 was chosen in part (b):
  slope $= \dfrac{(8-4)(\mathsf{T} \times 10^{-5})}{(82-35)(\mathsf{A/m})} = 8.51 \times 10^{-7}$ T · m/A.]

Solution 3

(c)(ii) Mark scheme

- Use the correct percent-error formula, substituting the accepted value ($\mu_0 = 1.26 \times 10^{-6}$ T · m/A) and the experimental value from part (c)i:
  $\%\,\text{error} = \frac{|acc - exp|}{acc} \times 100\% = \frac{|(1.26 \times 10^{-6}) - (1.18 \times 10^{-6})|}{1.26 \times 10^{-6}} \times 100\% \approx 6.35\%$
  (1 point) [as transcribed in the official guideline for Solenoid 1; for Solenoid 2 the analogous calculation gives $\frac{|(1.26 \times 10^{-6}) - (8.51 \times 10^{-7})|}{1.26 \times 10^{-6}} \times 100\% \approx 32.4\%$].

Solution 3

(d)(i)

Mark scheme

- 1 point for a correct explanation involving any extraneous (stray) magnetic field, e.g.: a component of Earth's magnetic field lies along the direction of the solenoid's (inductor's) own magnetic field, shifting the best-fit line so it does not pass through the origin.

Solution 3

(d)(ii)

Mark scheme

- State that either the solenoid (inductor) or the circuit has near-zero resistance (1 point); state that, as a result of this near-zero resistance, the current becomes very high/near-infinite (1 point). Example: the inductor has near-zero resistance, so without the resistor in series with it the inductor draws a near-infinite current, which the multimeter cannot handle — causing the multimeter to stop functioning.

Question 2

2016 ▪ Q2

[Circuit diagram: A rectangular circuit loop. The top branch contains an ideal voltmeter V in parallel with a resistor of resistance R (drawn as a resistor symbol labeled R, with the voltmeter V connected across it above). The left branch contains a source of variable emf $\mathcal{E}$ (battery symbol with an arrow through it, indicating variability). The right branch contains a sample of wire with resistance r (resistor symbol labeled r). The bottom branch contains an ideal ammeter A. All four branches connect the loop in series around the rectangle.]

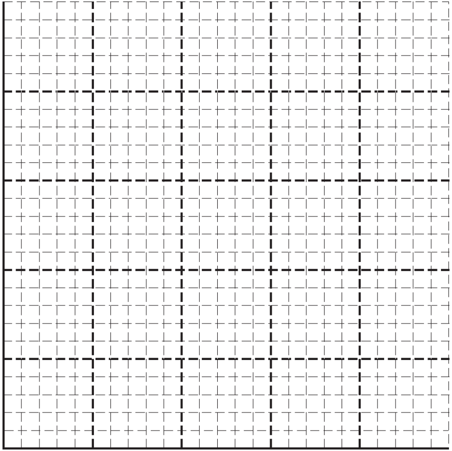
The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance R, and a sample of wire with resistance r.

(a) How does the current through the wire sample compare with the current through the resistor R?

Question 2

______ It is greater through R. ______ It is greater through the sample. ______ It is the same through both. ______ It depends on the resistance of the sample. Justify your answer.

Question 2



Question 2

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

Question 2