Past-paper walk-through

Capacitors ▪ 10.3

eXercise

Questions in this set

- 2025 Q1
- 2023 Set 1 Q3
- 2023 Set 2 Q3
- 2022 Set 1 Q2
- 2021 Q1
- 2019 Set 2 Q1
- 2018 Q2
- 2017 Q2
- 2015 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

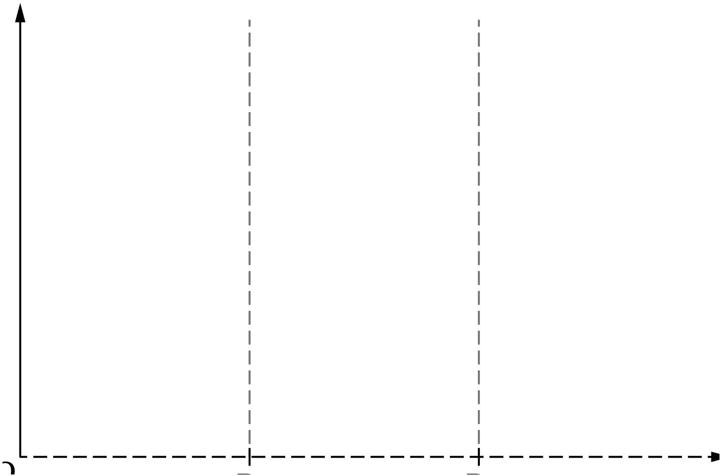
2025 ▪ Q1

An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.
[Figure 1: Two diagrams. "Side View" shows a coaxial cylindrical capacitor of length L, with the inner shell labeled $+\sigma_1$ and the outer shell labeled $-\sigma_2$. "Cross-Sectional View" shows two concentric circles, the inner one with radius R_1 and the outer one with radius R_2. Note: Figures not drawn to scale.]
(a)(i) Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region

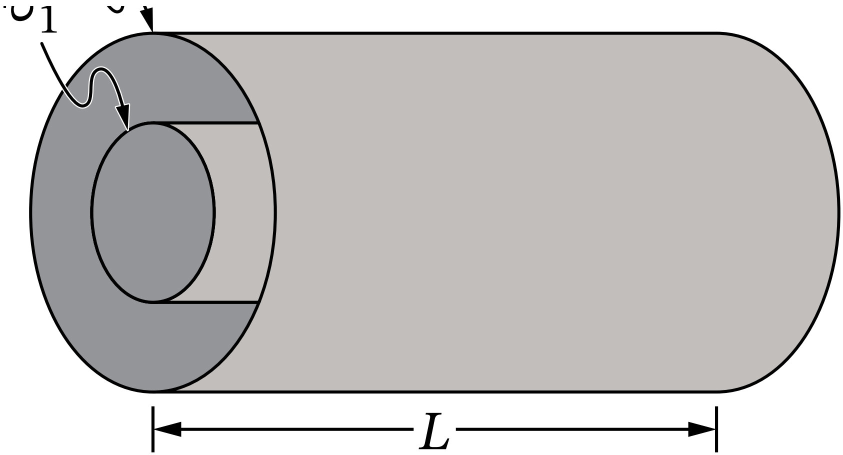
Question 1

$R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.
(a)(ii) **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
(a)(iii) On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.
[Figure 2: A blank set of axes with vertical axis labeled E and horizontal axis labeled r, origin O. Two vertical dashed reference lines are marked on the r-axis at positions labeled R_1 and R_2.]

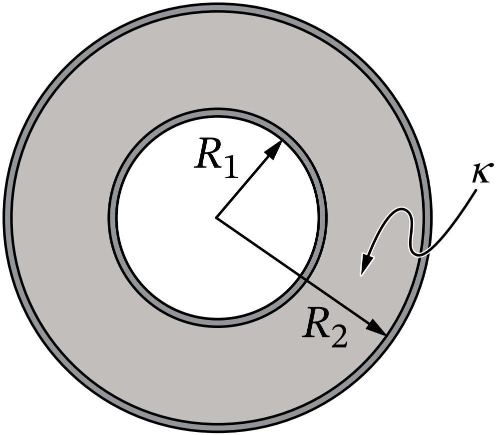
Question 1



Question 1



Question 1



Question 1

2025 ▪ Q1

An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.

[Figure 1: Two diagrams. "Side View" shows a coaxial cylindrical capacitor of length L, with the inner shell labeled $+\sigma_1$ and the outer shell labeled $-\sigma_2$. "Cross-Sectional View" shows two concentric circles, the inner one with radius R_1 and the outer one with radius R_2. Note: Figures not drawn to scale.]

(b) A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.

Question 1

[Figure 3: "Cross-Sectional View" showing two concentric circles, the inner one with radius R_1 and the outer one with radius R_2; the annular region between them is shaded and labeled with κ, indicating the dielectric material fills that region.]
Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Solution 1

(a)(i)

Mark scheme

- Apply Gauss's law: $\oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0. ChooseacylindricalGaussiansurfaceofradiusr(R_1 < r < R_2)andlength\ell, sothefluxtermisE(2\pi r\ell). Theenclosedchargeisonlyontheinnershell: q_{enc} = \sigma_1(2\pi R_1\ell). Substituting : E(2\pi r\ell) = \sigma_1(2\pi R_1\ell)/\varepsilon_0, givingE = \sigma_1 R_1/(\varepsilon_0 r). Points : (1)writingGauss'slaw, (2)correctGaussian - surfacearea2\pi r\ell forR_1 < r < R_2, (3)correctenclosedcharge\sigma_1(2\pi R_1\ell).$

Solution 1

(a)(ii) Mark scheme

- Substitute E from part (a)(i) into $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}. Then |\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r\varepsilon_0} dr = \frac{\sigma_1 R_1}{\varepsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr = \frac{\sigma_1 R_1}{\varepsilon_0} \ln(r)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right).Points: (1) substituting E into the potential - difference integral, (2) attempting/solving the integral with correct limits R_1 to R_2 (order of limits doesn$

Solution 1

(a)(iii)

Mark scheme

- Sketch E vs r: E = 0 for 0<r<R_1 $andforr > R_2(fieldiszeroinsidetheinnershellandoutsidetheoutershell, sinceaGaussiansurfacethereenclosesz$ $r < R_2, E = \sigma_1 R_1/(\varepsilon_0 r), acurvethatisdecreasingandconcaveup(doesnotneedtotouchthedashed$ $(1)zerofor 0 < r < R_1 andr > R_2, (2)decreasingandconcave - upcurvebetweenR_1 andR_2.$

Solution 1

(b) Mark scheme

- Start from C $= \mathrm{Q}/\Delta V. With the dielectric filling R_1 < r < R_2, the capacitance is \kappa times the capacitance without the dielectric: C = \kappa Q/\Delta V. Using Q = \sigma_1(2\pi L R_1) and \Delta V = \frac{\sigma_1 R_1}{\varepsilon_0}\ln(R_2/R_1) from part (a): without the dielectric, C = \frac{\sigma_1(2\pi L R_1)}{\frac{\sigma_1 R_1}{\varepsilon_0}\ln(R_2/R_1)} = \frac{2\pi L\varepsilon_0}{\ln(R_2/R_1)}. With the dielectric, C = \kappa\frac{Q}{\Delta V} = \frac{2\pi L\kappa\varepsilon_0}{\ln(R_2/R_1)}. Points: (1) C = Q/\Delta V, (2) C_{with dielectric} = \kappa \times C_{without}, (3) correct substitution of Q and \Delta V consistent with part (a).$

Question 3

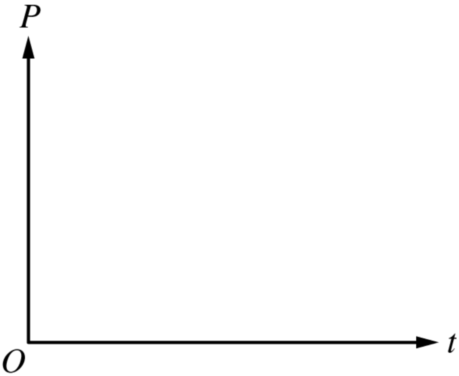
2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is

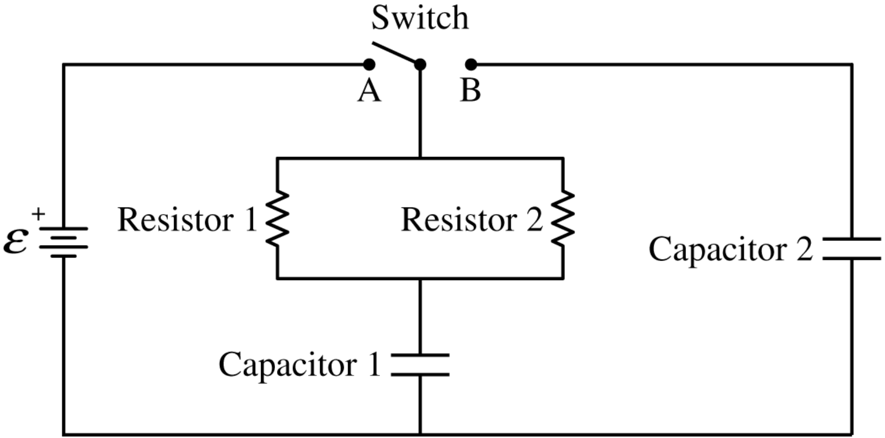
Question 3

closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

Question 3



Question 3



Switch
A
B
Resistor 1
Resistor 2
Capacitor 2
Capacitor 1
ε
+

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.

Question 3

[Graph 1: Blank axes, vertical axis σ, horizontal axis t, origin O, for the student to sketch surface charge density vs. time.]
[Graph 2: Blank axes, vertical axis P, horizontal axis t, origin O, for the student to sketch total power dissipated vs. time.]
A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(c)(i) At time t_1, the switch is closed to Position B.
Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

Question 3

(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

(c)(iii) Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(d) With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

Question 3

Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(e)(i) With the capacitors still charged as in part (d), the switch is now closed to Position A.

Question 3

Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.
(e)(ii) With the capacitors still charged as in part (d), the switch is now closed to Position A.
Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

Solution 3

(a) Mark scheme

- Loop rule around the circuit with Capacitor 1 and the two parallel resistors (each of resistance R, combining to an equivalent resistance $R/2$):
  $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$ (1 pt, for a loop-rule expression that includes terms for the equivalent resistance $R/2$ and the potential difference across the battery).
  Writing the capacitor voltage as $\Delta V_C = Q/C$ and the resistor-pair voltage as $\Delta V_{R,eq} = \dfrac{R}{2}\dfrac{dQ}{dt}$ (1 pt, for an expression that includes charge Q in the capacitor-voltage term and dQ/dt in the resistor-voltage term) gives the final differential equation: $\mathcal{E} - \dfrac{Q}{C} - \dfrac{R}{2}\dfrac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Two sketches vs. time t, from $t = 0$ until steady state is nearly reached: the surface charge density σ on the positive plate of Capacitor 1 increases and is concave down (rises quickly, then levels off toward a maximum) (1 pt); the total power P dissipated by the resistors decreases and is concave up (falls quickly, then levels off toward zero) (1 pt); both curves approach a slope of zero as time increases, i.e. both flatten out as steady state is approached (1 pt — this point can be earned even if the first two points are not).

Solution 3

(c)(i) Mark scheme

- The current in the switch immediately after t_1 is directed toward the right. Justification: while the switch was closed to Position A, Capacitor 1 charged so that its top plate accumulated positive charge (and the bottom plate negative), making the top plate's electric potential higher than the bottom plate's; immediately after moving the switch to Position B, current flows from high to low potential, i.e., away from the top plate through the switch toward the right (into the second loop with Capacitor 2), so conventional current is directed toward the right. 1 point for this indication with justification along the lines of the accumulated plate charge or the relative potentials of the plates.

Solution 3

(c)(ii) Mark scheme

- The total charge on the positive plate of Capacitor 2 a long time after t_1 is $\frac{2}{3}Q_0$. Reasoning: once connected in the second loop, Capacitor 1 and Capacitor 2 reach the same potential difference across them; since Capacitor 2 has twice the capacitance of Capacitor 1, it stores twice as much charge as Capacitor 1 in that final state. By conservation of the original charge Q_0 (all of which was on Capacitor 1 before t_1), splitting it in a 1:2 ratio between Capacitor 1 and Capacitor 2 gives Capacitor 2 a final charge of $\frac{2}{3}Q_0$ (and Capacitor 1 a final charge of $\frac{1}{3}Q_0$). 1 point; can be earned without supporting calculations.

Solution 3

(c)(iii) Mark scheme

- The energy dissipated by resistors 1 and 2 equals the decrease in total capacitor stored energy: $E_R = U_{0C} - U_C$, i.e., $\Delta E_R = U_C - U_{0C}$ with $E_R = -\Delta E_R$ (1 pt, for indicating the total energy dissipated is the difference between the initial electric PE stored at $t = t_1$ and the final electric PE stored once the new steady state is reached). Before t_1, only Capacitor 1 stores nonzero energy, $U_{0C} = U_{01} = \dfrac{1}{2}\dfrac{Q_0^2}{C}$; after the new steady state, both capacitors store nonzero energy, $U_C = U_1 + U_2$ (1 pt, for this indication, or an alternative consistent with part (c)(ii)). Substituting the steady-state charges from part (c)(ii), $Q_1 = Q_0/3$ on Capacitor 1 (capacitance C) and $Q_2 = 2Q_0/3$ on Capacitor 2 (capacitance $2C$) (1 pt, for

Solution 3

correct substitutions consistent with (c)(ii)):
$\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\frac{1}{2C}\left(\frac{2Q_0}{3}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \frac{Q_0^2}{6C} - \frac{1}{2}\frac{Q_0^2}{C} = -\frac{Q_0^2}{3C}$. So the resistors dissipate a total energy of $\frac{Q_0^2}{3C}$.

Solution 3

(d) Mark scheme

- $\dfrac{U_2}{U_1} = 1$. Justification (each part 1 pt): after the new steady state is reached, the potential difference across each capacitor is the same (equivalently, the charge stored on each capacitor is the same), $\Delta V_1 = \Delta V_2$ or $Q_1 = Q_2$ (1 pt, indicating equal potential difference or equal charge across the two capacitors in steady state); and since capacitance is inversely related to plate separation, doubling Capacitor 2's plate separation halves its capacitance, making it equal to Capacitor 1's capacitance in this new configuration (1 pt, recognizing the capacitances are

Solution 3

now equal). Then, since $U_C = \frac{1}{2}C(\Delta V)^2$ with equal C and equal ΔV for both capacitors, $\frac{U_2}{U_1} = 1$.

Solution 3

(e)(i) Mark scheme

- With the switch closed to Position A again (capacitors 1 and 2, each already charged as in part (d), now effectively act together across the battery loop with the pair of resistors), the loop rule is $\mathcal{E} - \Delta V_R - \Delta V_C = 0$ (1 pt, for a loop rule including terms for the battery emf, the potential difference across the pair of resistors, and the potential difference across Capacitor 1 / the capacitor combination). Using $\mathcal{E} = Q_0/C$, $\Delta V_R = I(R/2)$, and $\Delta V_C = Q_0/(2C)$ (the steady-state, equal charge/capacitor voltage from part (d)): $\dfrac{Q_0}{C} - I\dfrac{R}{2} - \dfrac{Q_0}{2C} = 0 \Rightarrow \dfrac{Q_0}{2C} = I\dfrac{R}{2} \Rightarrow \dfrac{Q_0}{C} = IR \Rightarrow I_0 = \dfrac{Q_0}{RC}$ (1 pt, for the correct answer $I_0 = Q_0/(RC)$).

Solution 3

(e)(ii)

Mark scheme

- A long time after the switch is closed to Position A, the capacitors reach a new steady state (fully charged) and, in DC steady state, no current flows into or out of a fully-charged capacitor branch. Therefore $I_\infty = 0$. 1 point for indicating the current is zero.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

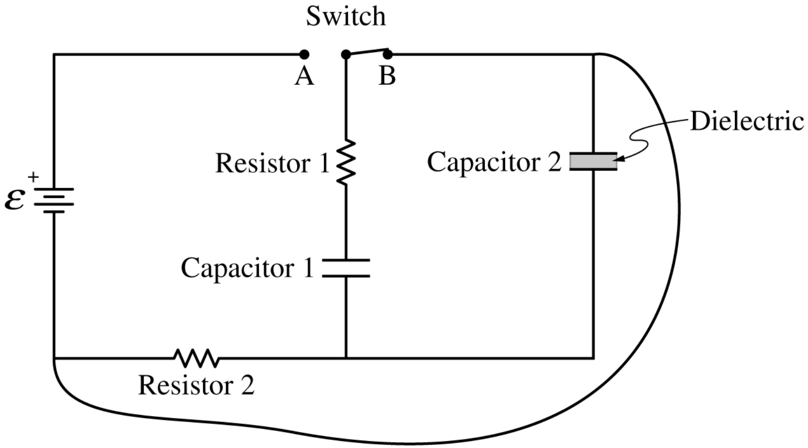
The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

Question 3

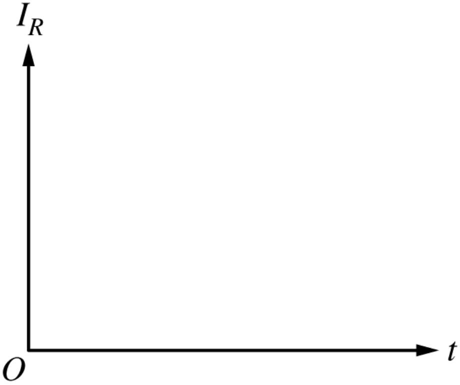
At time $t = 0$, the switch is closed to Position A.

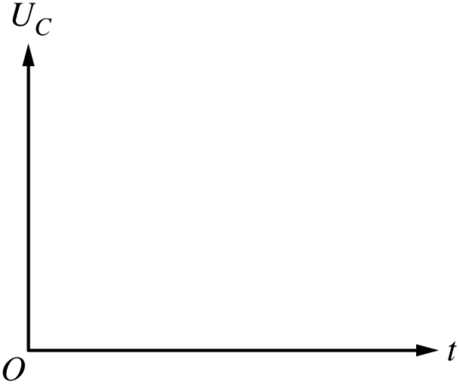
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

Question 3

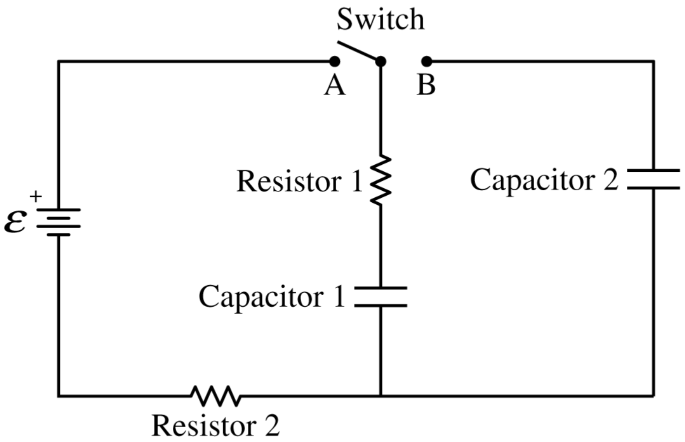

Switch
A
B
Dielectric
Resistor 1
Capacitor 2
+
ε
Capacitor 1
Resistor 2

Question 3





Question 3


Switch
A
B
Resistor 1
Capacitor 2
ε
+
Capacitor 1
Resistor 2

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.
[Graph 1: y-axis labeled "I_R", x-axis labeled "t", origin at O, no numeric scale shown.]
[Graph 2: y-axis labeled "U_C", x-axis labeled "t", origin at O, no numeric scale shown.]
A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.
At time t_1, the switch is closed to Position B.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(c)(i) Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

(c)(iii) Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
[Figure: The same circuit as before (battery $\mathcal{E}$, switch positions A/B, Resistor 1, Capacitor 1, Resistor 2, Capacitor 2), now with a dielectric slab labeled "Dielectric" shown inserted between the plates of Capacitor 2. An additional wire of negligible resistance is shown connected between two corners of the circuit (from the node between the switch and Resistor 1/Capacitor 1, curving around to the node below Capacitor 2/Resistor 2), as described below.]
With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(e)(i) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.
(e)(ii) Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

Solution 3

(a) Mark scheme

- Write, but do NOT solve, the differential equation for charge Q on the positive plate of Capacitor 1 vs time t after the switch closes to Position A, in terms of $\mathcal{E}, R, C, Q, t$. (1 point) Loop-rule expression that includes terms for the equivalent resistance $2R$ (the pair of resistors) and the potential difference across the battery. (1 point) An expression that includes charge Q in the term for the potential difference across the capacitor, and charge per unit time $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Result: $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0 \Rightarrow \mathcal{E} - \frac{Q}{C} - 2R\frac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 vs time t from $t = 0$ until steady state is nearly reached (RC charging curves). (1 point) I_R vs t is a CONCAVE UP, DECREASING curve starting at its maximum value at $t = 0$. (1 point) U_C vs t is a CONCAVE DOWN, INCREASING curve starting at zero. (1 point, can be earned even if the first two are not) Both curves approach a slope of zero (level off / asymptote) as time increases, consistent with exponential charging behavior.

Solution 3

(c)(i) Mark scheme

- Immediately after t_1 (switch moved to Position B, connecting the charged Capacitor 1 through Resistor 1 to the initially uncharged Capacitor 2), the current in Resistor 1 is directed UP. Justification (either form acceptable): positive charge has accumulated on the top plate of Capacitor 1 (and/or negative charge on its bottom plate) from being charged while the switch was at Position A, so at t_1 the higher potential top plate drives current up through Resistor 1 and down through Capacitor 2 to charge it. Equivalently: the top plate of Capacitor 1 is at higher electric potential than the bottom plate, so current flows from high to low potential, i.e. up through Resistor 1, around the loop through Capacitor 2.

Solution 3

(c)(ii) Mark scheme

- Determine the total charge on the positive plate of Capacitor 2 a long time after t_1: the answer is $\dfrac{Q_0}{2}$ (this point can be earned without supporting calculations). Justification: at new steady state, the potential difference across Capacitor 1 equals that across Capacitor 2 (parallel connection via Resistor 1 with no current, so no voltage drop across R1). Since Capacitor 2 has the same capacitance as Capacitor 1, it stores the same charge. By conservation of the total charge Q_0 originally on Capacitor 1, the two capacitors split it evenly, so Capacitor 2 stores $Q_0/2$.

Solution 3

(c)(iii) Mark scheme

- Derive an expression for the total energy dissipated by Resistor 1 from immediately after t_1 until the new steady state, in terms of C, Q_0. (1 point) Indicate that the energy dissipated equals the difference between the initial electric potential energy (at $t = t_1$) and the final electric potential energy (after new steady state): $\Delta E_R = U_C - U_{0C}$. (1 point) Indicate that only Capacitor 1 stores nonzero PE initially, and BOTH capacitors store PE after the new steady state (or an alternate response consistent with part (c)(ii)): $U_{0C} = U_{01}$ AND $U_C = U_1 + U_2$. (1 point) Correct substitution of the charges from part (c)(ii) ($Q_0/2$ on each capacitor). Full solution: $\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \frac{Q_0^2}{4C} - \frac{Q_0^2}{2C} = -\frac{Q_0^2}{4C}$. The

Solution 3

magnitude of energy dissipated by Resistor 1 is $\dfrac{Q_0^2}{4C}$ (the negative sign reflects the decrease in stored electric PE, which is dissipated as heat).

Solution 3

(d) Mark scheme

- Determine the charge on the positive plate of Capacitor 2 a long time after a dielectric $(\kappa = 2)$ is inserted between its plates, with the switch still at Position B: $Q_2 = \dfrac{2Q_0}{3}$ (this point can be earned without supporting calculations). Derivation: with $C_1 = C$ and $C_2 = \kappa C = 2C$, the plates are still connected so $\Delta V_{C1} = \Delta V_{C2} \Rightarrow \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_1 = \frac{1}{2}Q_2$. Total charge is conserved: $Q_1 + Q_2 = Q_0 \Rightarrow \frac{1}{2}Q_2 + Q_2 = Q_0 \Rightarrow Q_2 = \dfrac{2}{3}Q_0$.

Solution 3

(e)(i) Mark scheme

- Derive an expression for the current in Resistor 2 immediately after an additional (negligible-resistance) wire is connected into the circuit, in terms of R, C, Q_0. (1 point) Loop-rule equation with terms for the potential difference across Resistor 2 and the potential difference across (dielectric-filled) Capacitor 2: $\Delta V_{R,2} - \Delta V_{C,2} = 0$. (1 point) Correct substitution of IR for the potential difference across Resistor 2: $\Delta V_{R,2} = IR$. (1 point) Correct substitution of $2C$ for Capacitor 2's new (dielectric) capacitance and of the charge from part (d) $(2Q_0/3)$ into the expression for $\Delta V_{C,2} = \dfrac{Q_2}{C_2} = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right)$. Full solution: $IR = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right) = \dfrac{Q_0}{3C} \Rightarrow I = \dfrac{Q_0}{3RC}$.

Solution 3

Solution 3

(e)(ii)

Mark scheme

- Determine the current in Resistor 2 a long time after the wire is connected to the circuit: the current is ZERO. At the new steady state, Capacitor 2 is fully charged (no more charge flow needed to maintain the loop's voltage balance), so no current flows through Resistor 2.

Question 2

2022 Set 1 ▪ Q2

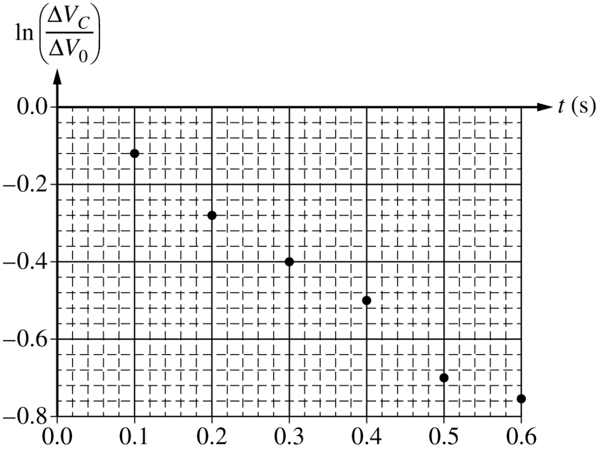
The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

Question 2

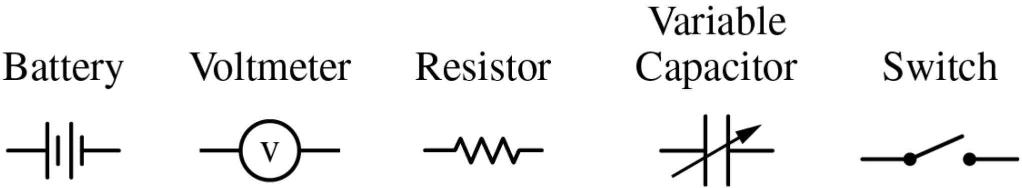
(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

Question 2



$\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$
t (s)
0.0
–0.2
–0.4
–0.6
–0.8
0.0
0.1
0.2
0.3
0.4
0.5
0.6

Question 2



Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(b) The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

Question 2

Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(c)(i) The experiment is performed using a resistor of $R = 150$ kΩ. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\dfrac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.

Question 2

[Scatter plot on graph paper. Vertical axis labeled $\ln\left(\dfrac{\Delta V_C}{\Delta V_0}\right)$, ranging from 0.0 to -0.8 in increments of 0.2. Horizontal axis labeled t (s), ranging from 0.0 to 0.6 in increments of 0.1. Data points approximately at: $(0.1, -0.10)$, $(0.2, -0.27)$, $(0.3, -0.38)$, $(0.4, -0.45)$, $(0.5, -0.63)$, $(0.6, -0.72)$.]

Draw the best-fit line for the data.

(c)(ii) Using the best-fit line, calculate a value for the unknown capacitance C.

Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second

Question 2

experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

____ More steep ____ Less steep ____ Unchanged

Briefly justify your answer.

Question 2

2022 Set 1 ▪ Q2

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.

[Diagram of circuit symbols with labels above each: "Battery" shown as a battery symbol; "Voltmeter" shown as a circle with V inside; "Resistor" shown as a zigzag resistor symbol; "Variable Capacitor" shown as a capacitor symbol with an arrow through it; "Switch" shown as a switch symbol.]

(e)(i) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

Question 2

Would the slope of the graph in this final experiment change compared to the graph in part (c)?

____ Yes ____ No

Briefly justify your answer.

(e)(ii) Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

____ Yes ____ No

Briefly justify your answer.

Solution 2

(a) Mark scheme

- Any schematic diagram satisfying all four criteria earns full credit: (1 pt) the capacitor in series with the resistor OR the resistor on a separate parallel path; (1 pt) the voltmeter connected in parallel with the capacitor; (1 pt) a switch used to connect the battery to the capacitor (to charge it); (1 pt) a switch that allows the capacitor to discharge through the resistor (bypassing the battery). Example circuit: a battery in series with switch S_1 and the resistor R, leading to capacitor C; the voltmeter V is wired in parallel across the capacitor; a second switch S_2 provides a path so the capacitor can discharge through R once S_1 is opened.

Solution 2

(b) Mark scheme

- Loop equation around the discharging RC loop: $V_C - V_R = 0$ (1 pt: appropriate loop equation). Substituting $\dfrac{q}{C}$ for V_C and IR for V_R: $\dfrac{q}{C} = IR$ (1 pt: correct substitution consistent with the loop equation). Using $I = -\dfrac{dq}{dt}$ for the discharging capacitor: $\dfrac{q}{C} = -\dfrac{dq}{dt}R \Rightarrow -\dfrac{1}{RC}dt = \dfrac{1}{q}dq$ (1 pt: differential expression consistent with the loop rule that includes the $I = -dq/dt$ substitution; sign is not graded). Integrating, $-\displaystyle\int_0^t \frac{1}{RC}dt' = \int_{q_0}^{q} \frac{1}{q'}dq'$ gives $-\dfrac{t}{RC} = \ln\left(\dfrac{q(t)}{q_0}\right)$, so $q(t) = q_0 e^{-t/RC}$, and dividing by C: $\Delta V_C(t) = \Delta V_0\, e^{-t/RC}$, as required.

Solution 2

(c)(i)

Mark scheme

- Draw a straight best-fit line through the plotted points of $\ln(\Delta V_C/\Delta V_0)$ vs. t — e.g. a line starting near $(0.0\text{ s}, 0.0)$ and sloping down to about $(0.6\text{ s}, -0.75)$, passing close to the scattered data points. Full credit for any reasonable straight best-fit line through the data.

Solution 2

(c)(ii) Mark scheme

- Start from $V(t) = V_0 e^{-t/RC} \Rightarrow \ln\left(\frac{V(t)}{V_0}\right) = -\frac{t}{RC}$ (1 pt: appropriate equation relating the unknown C to the data). Using two points on the best-fit line, e.g. $(0.30\text{ s}, -0.40)$ and $(0.54\text{ s}, -0.70)$: slope $= \frac{-0.70-(-0.40)}{0.54\text{ s}-0.30\text{ s}} = -1.25\text{ s}^{-1}$ (1 pt: correctly calculating the slope from two points on the best-fit line; the associated unit is not graded). Since slope $= -\frac{1}{RC}$: $C = \frac{-1}{\text{slope}\times R} = \frac{-1}{(-1.25\text{ s}^{-1})(150\text{ k}\Omega)} = 5\ \mu\text{F}$ (1 pt: correctly relating the slope to the unknown capacitance).

Solution 2

(d)

Mark scheme

- Answer: Less steep (1 pt, for the correct selection with an attempt at relevant justification). Increasing the plate area increases the capacitance C of the capacitor; since the slope of the graph is $-\dfrac{1}{RC}$, a larger C makes the magnitude of the slope smaller, so the best-fit line is less steep (1 pt: correctly relating the change in capacitance to the slope of the graph). (Scored consistently with the response to part (c).)

Solution 2

(e)(i)

Mark scheme

- Answer: No (1 pt, for the correct selection AND correctly describing the effect on the slope at $t = 0$). The capacitor still discharges only through resistor R — the battery and its internal resistance are disconnected from the loop during discharge — so the time constant RC, and hence the slope of the graph, is unchanged.

Solution 2

(e)(ii)

Mark scheme

- Answer: No (1 pt, for the correct selection AND correctly describing the effect on the vertical intercept/initial value). The best-fit line's intercept does not change, because the internal resistance of the battery does not affect the final potential difference across the capacitor once it is fully charged (no current flows through the internal resistance at that point, so there is no voltage drop across it). (Scored with consistency to the circuit drawn in part (a)/(b).)

Question 1

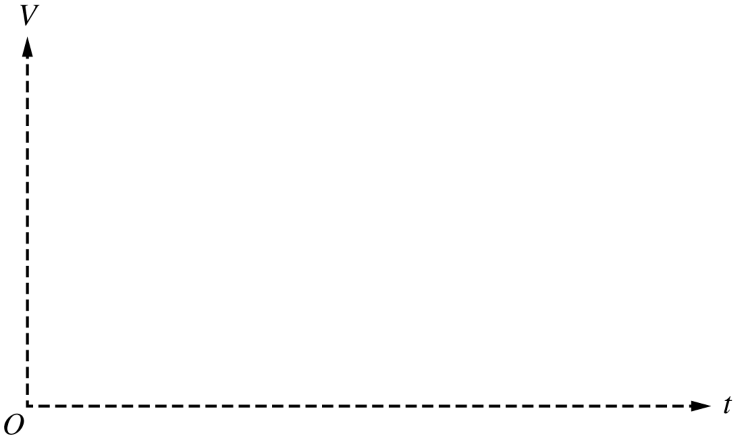
2021 ▪ Q1

[Circuit diagram: An ideal 10 V battery ($+$ terminal on top) connects through a switch S in series with resistor $R_1 = 100\ \Omega$. The circuit then splits into three parallel branches between the top and bottom rails: a branch with resistor $R_2 = 30\ \Omega$ (in series with switch S, both shown on the left branch), a branch with resistor $R_3 = 60\ \Omega$, a branch with two capacitors $C_1 = 10\ \mu$F and $C_2 = 15\ \mu$F stacked in series with each other, and a branch with capacitor $C_3 = 20\ \mu$F.]

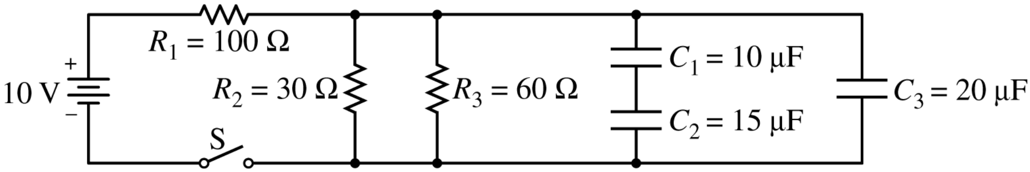
The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

Question 1



Question 1



$R_1 = 100\ \Omega$
10 V
+
−
$R_2 = 30\ \Omega$
$R_3 = 60\ \Omega$
S
$C_1 = 10\ \mu F$
$C_2 = 15\ \mu F$
$C_3 = 20\ \mu F$

Question 1

2021 ▪ Q1

[Circuit diagram: An ideal 10 V battery (+ terminal on top) connects through a switch S in series with resistor $R_1 = 100\ \Omega$. The circuit then splits into three parallel branches between the top and bottom rails: a branch with resistor $R_2 = 30\ \Omega$ (in series with switch S, both shown on the left branch), a branch with resistor $R_3 = 60\ \Omega$, a branch with two capacitors $C_1 = 10\ \mu$F and $C_2 = 15\ \mu$F stacked in series with each other, and a branch with capacitor $C_3 = 20\ \mu$F.]

The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(b) Switch S has been closed for a long time, and the circuit has reached a steady state.

Calculate the potential difference across R_1.

Question 1

2021 ▪ Q1

[Circuit diagram: An ideal 10 V battery (+ terminal on top) connects through a switch S in series with resistor $R_1 = 100\ \Omega$. The circuit then splits into three parallel branches between the top and bottom rails: a branch with resistor $R_2 = 30\ \Omega$ (in series with switch S, both shown on the left branch), a branch with resistor $R_3 = 60\ \Omega$, a branch with two capacitors $C_1 = 10\ \mu$F and $C_2 = 15\ \mu$F stacked in series with each other, and a branch with capacitor $C_3 = 20\ \mu$F.]

The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(c)(i) Calculate the charge stored on the positive plate of capacitor C_2.

(c)(ii) Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2?

______ Greater than ______ Less than ______ Equal to

Question 1

Justify your answer.

Question 1

2021 ▪ Q1

[Circuit diagram: An ideal 10 V battery (+ terminal on top) connects through a switch S in series with resistor $R_1 = 100\ \Omega$. The circuit then splits into three parallel branches between the top and bottom rails: a branch with resistor $R_2 = 30\ \Omega$ (in series with switch S, both shown on the left branch), a branch with resistor $R_3 = 60\ \Omega$, a branch with two capacitors $C_1 = 10\ \mu$F and $C_2 = 15\ \mu$F stacked in series with each other, and a branch with capacitor $C_3 = 20\ \mu$F.]

The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(d)(i) Switch S is then opened.

Determine the current through R_1 immediately after the switch is opened.

(d)(ii) Calculate the current through R_2 immediately after the switch is opened.

Question 1

2021 ▪ Q1

[Circuit diagram: An ideal 10 V battery (+ terminal on top) connects through a switch S in series with resistor $R_1 = 100\ \Omega$. The circuit then splits into three parallel branches between the top and bottom rails: a branch with resistor $R_2 = 30\ \Omega$ (in series with switch S, both shown on the left branch), a branch with resistor $R_3 = 60\ \Omega$, a branch with two capacitors $C_1 = 10\ \mu$F and $C_2 = 15\ \mu$F stacked in series with each other, and a branch with capacitor $C_3 = 20\ \mu$F.]

The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.

Question 1

[Graph axes: vertical axis V, horizontal axis t; origin labeled O; a dashed vertical guideline is shown near the vertical axis, and a dashed horizontal guideline extends across the plot area; no data plotted, blank axes for the student to sketch on.]

Question 1

2021 ▪ Q1

[Circuit diagram: An ideal 10 V battery (+ terminal on top) connects through a switch S in series with resistor $R_1 = 100\ \Omega$. The circuit then splits into three parallel branches between the top and bottom rails: a branch with resistor $R_2 = 30\ \Omega$ (in series with switch S, both shown on the left branch), a branch with resistor $R_3 = 60\ \Omega$, a branch with two capacitors $C_1 = 10\ \mu$F and $C_2 = 15\ \mu$F stacked in series with each other, and a branch with capacitor $C_3 = 20\ \mu$F.]

The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(f) Capacitor C_3 is replaced by two $10\ \mu$F capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$. For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e)?

Question 1

$A^{bove}{}_{B^{elow}{}_{T^{hesame}}}$

Justify your answer.

Solution 1

(a)

Mark scheme

- Immediately after S closes, the initially-uncharged capacitors act as short circuits, so $R_{EQ} = R_1 = 100\ \Omega$ (all current flows through R_1, bypassing R_2 and R_3). By Ohm's law, $I = \Delta V/R = (10\ \text{V})/(100\ \Omega) = 0.10\ \text{A}$. Points: (1) recognizing capacitors act as short circuits immediately after closing, (2) correct Ohm's-law calculation giving $I = 0.10\ \text{A}$.

Solution 1

(b)

Mark scheme

- At steady state the capacitors are fully charged (no current into them), so current flows through R_1 in series with the parallel combination of R_2 and R_3: $1/R_P = 1/R_2 + 1/R_3 = 1/30\ \Omega + 1/60\ \Omega \Rightarrow R_P = 20\ \Omega$, so $R_{EQ} = R_1 + R_P = 100 + 20 = 120\ \Omega$. Current through the battery: $I = \Delta V/R = (10\ \text{V})/(120\ \Omega) = 0.083\ \text{A}$. Potential difference across R_1: $\Delta V = (0.083\ \text{A})(100\ \Omega) = 8.3\ \text{V}$. Points: (1) correct steady-state equivalent resistance, (2) correct current/voltage calculation giving $\Delta V_{R_1} = 8.3\ \text{V}$.

Solution 1

(c)(i)

Mark scheme

- The potential difference across C_3 equals that across the parallel branch: $\Delta V_{C_3} = \Delta V_P = IR_P = (0.083\text{ A})(20\ \Omega) = 1.67$ V, which also equals the total p.d. across the series combination of C_1 and C_2 (that series pair is in parallel with C_3). Equivalent capacitance of C_1, C_2 in series: $1/C_S = 1/C_1 + 1/C_2 = 1/10\ \mu\text{F} + 1/15\ \mu\text{F} \Rightarrow C_S = 6.0\ \mu\text{F}$. Charge on C_2 (same as on C_1, series): $Q_2 = Q_S = C_S \Delta V_{C_{12}} = (1.67\text{ V})(6.0\ \mu\text{F}) = 10\ \mu\text{C}$. Points: (1) correct equation giving $\Delta V_{C_3} = 1.67$ V, (2) correct use of the series-equivalent capacitance to get $Q_2 = 10\ \mu\text{C}$.

Solution 1

(c)(ii) Mark scheme

- Greater than. Justification (per the scoring guidelines): The potential difference across the series combination of C_1 and C_2 is the same as the potential difference across C_3 (parallel branches share the same p.d.); thus capacitor C_3 has a greater potential difference across its plates than C_2 individually, and C_3 has a greater capacitance than C_2. Because $Q = C\Delta V$, C_3 stores a greater charge than C_2. (A justification via a full mathematical proof also earns full credit.) Points: (1) selecting 'Greater than' and attempting a relevant justification, (2) a fully correct justification.

Solution 1

(d)(i)

Mark scheme

- Immediately after S is opened, the current through R_1 is zero: $I_1 = 0$. Opening the switch removes R_1's branch from any closed conducting path, so no current flows through it (the capacitors instead discharge through R_2).

Solution 1

(d)(ii)

Mark scheme

- The potential difference across R_2 immediately after opening equals the p.d. across the capacitors just before the switch was opened (capacitor voltage cannot change instantaneously): $\Delta V_{R_2} = \Delta V_{C_{12}} = 1.67$ V. By Ohm's law: $I = \Delta V_{R_2}/R_2 = (1.67\text{ V})/(30\ \Omega) = 0.056$ A. Points: (1) correctly identifying $\Delta V_{R_2} = \Delta V_{C_{12}} = 1.67$ V, (2) correct Ohm's-law calculation giving $I = 0.056$ A.

Solution 1

(e)

Mark scheme

- Sketch an exponential-decay (RC discharge) curve: it must start at $t = 0$ at a nonzero, clearly labeled maximum value ($V = 0.67$ V — the steady-state voltage across C_2 before the switch opened, $Q_2/C_2 = 10\ \mu C/15\ \mu F$), then decrease as a concave-up curve that approaches the horizontal (t) axis asymptotically (never reaching $V = 0$) as $t \to \infty$ (discharge through R_2). Points: (1) curve starts at a nonzero, labeled maximum value, (2) concave-up curve with the horizontal axis as an asymptote.

Solution 1

(f)

Mark scheme

- Below. Justification: Replacing C_3 with two $10\ \mu$F capacitors in series (equivalent capacitance $5\ \mu F$, less than the original C_3) decreases the equivalent capacitance of the circuit; since the discharge time constant $\tau = RC$ decreases, the capacitors discharge more rapidly, so the new V-vs-t curve for C_2 (for $t > 0$) lies below the original curve from part (e). Points: (1) selecting 'Below' and attempting a relevant justification, (2) a fully correct justification.

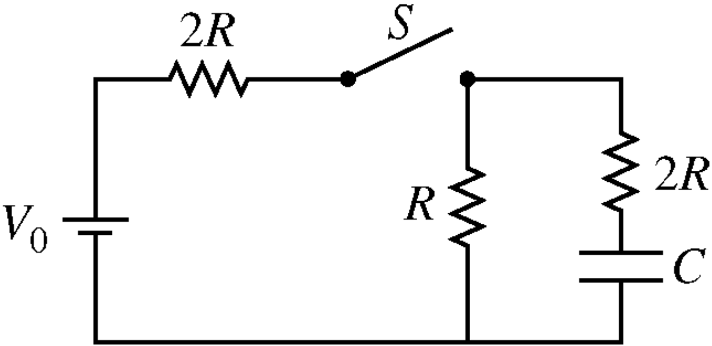
Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]
The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.
(a) Derive an expression for the steady-state current supplied by the battery.

Question 1

Question 1



Question 1

2019 Set 2 ▪ Q1

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The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(b) Derive an expression for the charge on the capacitor.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(c) Derive an expression for the energy stored in the capacitor.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(d) Now the switch is opened at time $t = 0$.

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(e)(i) Calculate the current in resistor R immediately after the switch is opened.

(e)(ii) On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

Question 1

[Graph with vertical axis labeled "Current" and horizontal axis labeled "Time"; origin labeled O; axes are blank/unscaled for the student to sketch on.]

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Solution 1

(a) Mark scheme

- At steady state the capacitor is fully charged and draws no current, so no current flows through the branch containing the second $2R$ resistor and C. The circuit reduces to the top $2R$ in series with R (effective resistance $3R$). By Ohm's law: $I = \frac{V_0}{R_{eff}} = \frac{V_0}{2R + R}$, so $I = \frac{V_0}{3R}$. (1 pt: using Ohm's law with the correct effective resistance; 1 pt: correct substitution leading to the correct answer.)

Solution 1

(b) Mark scheme

- The stored charge is $q = CV_C$. At steady state no current flows through the $2R$-C branch, so $V_C = V_R$ (the parallel branch's voltage). $V_R = IR = \left(\frac{V_0}{3R}\right) R = \frac{1}{3} V_0$. Therefore $q = CV_R = \frac{1}{3} CV_0$. (1 pt: using the equation relating stored charge to capacitance; 1 pt: correctly determining V_R and substituting.)

Solution 1

(c) Mark scheme

- Energy stored in a capacitor: $U = \dfrac{q^2}{2C}$. Substituting $q = \dfrac{1}{3}CV_0$ from part (b): $U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$. (1 pt: any correct equation for energy stored in a capacitor; 1 pt: correct substitution of charge and/or voltage from part (b).)

Solution 1

(d) Mark scheme

- After the switch opens, current can only flow around the loop containing R, the second $2R$, and C. Voltage loop equation: $V_C - V_R - V_{2R} = 0 \Rightarrow \frac{q(t)}{C} = I(R + 2R)$. Since the capacitor is discharging, $I = -\frac{dq}{dt}$, giving the differential equation $\frac{q(t)}{C} = -3R\frac{dq}{dt}$ (do not solve it). (1 pt: any correct voltage loop equation for when the switch is open; 1 pt: substituting $-dq/dt$ or dq/dt for the current, consistent with the loop equation.)

Solution 1

(e)(i) Mark scheme

- The capacitor voltage cannot change instantaneously, so immediately after the switch opens $V_C(0^+) = V_R(0^+) = \dfrac{V_0}{3}$ (from part (b)). Current now flows through the total resistance $R + 2R = 3R$: $I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$. (1 pt: using a voltage consistent with part (b) in $I = V/R$; 1 pt: substituting the correct total resistance $3R$ into Ohm's law.)

Solution 1

(e)(ii) Mark scheme

- The current starts at its maximum value $I_0 = \dfrac{V_0}{9R}$ (from part (e)(i)) at $t = 0$ and decays exponentially toward zero as the capacitor discharges: $I(t) = \dfrac{V_0}{9R}e^{-t/3RC}$. The sketch should show a curve that is concave up throughout, decreasing monotonically toward the horizontal (time) axis as an asymptote, with the initial value explicitly labeled $V_0/9R$ on the vertical axis at $t = 0$. (1 pt: curve concave up throughout the graph; 1 pt: horizontal axis as an asymptote; 1 pt: labeling the maximum current, consistent with part (e)(i).)

Solution 1

(f) Mark scheme

- Equal to. Justification: after the switch opens, the capacitor discharges all of its stored energy and charge; assuming no energy is lost in the connecting wires, the only circuit elements that dissipate this energy are the two resistors (R and 2R) in series with the capacitor, so the total energy dissipated in the resistors equals the energy that was stored in the capacitor (conservation of energy). (1 pt: selecting \"Equal to\"; 1 pt: a correct justification invoking conservation of energy.) [An alternate full-credit path selects \"Less than\" with justification that some energy is additionally lost due to resistance in the connecting wires and/or losses in the capacitor.]

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Question 2

Sheets of Paper	d (m)	C (F)
500	0.045	6.5×10^{-11}
400	0.036	7.4×10^{-11}
300	0.027	8.9×10^{-11}
200	0.018	11.9×10^{-11}
100	0.010	21.0×10^{-11}

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ________________

Horizontal axis: ________________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

Question 2

Sheets of Paper	d (m)	C (F)		
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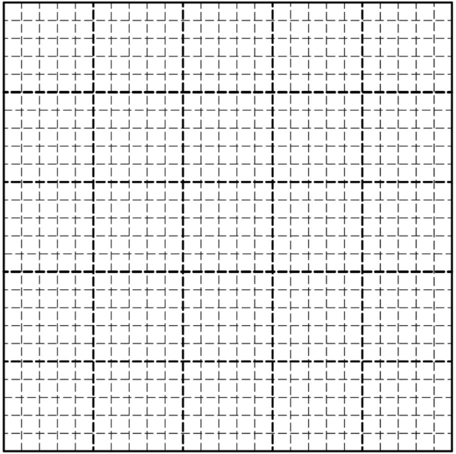
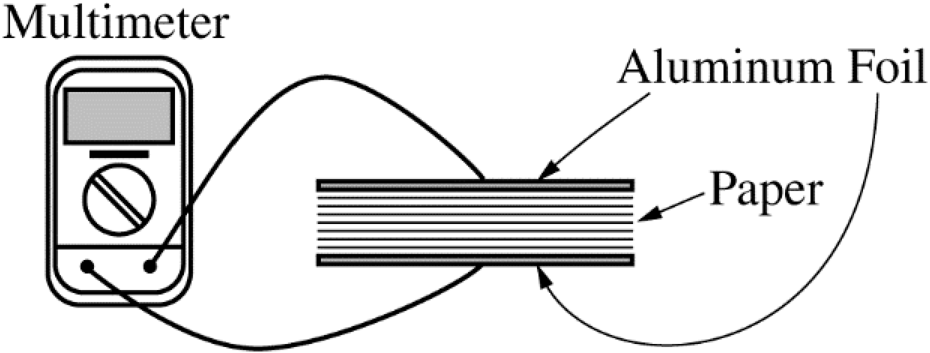
Question 2



Question 2



Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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Question 2

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.
[Graph: A blank grid (large square grid of fine gridlines with heavier gridlines every 5 squares) for the student to plot data points and draw a best-fit straight line.]

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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Question 2

(c) Using the straight line, calculate a dielectric constant for the paper.
[Circuit diagram: A 36 V battery (with $+$ terminal marked) in series with an open switch, connected to a circuit containing three identical $80\ \Omega$ resistors and an 18 nF capacitor. Two $80\ \Omega$ resistors are in the top branch (one in series with the main loop, one in a parallel branch also carrying the label $80\ \Omega$), in series with the 18 nF capacitor; a third $80\ \Omega$ resistor is in the bottom branch of the circuit.]
The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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100	0.010	21.0×10^{-11}

Question 2

(d) Calculate the current in the battery immediately after the switch is closed.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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100	0.010	21.0×10^{-11}

Question 2

(e) Determine the time constant for this circuit.

Question 2

2018 ▪ Q2

[Figure: A sketch of the experimental setup showing a Multimeter connected by two wires to a parallel-plate arrangement made of Aluminum Foil, with Paper sandwiched between the foil sheets.]

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \mathrm{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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Question 2

(f)(i) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.
Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?
_______ Larger than measured _______ Smaller than measured
Explain experimentally what could account for this.
(f)(ii) Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?
_______ Larger than measured _______ Smaller than measured
Explain experimentally what could account for this.

Solution 2

(a)

Mark scheme

- Vertical axis: C. Horizontal axis: $1/d$. (Reversed axes, or another algebraically valid straight-line combination, also earn full credit.) Rationale: $C = \dfrac{\kappa\varepsilon_0 A}{d}$, so plotting C vs. $1/d$ gives a straight line through the origin with slope $\kappa\varepsilon_0 A$.

Solution 2

(b)

Mark scheme

- Compute $1/d$ for each row: $d = 0.045$ m $\Rightarrow 1/d \approx 22.2$ m^{-1}; $d = 0.036 \Rightarrow 27.8$; $d = 0.027 \Rightarrow 37.0$; $d = 0.018 \Rightarrow 55.6$; $d = 0.010 \Rightarrow 100$. Plot $C\,(\mathrm{F} \times 10^{-11})$ on the vertical axis against these $1/d$ values: points approximately $(22.2, 6.5)$, $(27.8, 7.4)$, $(37.0, 8.9)$, $(55.6, 11.9)$, $(100, 21.0)$. Full credit needs: a scale that uses more than half the grid (1 point), correctly labeled axes with units (1 point), correctly plotted data points (1 point), and a single straight best-fit line consistent with the data (1 point).

Solution 2

(c) Mark scheme

- Read the slope from the best-fit line (not raw data points unless they lie on the line), e.g. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{(17-8)(\mathsf{F}\times 10^{-11})}{(80-32)(1/\mathsf{m})} \approx 0.19\ \mathsf{F}\cdot\mathsf{m}$ (linear regression gives $0.187\ \mathsf{F}\cdot\mathsf{m}$) (1 point). Relate the slope to the dielectric constant: slope $= \kappa\varepsilon_0 A$ (1 point). Solve: $\kappa = \dfrac{\text{slope}}{\varepsilon_0 A} = \dfrac{0.19\ \mathsf{F}\cdot\mathsf{m}}{(8.85\times 10^{-12}\ \mathsf{C}^2/\mathsf{N}\cdot\mathsf{m}^2)(0.060\ \mathsf{m}^2)} \approx 3.58$ (linear regression value: $\kappa \approx 3.52$) (1 point).

Solution 2

(d) Mark scheme

- Combine the two parallel $80\ \Omega$ resistors: $\frac{1}{R_P} = \frac{1}{80} + \frac{1}{80} = \frac{1}{40} \Rightarrow R_P = 40\ \Omega$ (1 point). Immediately after the switch closes the initially-uncharged capacitor has zero voltage across it, so Ohm's law applies to the series combination of R_P and the third $80\ \Omega$ resistor R_3: $I = \frac{V}{R_P + R_3}$ (1 point). Substitute values: $I = \frac{36\ \text{V}}{40\ \Omega + 80\ \Omega} = 0.30$ A (1 point).

Solution 2

(e)

Mark scheme

- Time constant uses the equivalent series resistance seen by the capacitor: $\tau = R_{eq}C = (R_P + R_3)C = (120\ \Omega)(18\ \text{nF})$ (1 point). Evaluate with correct units (consistent with part (d)): $\tau = 2.16\ \mu\text{s}$ (1 point).

Solution 2

(f)(i)

Mark scheme

- Answer: Larger than measured. Explanation: the battery is not ideal and has internal resistance, so the actual resistance in the circuit is larger than the measured resistance — a larger effective resistance produces a longer charging time, matching the observation.

Solution 2

(f)(ii)

Mark scheme

- Answer: Larger than measured. Explanation: some of the sheets of paper may be thinner than expected, so the actual capacitance in the circuit is larger than the measured capacitance — a larger effective capacitance also produces a longer charging time, matching the observation.

Question 2

2017 ▪ Q2

[Circuit diagram: an ideal battery of voltage V_0 is connected across a parallel combination of two branches, with an open switch S at the top of the circuit. One branch contains capacitor C_0 in series/parallel with resistor R_1 (capacitor C_0 on top, resistor R_1 below it, forming a middle branch), and a third branch on the right contains resistor R_2. The switch S connects the battery/C_0-R_1 branch node to the top rail that also connects to R_2.]

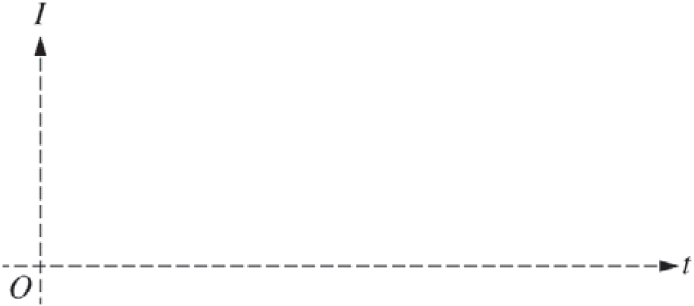
In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

Question 2

[Blank axes: vertical axis labeled q, horizontal axis labeled t, origin labeled O, dashed guide lines from the origin.]

Question 2

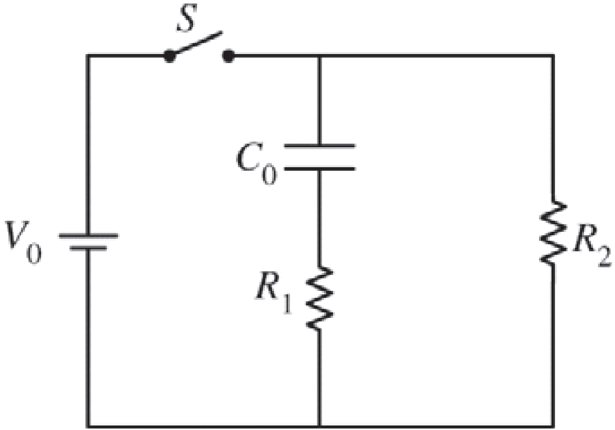


sing an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and

Question 2

time t. Explicitly label any intercepts, asymptotes, maxima, or minima with their expressions, as appropriate.

Question 2



Question 2

2017 ▪ Q2

[Circuit diagram: an ideal battery of voltage V_0 is connected across a parallel combination of two branches, with an open switch S at the top of the circuit. One branch contains capacitor C_0 in series/parallel with resistor R_1 (capacitor C_0 on top, resistor R_1 below it, forming a middle branch), and a third branch on the right contains resistor R_2. The switch S connects the battery/C_0-R_1 branch node to the top rail that also connects to R_2.]

In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

Question 2

[Blank axes: vertical axis labeled I, horizontal axis labeled t, origin labeled O, dashed guide lines from the origin.]

The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

Question 2

2017 ▪ Q2

[Circuit diagram: an ideal battery of voltage V_0 is connected across a parallel combination of two branches, with an open switch S at the top of the circuit. One branch contains capacitor C_0 in series/parallel with resistor R_1 (capacitor C_0 on top, resistor R_1 below it, forming a middle branch), and a third branch on the right contains resistor R_2. The switch S connects the battery/C_0-R_1 branch node to the top rail that also connects to R_2.]

In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(c) The time it takes to charge the capacitor to 50

$\Delta t_C > \Delta t_D$ $\Delta t_C = \Delta t_D$ $\Delta t_C < \Delta t_D$

Justify your answer.

Question 2

2017 ▪ Q2

[Circuit diagram: an ideal battery of voltage V_0 is connected across a parallel combination of two branches, with an open switch S at the top of the circuit. One branch contains capacitor C_0 in series/parallel with resistor R_1 (capacitor C_0 on top, resistor R_1 below it, forming a middle branch), and a third branch on the right contains resistor R_2. The switch S connects the battery/C_0-R_1 branch node to the top rail that also connects to R_2.]

In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(d)(i) Calculate the current through resistor R_2 immediately after the switch is opened.

(d)(ii) Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

_____ Increasing _____ Decreasing _____ Constant

Question 2

Justify your answer.

Question 2

2017 ▪ Q2

[Circuit diagram: an ideal battery of voltage V_0 is connected across a parallel combination of two branches, with an open switch S at the top of the circuit. One branch contains capacitor C_0 in series/parallel with resistor R_1 (capacitor C_0 on top, resistor R_1 below it, forming a middle branch), and a third branch on the right contains resistor R_2. The switch S connects the battery/C_0-R_1 branch node to the top rail that also connects to R_2.]

In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(e)(i) Calculate the energy stored in the capacitor immediately after the switch is opened.

(e)(ii) Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

Solution 2

(a)

Mark scheme

- Sketch of charge q on the capacitor vs. time t after the switch closes at $t = 0$: a concave-down curve starting at the origin, $q = 0$ at $t = 0$ (1 point); the curve approaches a horizontal asymptote as $t \to \infty$ (1 point); the asymptote is correctly labeled at $q = C_0 V_0$, the maximum (fully charged) charge (1 point).

Solution 2

(b)

Mark scheme

- Sketch of the currents I_1 (through R_1) and I_2 (through R_2) vs. time t. I_2 is drawn as a horizontal line, constant at $I_2 = V_0/R_2$ for all t, with the correct vertical intercept shown (1 point). I_1 is drawn as a concave-up curve that decays and is asymptotic to $I = 0$ as $t \to \infty$ (1 point), with its initial vertical intercept correctly labeled at $I_1 = V_0/R_1$ at $t = 0$ (1 point). The I_2 curve must be drawn everywhere above the I_1 curve (1 point) — per the guideline's own figure $V_0/R_2 > V_0/R_1$, so the constant I_2 line sits above the decaying I_1 curve for all $t > 0$.

Solution 2

(c)

Mark scheme

- Correct selection: $\Delta t_C < \Delta t_D$ (charging to 50

Solution 2

(d)(i)

Mark scheme

- Use a correct statement of Ohm's law / the loop rule with the switch open: $V = IR \Rightarrow I = \dfrac{V_C}{R_{tot}} = \dfrac{V_0}{R_1 + R_2}$ (1 point). Substitute the given values: $I = \dfrac{1.5\ \text{V}}{100\,\Omega + 150\,\Omega}$. Correct final answer: $I = 6.0$ mA (or 0.006 A without units) (1 point).

Solution 2

(d)(ii)

Mark scheme

- Correct selection: Decreasing. Justification (1 point): once the capacitor begins to discharge, the charge stored on it decreases, so the potential difference across it (and across R_2, which is in parallel with it) decreases; therefore the current through R_2 decreases.

Solution 2

(e)(i)

Mark scheme

- Substitute into the energy-stored-in-a-capacitor formula, including units: $U = \frac{1}{2}CV^2 = \frac{1}{2}(80\ \mu\text{F})(1.5\ \text{V})^2 = 90\ \mu\text{J}$ (1 point).

Solution 2

(e)(ii) Mark scheme

- Set up a valid equation/argument for the energy dissipated by R_1 as the capacitor fully discharges: since $E_{dis} = Pt = I^2Rt$ and the same current flows through both series resistors, the energy dissipated in each resistor is proportional to its resistance, so $E_{dis,1} = E_{dis,tot} \cdot \dfrac{R_1}{R_1 + R_2}$ (1 point). Substitute values: $E_{dis,1} = (90\ \mu\text{J}) \cdot \dfrac{150\,\Omega}{150\,\Omega + 100\,\Omega} = 54\ \mu\text{J}$ (1 point). [Alternate solution, also full credit: derive $I(t) = (6.0\text{ mA})e^{-t/0.02}$ and integrate $E = \int I(t)^2 R_1\, dt$ from $t = 0$ to ∞, which likewise gives $E = 54\ \mu\text{J}$.]

Question 1

2015 ▪ Q1

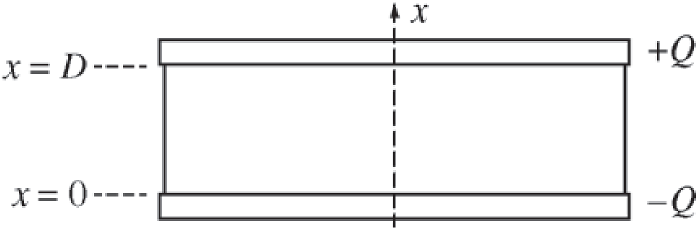
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.
[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]
(a)(i) On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
(a)(ii) On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

Question 1

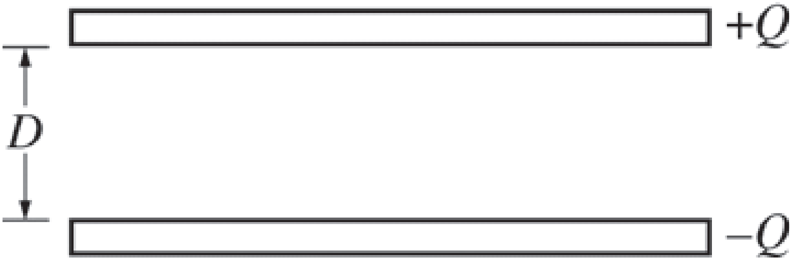
(a)(iii) Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.
[Figure: A vertical x-axis points upward. The bottom plate is at $x = 0$, labeled $-Q$; the top plate is at $x = D$, labeled $+Q$. A dashed vertical center line runs between the plates.]
The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

Question 1

of the electric field E between the plates. Express your answer in term
constants, as appropriate.



Question 1



Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(b) Determine an expression for the dielectric constant κ as a function of x.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(c)(i) Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

(c)(ii) Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(d) Determine the capacitance of the capacitor.

Question 1

2015 ▪ Q1

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

[Figure: Two parallel horizontal plates, separated by distance D (shown with a vertical double arrow between them on the left). The top plate is labeled $+Q$, the bottom plate is labeled $-Q$.]

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

______ $U_V < U_C$ ______ $U_V > U_C$ ______ $U_V = U_C$

Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The electric field between two oppositely charged parallel plates points from the positive plate toward the negative plate. Since the $+Q$ plate is at $x = D$ (top) and the $-Q$ plate is at $x = 0$ (bottom), the field points downward, from the top plate to the bottom plate. Credit (1 point) requires at least one arrow between the plates pointing downward (from $+Q$ toward $-Q$) with no extraneous arrows pointing in any other direction.

Solution 1

(a)(ii)

Mark scheme

- Draw a Gaussian pillbox (a small rectangular box with flat faces parallel to the plates) straddling one of the plates so that it encloses at least the inner edge of that plate, with its flat end-faces parallel to the plate surfaces (perpendicular to $\vec{E}$). Credit (1 point) requires an appropriate Gaussian surface enclosing at least the inner edge of one plate that can be used to determine E between the plates.

Solution 1

(a)(iii) Mark scheme

- Start from Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\varepsilon_0}$ (1 point for a correct statement of Gauss's law). Apply it to the pillbox from part (a)-ii: only the flat end-face between the plates carries flux (the field is uniform and perpendicular to that face, and zero elsewhere on the surface), so $E A_{GS} = \dfrac{q_{enc}}{\varepsilon_0}$, giving $E = \dfrac{q_{enc}}{\varepsilon_0 A_{GS}}$; writing the surface charge density as $\sigma = \dfrac{q_{enc}}{A_{GS}} = \dfrac{Q}{A}$ (enclosed charge and Gaussian area consistent with the plate) gives $E = \dfrac{\sigma}{\varepsilon_0}$ (1 point for applying Gauss's law with enclosed charge and area consistent with the surface drawn in part (a)-ii). The

Solution 1

final result, with work shown, is $E = \dfrac{Q}{\varepsilon_0 A}$ (1 point for a correct answer with work shown).

Solution 1

(b)

Mark scheme

- Compare the standard parallel-plate field with a uniform dielectric, $E = \dfrac{Q}{\kappa\varepsilon_0 A}$, to the given expression for the field with the varying dielectric in place, $E = \dfrac{Q}{\varepsilon_0\kappa_0 e^{-x/D}A}$. Matching the two expressions term by term gives $\kappa = \kappa_0 e^{-x/D}$ (1 point; the answer must be consistent with the expression for E found in part (a)-iii).

Solution 1

(c)(i) Mark scheme

- Use the relation between electric field and potential, $E = -\frac{dV}{dx}$, and substitute the field from part (b): $\frac{dV}{dx} = -\left(-\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\right) = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$ (1 point for a correct differential equation; it does not need to be solved yet). An equivalent alternate approach accepted by the scoring guidelines uses $\Delta V = -\int \vec{E} \cdot d\vec{r}$ directly, giving $\Delta V = \int \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\, dx$, which earns the same 1 point.

Solution 1

(c)(ii) Mark scheme

- Separate variables in the differential equation from part (c)-i: $dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) e^{x/D}\, dx$. Integrate using the correct limits (V runs from V_0 to V_D as x runs from 0 to D): $\int_{V_0}^{V_D} dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \int_0^D e^{x/D}\, dx$ (1 point for using the correct limits of integration). Carrying out the integration: $[V]_{V_0}^{V_D} = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \left[De^{x/D}\right]_0^D$, so $(V_D - V_0) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)\left(e^{D/D} - e^0\right) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e - 1)$ (1 point for correctly

Solution 1

integrating the equation). Taking the magnitude gives $\Delta V = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)$ (1 point for the correct absolute value of the potential difference between the plates), and this value must be reported as positive (1 point for having the potential difference be positive). Numerically, $\Delta V = \frac{1.72\, QD}{\varepsilon_0 \kappa_0 A}$.

Solution 1

(d) Mark scheme

- Use the definition of capacitance, $C = \dfrac{Q}{\Delta V}$, and substitute the result from part (c)-ii: $C = \dfrac{Q}{\left(\dfrac{QD}{\varepsilon_0\kappa_0 A}\right)(e-1)} = \dfrac{\varepsilon_0\kappa_0 A}{D(e-1)} = \dfrac{\varepsilon_0\kappa_0 A}{1.72D}$ (1 point; the answer must be consistent with part (c)-ii).

Solution 1

(e) Mark scheme

- Select $U_V > U_C$ (1 point). Justification: from part (d), a capacitor with the uniform dielectric constant κ_0 throughout has a larger capacitance than the varying-dielectric capacitor, i.e. $C_C > C_V$, because $\kappa = \kappa_0 e^{-x/D} \leq \kappa_0$ everywhere reduces the effective capacitance compared to a full-κ_0 fill (1 point for correctly comparing the capacitance, or equivalently the potential difference, of the varying-dielectric capacitor to that of the constant-dielectric capacitor). Since both capacitors are given the same charge Q and $U = \dfrac{Q^2}{2C}$, the smaller capacitance stores more energy; because $C_C > C_V$, it follows that $U_V > U_C$ (1 point for correctly comparing the two stored energies consistent with the

Solution 1

capacitance/potential-difference comparison). Example from the scoring guidelines: 'According to the equation from part (d), $C_C > C_V$. Since $U = Q^2/2C$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.'