Past-paper walk-through

Dielectrics ▪ 10.4

eXercise

Questions in this set

- 2025 Q1
- 2023 Set 2 Q3
- 2018 Q2
- 2015 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2025 ▪ Q1

An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.
[Figure 1: Two diagrams. "Side View" shows a coaxial cylindrical capacitor of length L, with the inner shell labeled $+\sigma_1$ and the outer shell labeled $-\sigma_2$. "Cross-Sectional View" shows two concentric circles, the inner one with radius R_1 and the outer one with radius R_2. Note: Figures not drawn to scale.]
(a)(i) Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region

Question 1

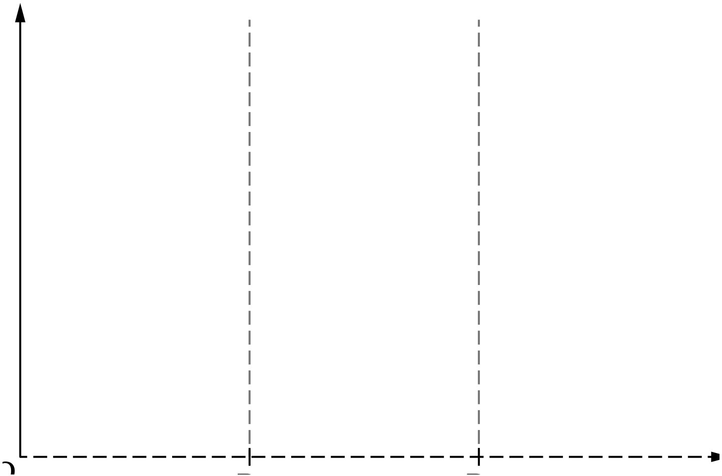
$R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

(a)(ii) **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

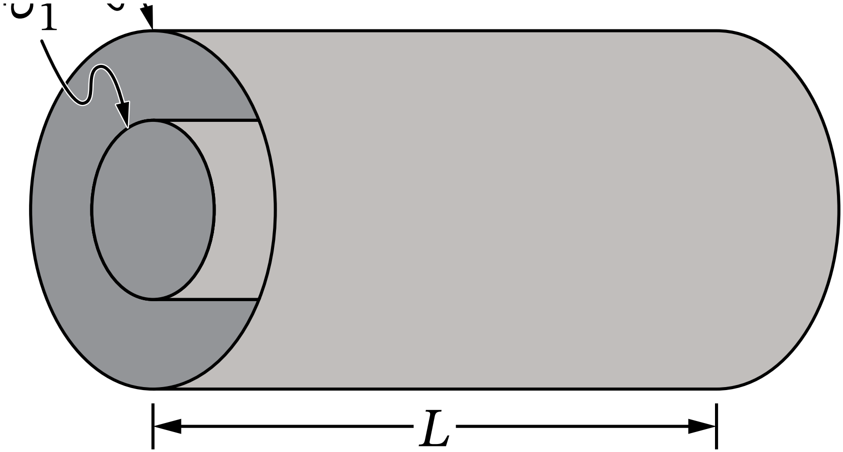
(a)(iii) On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

[Figure 2: A blank set of axes with vertical axis labeled E and horizontal axis labeled r, origin O. Two vertical dashed reference lines are marked on the r-axis at positions labeled R_1 and R_2.]

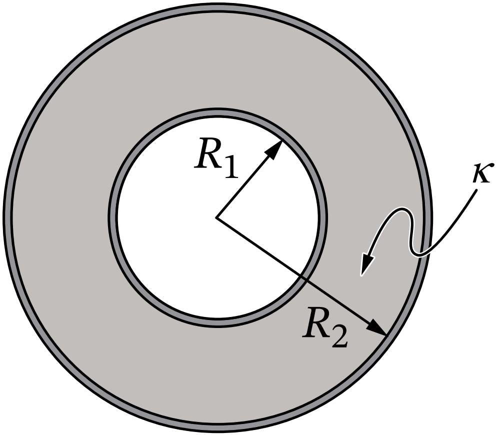
Question 1



Question 1



Question 1



Question 1

2025 ▪ Q1

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[Figure 1: Two diagrams. "Side View" shows a coaxial cylindrical capacitor of length L, with the inner shell labeled $+\sigma_1$ and the outer shell labeled $-\sigma_2$. "Cross-Sectional View" shows two concentric circles, the inner one with radius R_1 and the outer one with radius R_2. Note: Figures not drawn to scale.]

(b) A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.

Question 1

[Figure 3: "Cross-Sectional View" showing two concentric circles, the inner one with radius R_1 and the outer one with radius R_2; the annular region between them is shaded and labeled with κ, indicating the dielectric material fills that region.]

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Solution 1

(a)(i)

Mark scheme

- Apply Gauss's law: $\oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0. ChooseacylindricalGaussiansurfaceofradiusr(R_1 < r < R_2)andlength\ell, sothefluxtermisE(2\pi r\ell). Theenclosedchargeisonlyontheinnershell: q_{enc} = \sigma_1(2\pi R_1\ell). Substituting : E(2\pi r\ell) = \sigma_1(2\pi R_1\ell)/\varepsilon_0, givingE = \sigma_1 R_1/(\varepsilon_0 r). Points : (1)writingGauss's law, (2)correctGaussian - surfacearea2\pi r\ell forR_1 < r < R_2, (3)correctenclosedcharge\sigma_1(2\pi R_1\ell).$

Solution 1

(a)(ii) Mark scheme

- Substitute E from part (a)(i) into $\Delta V = -\int_a^b \vec{E}\cdot d\vec{r}. Then |\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r\varepsilon_0} dr = \frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r} dr = \frac{\sigma_1 R_1}{\varepsilon_0} \ln(r)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right).Points: (1) substituting E into the potential - difference integral, (2) attempting/solving the integral with correct limits R_1 to R_2 (order of limits doesn$

Solution 1

(a)(iii)

Mark scheme

- Sketch E vs r: E = 0 for 0<r<R_1 $andforr > R_2(fieldiszeroinsidetheinnershellandoutsidetheoutershell, sinceaGaussiansurfacethereenclosesz$ $r < R_2, E = \sigma_1 R_1/(\varepsilon_0 r), acurvethatisdecreasingandconcaveup(doesnotneedtotouchthedashec$ $(1) zerofor 0 < r < R_1 andr > R_2, (2) decreasingandconcave - upcurvebetweenR_1 andR_2.$

Solution 1

(b) Mark scheme

- Start from C = Q/$\Delta V. WiththedielectricfillingR_1 < r < R_2, thecapacitanceis\kappa timesthecapacitancewithoutthedielectric: C = \kappa Q/\Delta V. UsingQ = \sigma_1(2\pi LR_1)and\Delta V = \frac{\sigma_1 R_1}{\varepsilon_0}\ln(R_2/R_1)frompart(a): withoutthedielectric, C = \frac{\sigma_1(2\pi LR_1)}{\frac{\sigma_1 R_1}{\varepsilon_0}\ln(R_2/R_1)} = \frac{2\pi L\varepsilon_0}{\ln(R_2/R_1)}. Withthedielectric, C = \kappa\frac{Q}{\Delta V} = \frac{2\pi L\kappa\varepsilon_0}{\ln(R_2/R_1)}. Points: (1)C = Q/\Delta V, (2)C_{withdielectric} = \kappa \times C_{without}, (3)correctsubstitutionofQand\Delta Vconsistentwithpart(a).$

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

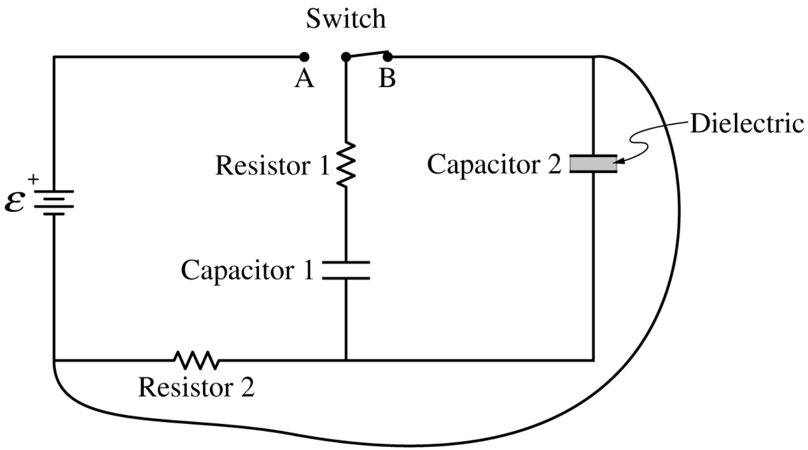
The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

Question 3

At time $t = 0$, the switch is closed to Position A.

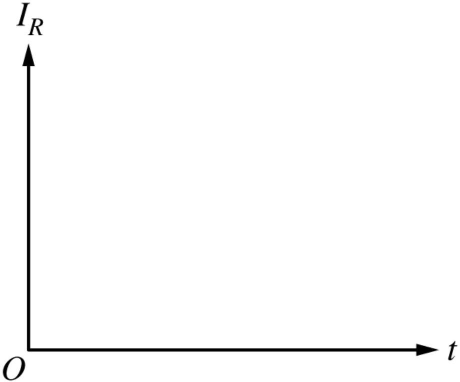
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

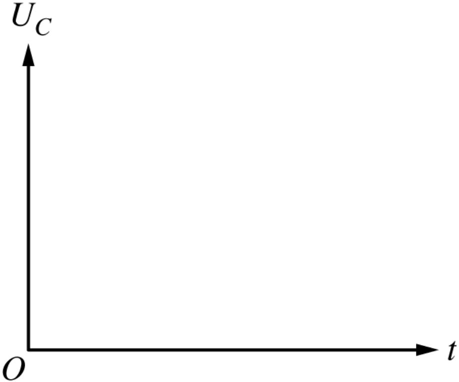
Question 3



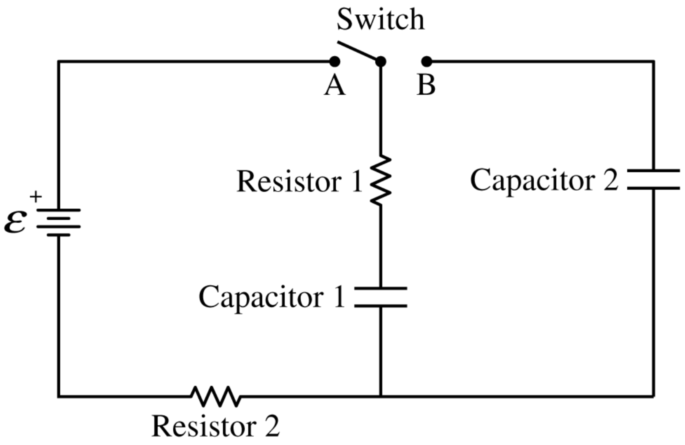
Switch
A
B
Dielectric
Resistor 1
Capacitor 2
$\mathcal{E}$
+
Capacitor 1
Resistor 2

Question 3





Question 3


Switch
A
B
Resistor 1
Capacitor 2
$\mathcal{E}$
+
Capacitor 1
Resistor 2

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.
[Graph 1: y-axis labeled "I_R", x-axis labeled "t", origin at O, no numeric scale shown.]
[Graph 2: y-axis labeled "U_C", x-axis labeled "t", origin at O, no numeric scale shown.]
A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.
At time t_1, the switch is closed to Position B.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(c)(i) Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.
(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
(c)(iii) Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.
With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
[Figure: The same circuit as before (battery $\mathcal{E}$, switch positions A/B, Resistor 1, Capacitor 1, Resistor 2, Capacitor 2), now with a dielectric slab labeled "Dielectric" shown inserted between the plates of Capacitor 2. An additional wire of negligible resistance is shown connected between two corners of the circuit (from the node between the switch and Resistor 1/Capacitor 1, curving around to the node below Capacitor 2/Resistor 2), as described below.]
With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(e)(i) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.
(e)(ii) Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

Solution 3

(a) Mark scheme

- Write, but do NOT solve, the differential equation for charge Q on the positive plate of Capacitor 1 vs time t after the switch closes to Position A, in terms of $\mathcal{E}, R, C, Q, t$. (1 point) Loop-rule expression that includes terms for the equivalent resistance $2R$ (the pair of resistors) and the potential difference across the battery. (1 point) An expression that includes charge Q in the term for the potential difference across the capacitor, and charge per unit time $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Result: $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0 \;\Rightarrow\; \mathcal{E} - \frac{Q}{C} - 2R\frac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 vs time t from $t = 0$ until steady state is nearly reached (RC charging curves). (1 point) I_R vs t is a CONCAVE UP, DECREASING curve starting at its maximum value at $t = 0$. (1 point) U_C vs t is a CONCAVE DOWN, INCREASING curve starting at zero. (1 point, can be earned even if the first two are not) Both curves approach a slope of zero (level off / asymptote) as time increases, consistent with exponential charging behavior.

Solution 3

(c)(i) Mark scheme

- Immediately after t_1 (switch moved to Position B, connecting the charged Capacitor 1 through Resistor 1 to the initially uncharged Capacitor 2), the current in Resistor 1 is directed UP. Justification (either form acceptable): positive charge has accumulated on the top plate of Capacitor 1 (and/or negative charge on its bottom plate) from being charged while the switch was at Position A, so at t_1 the higher potential top plate drives current up through Resistor 1 and down through Capacitor 2 to charge it. Equivalently: the top plate of Capacitor 1 is at higher electric potential than the bottom plate, so current flows from high to low potential, i.e. up through Resistor 1, around the loop through Capacitor 2.

Solution 3

(c)(ii) Mark scheme

- Determine the total charge on the positive plate of Capacitor 2 a long time after t_1: the answer is $\frac{Q_0}{2}$ (this point can be earned without supporting calculations). Justification: at new steady state, the potential difference across Capacitor 1 equals that across Capacitor 2 (parallel connection via Resistor 1 with no current, so no voltage drop across R1). Since Capacitor 2 has the same capacitance as Capacitor 1, it stores the same charge. By conservation of the total charge Q_0 originally on Capacitor 1, the two capacitors split it evenly, so Capacitor 2 stores $Q_0/2$.

Solution 3

(c)(iii) Mark scheme

- Derive an expression for the total energy dissipated by Resistor 1 from immediately after t_1 until the new steady state, in terms of C, Q_0. (1 point) Indicate that the energy dissipated equals the difference between the initial electric potential energy (at $t = t_1$) and the final electric potential energy (after new steady state): $\Delta E_R = U_C - U_{0C}$. (1 point) Indicate that only Capacitor 1 stores nonzero PE initially, and BOTH capacitors store PE after the new steady state (or an alternate response consistent with part (c)(ii)): $U_{0C} = U_{01}$ AND $U_C = U_1 + U_2$. (1 point) Correct substitution of the charges from part (c)(ii) ($Q_0/2$ on each capacitor). Full solution: $\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \frac{Q_0^2}{4C} - \frac{Q_0^2}{2C} = -\frac{Q_0^2}{4C}$. The

Solution 3

magnitude of energy dissipated by Resistor 1 is $\frac{Q_0^2}{4C}$ (the negative sign reflects the decrease in stored electric PE, which is dissipated as heat).

Solution 3

(d) Mark scheme

- Determine the charge on the positive plate of Capacitor 2 a long time after a dielectric ($\kappa = 2$) is inserted between its plates, with the switch still at Position B: $Q_2 = \dfrac{2Q_0}{3}$ (this point can be earned without supporting calculations). Derivation: with $C_1 = C$ and $C_2 = \kappa C = 2C$, the plates are still connected so $\Delta V_{C1} = \Delta V_{C2} \Rightarrow \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_1 = \frac{1}{2}Q_2$. Total charge is conserved: $Q_1 + Q_2 = Q_0 \Rightarrow \frac{1}{2}Q_2 + Q_2 = Q_0 \Rightarrow Q_2 = \dfrac{2}{3}Q_0$.

Solution 3

(e)(i) Mark scheme

- Derive an expression for the current in Resistor 2 immediately after an additional (negligible-resistance) wire is connected into the circuit, in terms of R, C, Q_0. (1 point) Loop-rule equation with terms for the potential difference across Resistor 2 and the potential difference across (dielectric-filled) Capacitor 2: $\Delta V_{R,2} - \Delta V_{C,2} = 0$. (1 point) Correct substitution of IR for the potential difference across Resistor 2: $\Delta V_{R,2} = IR$. (1 point) Correct substitution of $2C$ for Capacitor 2's new (dielectric) capacitance and of the charge from part (d) $(2Q_0/3)$ into the expression for $\Delta V_{C,2} = \dfrac{Q_2}{C_2} = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right)$. Full solution: $IR = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right) = \dfrac{Q_0}{3C} \Rightarrow I = \dfrac{Q_0}{3RC}$.

Solution 3

Solution 3

(e)(ii)

Mark scheme

- Determine the current in Resistor 2 a long time after the wire is connected to the circuit: the current is ZERO. At the new steady state, Capacitor 2 is fully charged (no more charge flow needed to maintain the loop's voltage balance), so no current flows through Resistor 2.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Question 2

Sheets of Paper	d (m)	C (F)
500	0.045	6.5×10^{-11}
400	0.036	7.4×10^{-11}
300	0.027	8.9×10^{-11}
200	0.018	11.9×10^{-11}
100	0.010	21.0×10^{-11}

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________________

Horizontal axis: ____________________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

Question 2

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Question 2

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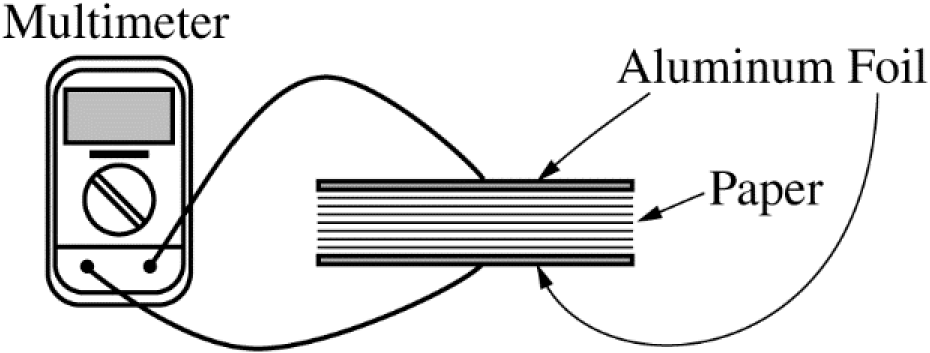

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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Question 2

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.
[Graph: A blank grid (large square grid of fine gridlines with heavier gridlines every 5 squares) for the student to plot data points and draw a best-fit straight line.]

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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Question 2

(c) Using the straight line, calculate a dielectric constant for the paper.
[Circuit diagram: A 36 V battery (with + terminal marked) in series with an open switch, connected to a circuit containing three identical $80\ \Omega$ resistors and an 18 nF capacitor. Two $80\ \Omega$ resistors are in the top branch (one in series with the main loop, one in a parallel branch also carrying the label $80\ \Omega$), in series with the 18 nF capacitor; a third $80\ \Omega$ resistor is in the bottom branch of the circuit.]
The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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Question 2

(d) Calculate the current in the battery immediately after the switch is closed.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)
500	0.045	6.5×10^{-11}
400	0.036	7.4×10^{-11}
300	0.027	8.9×10^{-11}
200	0.018	11.9×10^{-11}
100	0.010	21.0×10^{-11}

Question 2

(e) Determine the time constant for this circuit.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

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Question 2

(f)(i) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.
Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?
_______ Larger than measured _______ Smaller than measured
Explain experimentally what could account for this.
(f)(ii) Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?
_______ Larger than measured _______ Smaller than measured
Explain experimentally what could account for this.

Solution 2

(a)

Mark scheme

- Vertical axis: C. Horizontal axis: $1/d$. (Reversed axes, or another algebraically valid straight-line combination, also earn full credit.) Rationale: $C = \dfrac{\kappa\varepsilon_0 A}{d}$, so plotting C vs. $1/d$ gives a straight line through the origin with slope $\kappa\varepsilon_0 A$.

Solution 2

(b)

Mark scheme

- Compute $1/d$ for each row: $d = 0.045$ m $\Rightarrow 1/d \approx 22.2$ m^{-1}; $d = 0.036 \Rightarrow 27.8$; $d = 0.027 \Rightarrow 37.0$; $d = 0.018 \Rightarrow 55.6$; $d = 0.010 \Rightarrow 100$. Plot $C\,(\mathrm{F} \times 10^{-11})$ on the vertical axis against these $1/d$ values: points approximately $(22.2, 6.5)$, $(27.8, 7.4)$, $(37.0, 8.9)$, $(55.6, 11.9)$, $(100, 21.0)$. Full credit needs: a scale that uses more than half the grid (1 point), correctly labeled axes with units (1 point), correctly plotted data points (1 point), and a single straight best-fit line consistent with the data (1 point).

Solution 2

(c) Mark scheme

- Read the slope from the best-fit line (not raw data points unless they lie on the line), e.g. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{(17-8)(\mathsf{F}\times 10^{-11})}{(80-32)(1/\mathsf{m})} \approx 0.19\ \mathsf{F}\cdot\mathsf{m}$ (linear regression gives $0.187\ \mathsf{F}\cdot\mathsf{m}$) (1 point). Relate the slope to the dielectric constant: slope $= \kappa\varepsilon_0 A$ (1 point). Solve: $\kappa = \dfrac{\text{slope}}{\varepsilon_0 A} = \dfrac{0.19\ \mathsf{F}\cdot\mathsf{m}}{(8.85\times 10^{-12}\ \mathsf{C}^2/\mathsf{N}\cdot\mathsf{m}^2)(0.060\ \mathsf{m}^2)} \approx 3.58$ (linear regression value: $\kappa \approx 3.52$) (1 point).

Solution 2

(d) Mark scheme

- Combine the two parallel $80\ \Omega$ resistors: $\dfrac{1}{R_P} = \dfrac{1}{80} + \dfrac{1}{80} = \dfrac{1}{40} \Rightarrow R_P = 40\ \Omega$ (1 point). Immediately after the switch closes the initially-uncharged capacitor has zero voltage across it, so Ohm's law applies to the series combination of R_P and the third $80\ \Omega$ resistor R_3: $I = \dfrac{V}{R_P + R_3}$ (1 point). Substitute values: $I = \dfrac{36\ \text{V}}{40\ \Omega + 80\ \Omega} = 0.30\ \text{A}$ (1 point).

Solution 2

(e)

Mark scheme

- Time constant uses the equivalent series resistance seen by the capacitor: $\tau = R_{eq}C = (R_P + R_3)C = (120\ \Omega)(18\ \text{nF})$ (1 point). Evaluate with correct units (consistent with part (d)): $\tau = 2.16\ \mu\text{s}$ (1 point).

Solution 2

(f)(i)

Mark scheme

- Answer: Larger than measured. Explanation: the battery is not ideal and has internal resistance, so the actual resistance in the circuit is larger than the measured resistance — a larger effective resistance produces a longer charging time, matching the observation.

Solution 2

(f)(ii)

Mark scheme

- Answer: Larger than measured. Explanation: some of the sheets of paper may be thinner than expected, so the actual capacitance in the circuit is larger than the measured capacitance — a larger effective capacitance also produces a longer charging time, matching the observation.

Question 1

2015 ▪ Q1

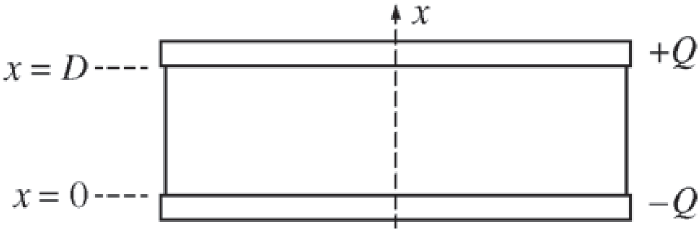
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.
[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]
(a)(i) On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
(a)(ii) On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

Question 1

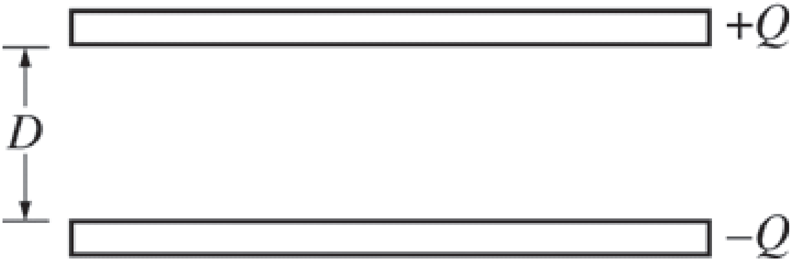
(a)(iii) Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.
[Figure: A vertical x-axis points upward. The bottom plate is at $x = 0$, labeled $-Q$; the top plate is at $x = D$, labeled $+Q$. A dashed vertical center line runs between the plates.]
The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

Question 1

of the electric field E between the plates. Express your answer in term
constants, as appropriate.



Question 1



Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(b) Determine an expression for the dielectric constant κ as a function of x.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(c)(i) Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

(c)(ii) Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(d) Determine the capacitance of the capacitor.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

_____ $U_V < U_C$ _____ $U_V > U_C$ _____ $U_V = U_C$

Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The electric field between two oppositely charged parallel plates points from the positive plate toward the negative plate. Since the $+Q$ plate is at $x = D$ (top) and the $-Q$ plate is at $x = 0$ (bottom), the field points downward, from the top plate to the bottom plate. Credit (1 point) requires at least one arrow between the plates pointing downward (from $+Q$ toward $-Q$) with no extraneous arrows pointing in any other direction.

Solution 1

(a)(ii)

Mark scheme

- Draw a Gaussian pillbox (a small rectangular box with flat faces parallel to the plates) straddling one of the plates so that it encloses at least the inner edge of that plate, with its flat end-faces parallel to the plate surfaces (perpendicular to $\vec{E}$). Credit (1 point) requires an appropriate Gaussian surface enclosing at least the inner edge of one plate that can be used to determine E between the plates.

Solution 1

(a)(iii) Mark scheme

- Start from Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\varepsilon_0}$ (1 point for a correct statement of Gauss's law). Apply it to the pillbox from part (a)-ii: only the flat end-face between the plates carries flux (the field is uniform and perpendicular to that face, and zero elsewhere on the surface), so $EA_{GS} = \dfrac{q_{enc}}{\varepsilon_0}$, giving $E = \dfrac{q_{enc}}{\varepsilon_0 A_{GS}}$; writing the surface charge density as $\sigma = \dfrac{q_{enc}}{A_{GS}} = \dfrac{Q}{A}$ (enclosed charge and Gaussian area consistent with the plate) gives $E = \dfrac{\sigma}{\varepsilon_0}$ (1 point for applying Gauss's law with enclosed charge and area consistent with the surface drawn in part (a)-ii). The

Solution 1

final result, with work shown, is $E = \dfrac{Q}{\varepsilon_0 A}$ (1 point for a correct answer with work shown).

Solution 1

(b)

Mark scheme

- Compare the standard parallel-plate field with a uniform dielectric, $E = \dfrac{Q}{\kappa \varepsilon_0 A}$, to the given expression for the field with the varying dielectric in place, $E = \dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$. Matching the two expressions term by term gives $\kappa = \kappa_0 e^{-x/D}$ (1 point; the answer must be consistent with the expression for E found in part (a)-iii).

Solution 1

(c)(i) Mark scheme

- Use the relation between electric field and potential, $E = -\dfrac{dV}{dx}$, and substitute the field from part (b): $\dfrac{dV}{dx} = -\left(-\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\right) = \dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$ (1 point for a correct differential equation; it does not need to be solved yet). An equivalent alternate approach accepted by the scoring guidelines uses $\Delta V = -\int \vec{E} \cdot d\vec{r}$ directly, giving $\Delta V = \displaystyle\int \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\, dx$, which earns the same 1 point.

Solution 1

(c)(ii) Mark scheme

- Separate variables in the differential equation from part (c)-i: $dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) e^{x/D}\, dx$. Integrate using the correct limits (V runs from V_0 to V_D as x runs from 0 to D): $\int_{V_0}^{V_D} dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \int_0^D e^{x/D}\, dx$ (1 point for using the correct limits of integration). Carrying out the integration: $[V]_{V_0}^{V_D} = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \left[De^{x/D}\right]_0^D$, so $(V_D - V_0) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) \left(e^{D/D} - e^0\right) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right) (e - 1)$ (1 point for correctly

Solution 1

integrating the equation). Taking the magnitude gives $\Delta V = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)$ (1 point for the correct absolute value of the potential difference between the plates), and this value must be reported as positive (1 point for having the potential difference be positive). Numerically, $\Delta V = \frac{1.72\, QD}{\varepsilon_0 \kappa_0 A}$.

Solution 1

(d) Mark scheme

- Use the definition of capacitance, $C = \dfrac{Q}{\Delta V}$, and substitute the result from part (c)-ii: $C = \dfrac{Q}{\left(\dfrac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)} = \dfrac{\varepsilon_0 \kappa_0 A}{D(e-1)} = \dfrac{\varepsilon_0 \kappa_0 A}{1.72D}$ (1 point; the answer must be consistent with part (c)-ii).

Solution 1

(e) Mark scheme

- Select $U_V > U_C$ (1 point). Justification: from part (d), a capacitor with the uniform dielectric constant κ_0 throughout has a larger capacitance than the varying-dielectric capacitor, i.e. $C_C > C_V$, because $\kappa = \kappa_0 e^{-x/D} \leq \kappa_0$ everywhere reduces the effective capacitance compared to a full-κ_0 fill (1 point for correctly comparing the capacitance, or equivalently the potential difference, of the varying-dielectric capacitor to that of the constant-dielectric capacitor). Since both capacitors are given the same charge Q and $U = \dfrac{Q^2}{2C}$, the smaller capacitance stores more energy; because $C_C > C_V$, it follows that $U_V > U_C$ (1 point for correctly comparing the two stored energies consistent with the

Solution 1

capacitance/potential-difference comparison). Example from the scoring guidelines: 'According to the equation from part (d), $C_C > C_V$. Since $U = Q^2/2C$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.'