Past-paper walk-through

Redistribution of Charge Between Conductors ▪ 10.2

eXercise

Questions in this set

- 2023 Set 1 Q3
- 2023 Set 2 Q1
- 2022 Set 1 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

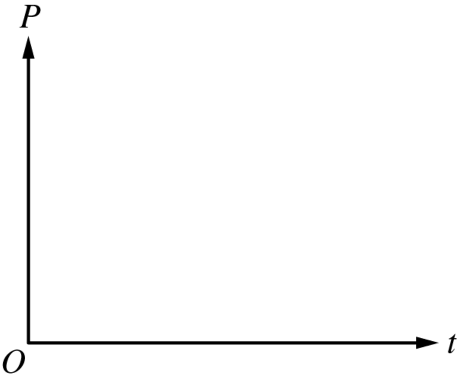
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is

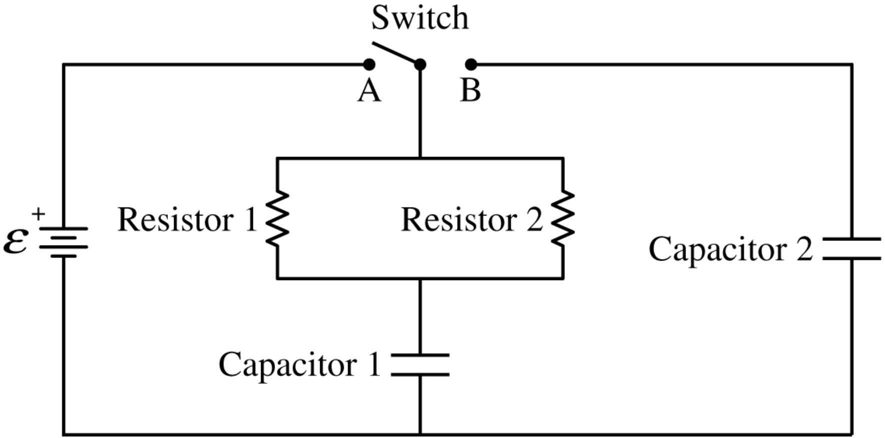
Question 3

closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

Question 3



Question 3


Switch
A
B
Resistor 1
Resistor 2
Capacitor 2
Capacitor 1
+
$\mathcal{E}$

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.

Question 3

[Graph 1: Blank axes, vertical axis σ, horizontal axis t, origin O, for the student to sketch surface charge density vs. time.]
[Graph 2: Blank axes, vertical axis P, horizontal axis t, origin O, for the student to sketch total power dissipated vs. time.]
A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(c)(i) At time t_1, the switch is closed to Position B.

Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

Question 3

(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

(c)(iii) Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.
[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(d) With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

Question 3

Determine the ratio $\dfrac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

Question 3

2023 Set 1 ▪ Q3

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.
At time $t = 0$, the switch is closed to Position A.

[Diagram: A circuit with a battery of emf $\mathcal{E}$ on the left. A switch at the top can connect to Position A or Position B. Resistor 1 and Resistor 2 (each resistance R) are in parallel branches below the switch positions. Capacitor 1 (capacitance C) is connected at the bottom of the Resistor 1 branch. Capacitor 2 (capacitance $2C$) is connected on the right side of the circuit, in the branch reached via Position B and Resistor 2.]

(e)(i) With the capacitors still charged as in part (d), the switch is now closed to Position A.

Question 3

Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.
(e)(ii) With the capacitors still charged as in part (d), the switch is now closed to Position A.
Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

Solution 3

(a) Mark scheme

- Loop rule around the circuit with Capacitor 1 and the two parallel resistors (each of resistance R, combining to an equivalent resistance $R/2$):
  $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$ (1 pt, for a loop-rule expression that includes terms for the equivalent resistance $R/2$ and the potential difference across the battery).
  Writing the capacitor voltage as $\Delta V_C = Q/C$ and the resistor-pair voltage as $\Delta V_{R,eq} = \dfrac{R}{2}\dfrac{dQ}{dt}$ (1 pt, for an expression that includes charge Q in the capacitor-voltage term and dQ/dt in the resistor-voltage term) gives the final differential equation: $\mathcal{E} - \dfrac{Q}{C} - \dfrac{R}{2}\dfrac{dQ}{dt} = 0$.

Solution 3

(b)

Mark scheme

- Two sketches vs. time t, from $t = 0$ until steady state is nearly reached: the surface charge density σ on the positive plate of Capacitor 1 increases and is concave down (rises quickly, then levels off toward a maximum) (1 pt); the total power P dissipated by the resistors decreases and is concave up (falls quickly, then levels off toward zero) (1 pt); both curves approach a slope of zero as time increases, i.e. both flatten out as steady state is approached (1 pt — this point can be earned even if the first two points are not).

Solution 3

(c)(i) Mark scheme

- The current in the switch immediately after t_1 is directed toward the right. Justification: while the switch was closed to Position A, Capacitor 1 charged so that its top plate accumulated positive charge (and the bottom plate negative), making the top plate's electric potential higher than the bottom plate's; immediately after moving the switch to Position B, current flows from high to low potential, i.e., away from the top plate through the switch toward the right (into the second loop with Capacitor 2), so conventional current is directed toward the right. 1 point for this indication with justification along the lines of the accumulated plate charge or the relative potentials of the plates.

Solution 3

(c)(ii) Mark scheme

- The total charge on the positive plate of Capacitor 2 a long time after t_1 is $\frac{2}{3}Q_0$. Reasoning: once connected in the second loop, Capacitor 1 and Capacitor 2 reach the same potential difference across them; since Capacitor 2 has twice the capacitance of Capacitor 1, it stores twice as much charge as Capacitor 1 in that final state. By conservation of the original charge Q_0 (all of which was on Capacitor 1 before t_1), splitting it in a 1:2 ratio between Capacitor 1 and Capacitor 2 gives Capacitor 2 a final charge of $\frac{2}{3}Q_0$ (and Capacitor 1 a final charge of $\frac{1}{3}Q_0$). 1 point; can be earned without supporting calculations.

Solution 3

(c)(iii) Mark scheme

- The energy dissipated by resistors 1 and 2 equals the decrease in total capacitor stored energy: $E_R = U_{0C} - U_C$, i.e., $\Delta E_R = U_C - U_{0C}$ with $E_R = -\Delta E_R$ (1 pt, for indicating the total energy dissipated is the difference between the initial electric PE stored at $t = t_1$ and the final electric PE stored once the new steady state is reached). Before t_1, only Capacitor 1 stores nonzero energy, $U_{0C} = U_{01} = \dfrac{1}{2}\dfrac{Q_0^2}{C}$; after the new steady state, both capacitors store nonzero energy, $U_C = U_1 + U_2$ (1 pt, for this indication, or an alternative consistent with part (c)(ii)). Substituting the steady-state charges from part (c)(ii), $Q_1 = Q_0/3$ on Capacitor 1 (capacitance C) and $Q_2 = 2Q_0/3$ on Capacitor 2 (capacitance $2C$) (1 pt, for

Solution 3

correct substitutions consistent with (c)(ii)):
$\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\frac{1}{2C}\left(\frac{2Q_0}{3}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \frac{Q_0^2}{6C} - \frac{1}{2}\frac{Q_0^2}{C} = -\frac{Q_0^2}{3C}$. So the resistors dissipate a total energy of $\frac{Q_0^2}{3C}$.

Solution 3

(d) Mark scheme

- $\dfrac{U_2}{U_1} = 1$. Justification (each part 1 pt): after the new steady state is reached, the potential difference across each capacitor is the same (equivalently, the charge stored on each capacitor is the same), $\Delta V_1 = \Delta V_2$ or $Q_1 = Q_2$ (1 pt, indicating equal potential difference or equal charge across the two capacitors in steady state); and since capacitance is inversely related to plate separation, doubling Capacitor 2's plate separation halves its capacitance, making it equal to Capacitor 1's capacitance in this new configuration (1 pt, recognizing the capacitances are

Solution 3

now equal). Then, since $U_C = \frac{1}{2}C(\Delta V)^2$ with equal C and equal ΔV for both capacitors, $\frac{U_2}{U_1} = 1$.

Solution 3

(e)(i) Mark scheme

- With the switch closed to Position A again (capacitors 1 and 2, each already charged as in part (d), now effectively act together across the battery loop with the pair of resistors), the loop rule is $\mathcal{E} - \Delta V_R - \Delta V_C = 0$ (1 pt, for a loop rule including terms for the battery emf, the potential difference across the pair of resistors, and the potential difference across Capacitor 1 / the capacitor combination). Using $\mathcal{E} = Q_0/C$, $\Delta V_R = I(R/2)$, and $\Delta V_C = Q_0/(2C)$ (the steady-state, equal charge/capacitor voltage from part (d)): $\dfrac{Q_0}{C} - I\dfrac{R}{2} - \dfrac{Q_0}{2C} = 0 \Rightarrow \dfrac{Q_0}{2C} = I\dfrac{R}{2} \Rightarrow \dfrac{Q_0}{C} = IR \Rightarrow I_0 = \dfrac{Q_0}{RC}$ (1 pt, for the correct answer $I_0 = Q_0/(RC)$).

Solution 3

(e)(ii)

Mark scheme

- A long time after the switch is closed to Position A, the capacitors reach a new steady state (fully charged) and, in DC steady state, no current flows into or out of a fully-charged capacitor branch. Therefore $I_\infty = 0$. 1 point for indicating the current is zero.

Question 1

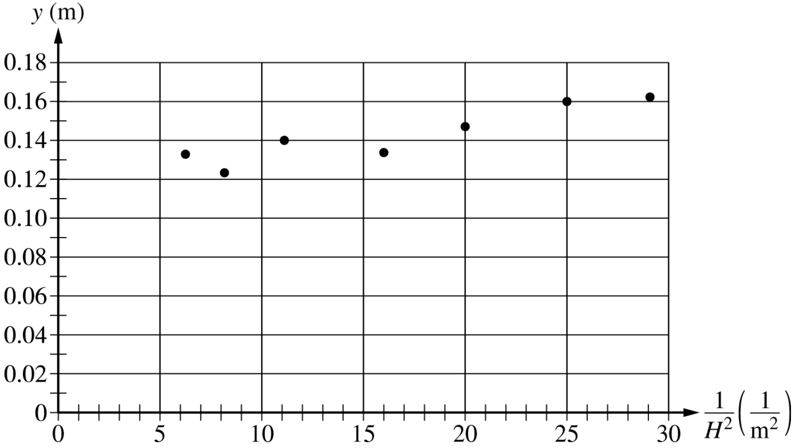
2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Question 1

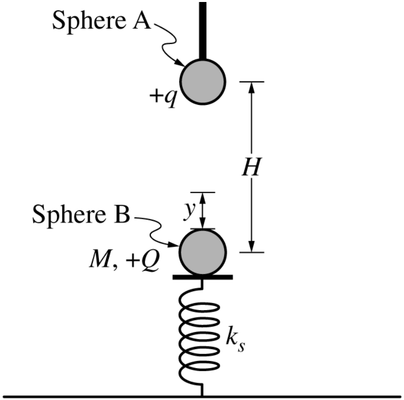
Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.
(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.
[Diagram: a single dot with a short dashed horizontal line and a short dashed vertical line crossing through it, indicating axes for drawing force vectors. No forces are pre-drawn; the student draws them.]

Question 1



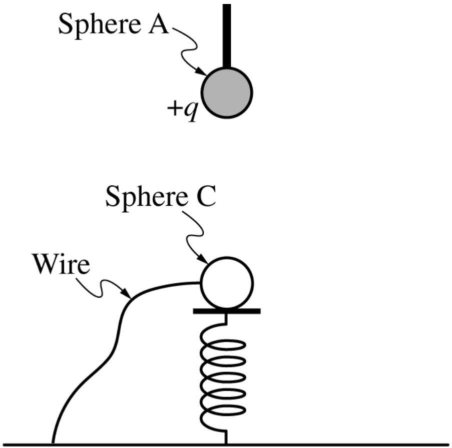
y (m)
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
$\frac{1}{H^2}\left(\frac{1}{\mathrm{m}^2}\right)$

Question 1



Note: Figure not drawn to scale.

Question 1



Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]

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Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(b) Derive the relationship between y and H to show that

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}.$$

Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
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Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(c)(i) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph. [Graph: y-axis labeled "y (m)" ranging from 0 to 0.18 in increments of 0.02 (gridlines at 0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18); x-axis labeled "$\frac{1}{H^2}\left(\frac{1}{\text{m}^2}\right)$" ranging from 0 to 30 in increments of 5. Data points (approximate): (0, 0.10), (7, 0.125), (10, 0.12), (13, 0.135), (17, 0.14), (20, 0.135), (23, 0.16), (27, 0.165), (29, 0.165).]
Draw the best-fit line for the data.
(c)(ii) Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.
(c)(iii) Using the best-fit line, calculate an experimental value for the mass of Sphere B.

Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
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Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(d)(i) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.
In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.
Is y_2 greater than, less than, or equal to y_1?
______ $y_2 > y_1$ ______ $y_2 < y_1$ ______ $y_2 = y_1$
Justify your answer.
(d)(ii) [Figure: Sphere A (charge $+q$) hangs above Sphere C. Sphere C rests on the platform above the spring, with a wire connected from Sphere C down to the ground/base.]

Question 1

Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."

Solution 1

(a)

Mark scheme

- Free-body diagram on Sphere B (dot), three vectors all starting at the dot: electrostatic force F_E pointing DOWNWARD (repulsion from Sphere A), normal force F_N pointing UPWARD (platform/spring pushing up on B; label F_S is also acceptable), and gravitational force F_g pointing DOWNWARD. 1 point for correctly drawing and labeling F_E downward. 1 point for drawing F_N upward AND F_g downward together. Scoring note: a maximum of 1 point may be earned if extraneous forces are included.

Solution 1

(b) Mark scheme

- Derive $y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$ from equilibrium of Sphere B. Start from the equilibrium equation consistent with the part (a) diagram: $F_N - F_E - F_g = 0$ (1 point). Substitute the spring/normal force $F_N = k_s y$ (1 point). Substitute the electrostatic (Coulomb) force $F_E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{H^2}$ (1 point). Substitute the gravitational force $F_g = Mg$ (1 point). Full derivation: $F_N = F_E + F_g \Rightarrow k_s y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{H^2} + Mg \Rightarrow y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$, matching the given result.

Solution 1

(c)(i)

Mark scheme

- Draw a straight line (best-fit/trend line) through the plotted y vs $1/H^2$ data that reasonably approximates the trend of the scattered points (data run roughly from $y \approx 0.10$–0.12 m at $1/H^2 = 0$ up to $y \approx 0.16$–0.165 m at $1/H^2 = 30\ \mathrm{m}^{-2}$). The line need not pass through the origin or any specific data point; it must simply approximate the overall upward trend of the data (e.g. rising from about (0, 0.12) to about (30, 0.16)).

Solution 1

(c)(ii) Mark scheme

- Calculate an experimental value of ε_0 from the graph, given $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m. (1) Use two points ON the student's trend line to compute the slope: slope $= \dfrac{\Delta y}{\Delta(1/H^2)}$ (e.g. using (0, 0.12 m) and (20, 0.15 m): slope $= \dfrac{0.15 - 0.12}{20 - 0} = 0.0015\ \mathrm{m}^3$). (2) Relate the slope to ε_0 from $y = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Qq}{k_s H^2} + \dfrac{Mg}{k_s}$: slope $= \dfrac{Qq}{4\pi\varepsilon_0 k_s}$. (3) Substitute the slope and given quantities: $\varepsilon_0 = \dfrac{Qq}{4\pi k_s(\text{slope})} = \dfrac{(2 \times 10^{-6}\ \mathrm{C})^2}{4\pi(25\,\mathrm{N/m})(0.0015\,\mathrm{m}^3)} \approx 8.5 \times 10^{-12}\ \mathrm{C}^2/(\mathrm{N}\cdot\mathrm{m}^2)$. A

Solution 1

numerical answer without units may earn the third point; exact value depends on the student's own trend line and chosen points.

Solution 1

(c)(iii) Mark scheme

- Calculate an experimental value of the mass M of Sphere B from the graph's y-intercept. (1) Identify that the y-intercept equals $\frac{Mg}{k_s}$. (2) Substitute the graph's y-intercept (approximately 0.12 m) and given quantities: $M = \frac{(\text{y-intercept})\,k_s}{g} = \frac{(0.12\,\mathrm{m})(25\,\mathrm{N/m})}{9.8\,\mathrm{m/s^2}} \approx 0.30\ \mathrm{kg}$. A numerical answer without units may earn the second point; exact value depends on the student's own trend line's intercept.

Solution 1

(d)(i) Mark scheme

- Compare y_2 (conducting Sphere C, modified experiment) to y_1 (nonconducting Sphere B, original experiment) at the same separation H_1: the correct answer is $y_2 < y_1$ (1 point for selecting this with an attempted relevant justification). Justification (1 point): because Sphere C is conducting, its charges are free to redistribute. Electrons in Sphere C are attracted toward the excess positive charge on Sphere A and shift toward A, leaving the far side of C more positively charged. This charge rearrangement (polarization) results in LESS net electric repulsion between the spheres than in the nonconducting case, so the spring compresses less, giving $y_2 < y_1$.

Solution 1

(d)(ii) Mark scheme

- Draw an arrow indicating the platform moves UPWARD immediately after Sphere C is grounded (1 point; the SG's example response shows a single upward-pointing arrow at the spring/platform). Reasoning: grounding forces Sphere C's electric potential to zero; since C sits in the positive potential produced by nearby charged Sphere A, reaching zero net potential requires C's own net charge to decrease (effectively negative charge flows onto C from ground, or positive charge flows off to ground). This reduces the electrostatic repulsion between A and C (and can even become attractive), decreasing the downward force on C, so the spring relaxes and the platform rises.

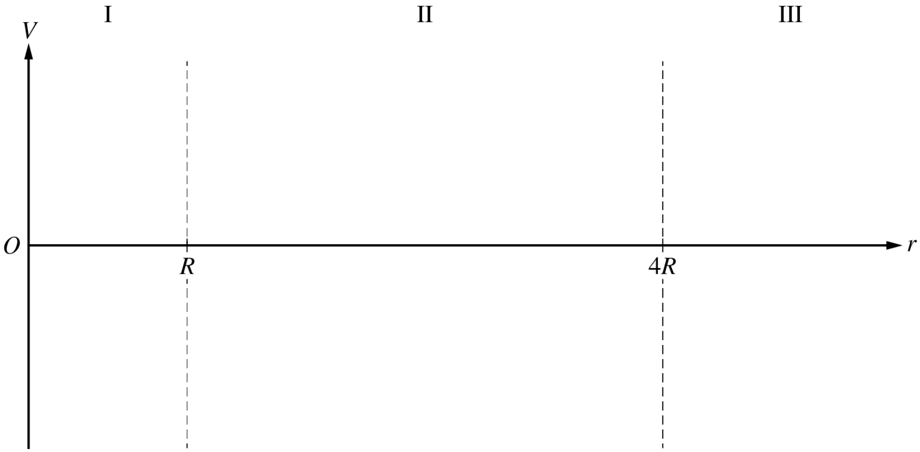
Question 1

2022 Set 1 ▪ Q1

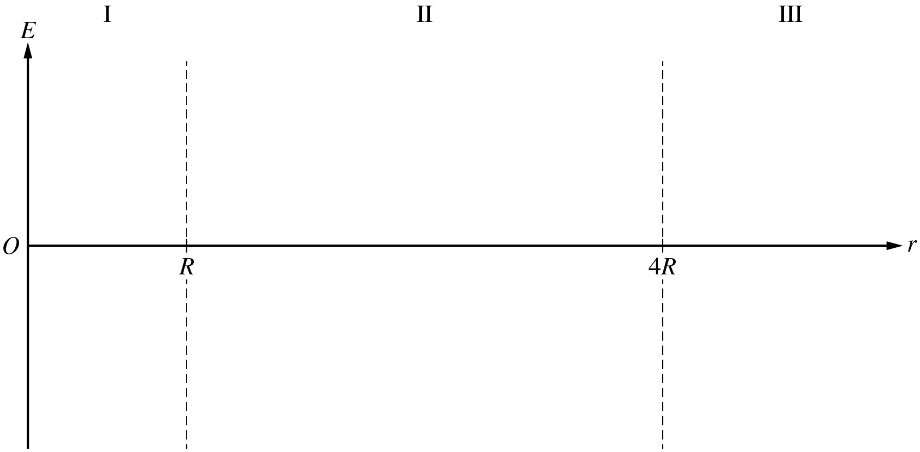
A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.
[Two figures side by side, captioned "Note: Figures not drawn to scale." Left figure, labeled "Cutout View": a large shaded circular shell with a wedge cut out to reveal a smaller shaded sphere at the center. Right figure, labeled "Cross-Section View": a large circle (the shell) with a smaller circle at the center labeled R, and an arrow from the center to the outer circle labeled $4R$.]

(a) Determine the charge on the outer surface of the shell.

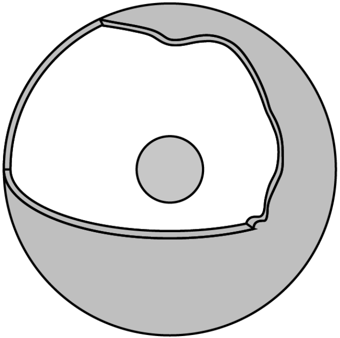
Question 1



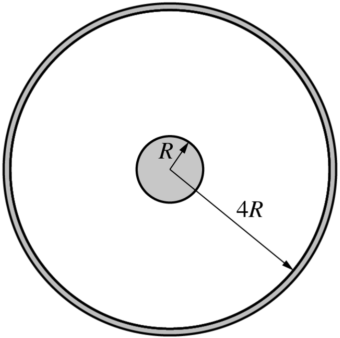
Question 1



Question 1



Cutout View



Cross-Section View

Note: Figures not drawn to scale.

Question 1

2022 Set 1 ▪ Q1

A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

[Two figures side by side, captioned "Note: Figures not drawn to scale." Left figure, labeled "Cutout View": a large shaded circular shell with a wedge cut out to reveal a smaller shaded sphere at the center. Right figure, labeled "Cross-Section View": a large circle (the shell) with a smaller circle at the center labeled R, and an arrow from the center to the outer circle labeled $4R$.]

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

Question 1

2022 Set 1 ▪ Q1

A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

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(c) The magnitude of the electric field at $r = R$ is 8 N/C. Calculate the value of the electric field at $r = 2R$.

Question 1

2022 Set 1 ▪ Q1

A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

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(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

Question 1

2022 Set 1 ▪ Q1

A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

[Two figures side by side, captioned "Note: Figures not drawn to scale." Left figure, labeled "Cutout View": a large shaded circular shell with a wedge cut out to reveal a smaller shaded sphere at the center. Right figure, labeled "Cross-Section View": a large circle (the shell) with a smaller circle at the center labeled R, and an arrow from the center to the outer circle labeled $4R$.]

(e)(i) On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.

Question 1

[Graph axes: horizontal axis labeled r, with vertical dashed reference lines at $r = R$ and $r = 4R$ dividing the plot into three labeled regions I, II, III (left to right). Vertical axis labeled E, origin marked O. Axes are blank, to be sketched on by the student.]

(e)(ii) On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.

[Graph axes: horizontal axis labeled r, with vertical dashed reference lines at $r = R$ and $r = 4R$ dividing the plot into three labeled regions I, II, III (left to right). Vertical axis labeled V, origin marked O. Axes are blank, to be sketched on by the student.]

Solution 1

(a)

Mark scheme

- $q_{net} = q_{inner} + q_{outer}$, so $q_{outer} = q_{net} - q_{inner} = +3Q - (+Q) = +2Q$. The charge on the outer surface of the shell is $+2Q$. (A correct final answer earns the point even with no work shown.)

Solution 1

(b) Mark scheme

- Apply Gauss's law: $\frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA = E(4\pi r^2)$ (1 pt: using Gauss's law, substituting for area or enclosed charge). The sphere has uniform charge density $\rho = \frac{-Q}{\frac{4}{3}\pi R^3}$ (net charge on the sphere is $-Q$), so $q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$ (1 pt: correct $q_{enc}(r)$, regardless of sign). Then $E = q_{enc}\frac{1}{\varepsilon_0(4\pi r^2)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0(4\pi r^2)}$ (1 pt: using the correct area), giving $E(r) = -\frac{Qr}{4\pi\varepsilon_0 R^3}$ for $r < R$ (sign is not graded).

Solution 1

Solution 1

(c)

Mark scheme

- E varies as $E \propto \frac{1}{r^2}$ from the center of the nonconducting sphere (1 pt: recognizing the inverse-square dependence). Since $r = 2R$ is twice $r = R$, where $E = 8$ N/C: $E_{new} = \frac{E_{old}}{4} = \frac{8 \text{ N/C}}{4} = 2$ N/C (1 pt: correct magnitude, with units).

Solution 1

(d) Mark scheme

- $V_f - V_i = -\int_{s_i}^{s_f} E\,dr$, so $\Delta V = -\int_{r=R}^{r=4R} E\,dr$ (1 pt: relating ΔV to E with an attempt at integration limits or evaluating the integral). Substituting $E = -\dfrac{Q}{4\pi\varepsilon_0 r^2}$ (consistent with part (c)): $\Delta V = -\int_R^{4R}\left(-\dfrac{Q}{4\pi\varepsilon_0 r^2}\right)dr = \dfrac{Q}{4\pi\varepsilon_0}\int_R^{4R}\dfrac{1}{r^2}dr$ (1 pt: substituting the correct E expression). Integrating with the correct limits: $\Delta V = \dfrac{Q}{4\pi\varepsilon_0}\left[-\dfrac{1}{r}\right]_R^{4R} = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{4R}\right) = \dfrac{Q}{4\pi\varepsilon_0}\cdot\dfrac{4-1}{4R}$ (1 pt: correctly

Solution 1

integrating with correct limits), so $\Delta V = \dfrac{3Q}{16\pi\varepsilon_0 R}$. (Alternate solution earning the same 3 points: treat the inner sphere as a point charge Q by symmetry, $\Delta V = \dfrac{Q}{4\pi\varepsilon_0 R} - \dfrac{Q}{4\pi\varepsilon_0 (4R)} = \dfrac{3Q}{16\pi\varepsilon_0 R}$, using charge Q — not $-Q$ or $3Q$ — on both terms and the correct r values.)

Solution 1

(e)(i)

Mark scheme

- Sketch of E vs. r (electric field, taking outward as positive): (1 pt) E is negative and its magnitude increases linearly from 0 at the center to a maximum magnitude at $r = R$ (region I); (1 pt) the curve is continuous at R and asymptotically approaches zero across $R < r < 4R$ (region II, negative and shrinking in magnitude); (1 pt) for $r > 4R$ (region III) the curve is positive, concave-up, and decreasing (falling off with distance from the now-enclosed total charge $+2Q$).

Solution 1

(e)(ii)

Mark scheme

- Sketch of V vs. r (electric potential): (1 pt) V is always increasing from $r = 0$ to $r = 4R$ (across regions I and II); (1 pt) V is decreasing for $r > 4R$ (region III); (1 pt) the graph is continuous across all three regions. (Scoring note: the vertical-axis intercept and where the curve crosses the horizontal axis are irrelevant — the curve may lie entirely below the axis as long as the shape criteria above are met.)