Past-paper walk-through

Electrostatics with Conductors ▪ 10.1

eXercise

Questions in this set

- 2023 Set 1 Q1
- 2023 Set 2 Q1
- 2022 Set 2 Q1
- 2021 Q2
- 2018 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

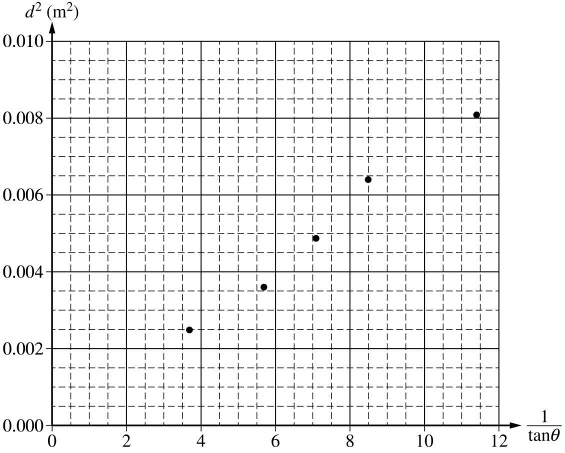
2023 Set 1 ▪ Q1

Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical. The experiment is repeated.
[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

Question 1

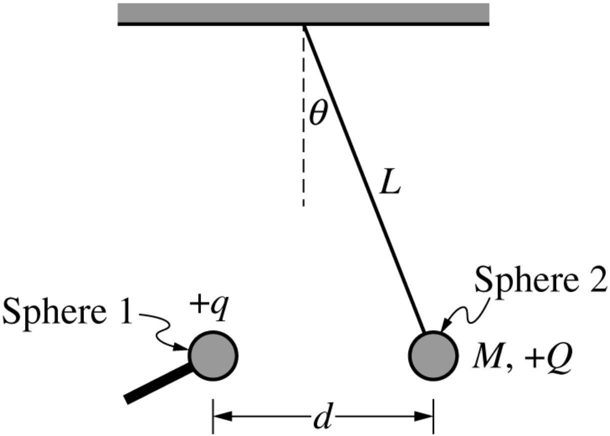
(a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. [Diagram: A single dot with a dashed horizontal line and a dashed vertical line through it, representing Sphere 2 at its equilibrium position, for the student to draw force vectors on.]

Question 1

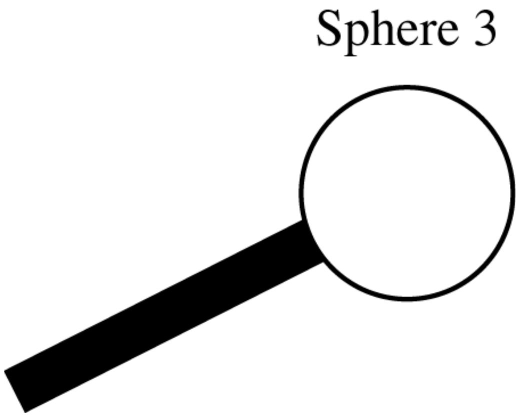


d^2 (m^2)
0.010
0.008
0.006
0.004
0.002
0.000
0
2
4
6
8
10
12
$\frac{1}{\tan\theta}$

Question 1



Question 1



Question 1

2023 Set 1 ▪ Q1

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[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

Question 1

(b) Derive the relationship between the distance d and the angle θ to show that

$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}.$$

Question 1

2023 Set 1 ▪ Q1

Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical. The experiment is repeated.

[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

(c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8}$ C, $\theta = 12°$, and $d = 0.057$ m. Calculate the expected force of tension exerted on Sphere 2 by the string.

Question 1

2023 Set 1 ▪ Q1

Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical. The experiment is repeated.

[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

(d)(i) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\dfrac{1}{\tan\theta}$.

Question 1

[Graph: y-axis d^2 (m^2) from 0.000 to 0.010 in increments of 0.002; x-axis $\dfrac{1}{\tan\theta}$ from 0 to 12 in increments of 2. Data points (approximate grid positions): (1.5, 0.0015), (2.3, 0.0022), (4.3, 0.0038), (6.2, 0.0050), (8.0, 0.0068), (11.3, 0.0087).]
Draw the best-fit line for the data.

(d)(ii) Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $M = 0.0050\,\mathrm{kg}$ and $Q = q = 6.0 \times 10^{-8}\,\mathrm{C}$.

Question 1

2023 Set 1 ▪ Q1

Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical. The experiment is repeated.

[Diagram: A rigid horizontal support at the top with a string of length L hanging at angle θ from the vertical, ending at Sphere 2 (nonconducting, mass M, charge $+Q$). Sphere 1 (nonconducting, charge $+q$) is mounted on an insulating rod held near Sphere 2 at the same height, separated by horizontal distance d.]

(e)(i) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

Question 1

The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single "+" sign to represent the location of highest concentration of the excess positive charges.
[Diagram: A circle labeled "Sphere 3" mounted on a thick insulating rod (shown as a solid black wedge), positioned as Sphere 1 was in the original figure, for the student to mark the "+" location.]
(e)(ii) Briefly explain your reasoning for the sketch drawn in part (e)(i).
(e)(iii) In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.
Is θ_2 greater than, less than, or equal to θ_1?
______ $\theta_2 > \theta_1$ ______ $\theta_2 < \theta_1$ ______ $\theta_2 = \theta_1$

Question 1

Briefly justify your answer.

Solution 1

(a)

Mark scheme

- Three forces act on Sphere 2: the electrostatic (Coulomb) force F_E directed horizontally away from Sphere 1 (to the right, since both spheres carry like positive charge and repel), the string tension F_T directed up and to the left along the string, and gravity F_g directed straight down. 1 point: correctly draw and label F_E pointing to the right. 1 point: draw F_T up-and-to-the-left and F_g straight down. Scoring note: a maximum of 1 point may be earned if extraneous (extra, non-existent) forces are included in the diagram.

Solution 1

(b) Mark scheme

- Using $\Sigma F_y = 0$: $F_{Ty} = F_g \Rightarrow F_T \cos\theta = Mg$ (1 pt, equating vertical tension component to gravity). Using $\Sigma F_x = 0$: $F_{Tx} = F_E \Rightarrow F_T \sin\theta = F_E$ (1 pt, equating horizontal tension component to the electrostatic force). Coulomb's law gives $F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}$, so $F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}$. Substituting into the vertical equation and solving simultaneously (1 pt, attempt to simultaneously solve the two force equations):
$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{\cos\theta}{\sin\theta} = Mg \Rightarrow d^2 = \frac{Qq\cos\theta}{4\pi\varepsilon_0 Mg\sin\theta} \Rightarrow d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$, as required.

Solution 1

(c) Mark scheme

- From Coulomb's law and the horizontal force balance, $F_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{d^2} = F_T \sin\theta$ (1 pt, applying Coulomb's law to determine tension; this point may be earned using the vertical component of tension instead). So $F_T = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{d^2 \sin\theta}$ (1 pt, correct substitution into an expression for tension consistent with part (b), or a correct tension expression). Substituting $Q = q = 6.0 \times 10^{-8}\,\mathrm{C}$, $d = 0.057\,\mathrm{m}$, $\theta = 12°$: $F_T = \dfrac{1}{4\pi(8.85 \times 10^{-12})}\dfrac{(6.0 \times 10^{-8})^2}{(0.057)^2 \sin(12°)} = 0.048\,\mathrm{N}$.

Solution 1

(d)(i)

Mark scheme

- Draw a single straight best-fit line through the cluster of six data points on the d^2 vs. $\dfrac{1}{\tan\theta}$ graph, approximating the overall linear trend (passing near the data points across their full range, roughly extrapolating toward the origin). 1 point for a line that reasonably approximates the trend of the data (does not need to pass through every point or through the origin exactly).

Solution 1

(d)(ii) Mark scheme

- Using two points that lie directly on the student's drawn best-fit line (not raw data points unless they fall exactly on the line):
  slope $= \dfrac{\Delta(d^2)}{\Delta(1/\tan\theta)} = \dfrac{0.0075\,\mathrm{m}^2 - 0.001\,\mathrm{m}^2}{10.5-2} = 7.647\times10^{-4}\,\mathrm{m}^2$ (1 pt, using two points from the trend line to calculate the slope). Relating the slope to the theoretical equation $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$: squaring gives $d^2 = \left(\dfrac{Qq}{4\pi\varepsilon_0 Mg}\right)\dfrac{1}{\tan\theta}$, so slope $= \dfrac{Qq}{4\pi\varepsilon_0 Mg}$ (1 pt, correctly relating slope to the equation). Solving for and substituting to get $\varepsilon_0 = \dfrac{Qq}{4\pi Mg(\text{slope})} =$

Solution 1

$$\frac{(6.0\times 10^{-8})^2}{4\pi(0.0050\,\mathsf{kg})(9.8\,\mathsf{m/s}^2)(7.647\times 10^{-4}\,\mathsf{m}^2)} \approx 7.6\times 10^{-12}\,\frac{\mathsf{C}^2}{\mathsf{N}\cdot\mathsf{m}^2}$$ (1 pt, substituting the slope value to calculate an experimental ε_0).

Solution 1

(e)(i)

Mark scheme

- Draw a single "+" sign on the left side of the circle representing conducting Sphere 3 — the side farther from (facing away from) Sphere 2 — indicating where the excess positive charge concentrates. 1 point for the "+" placed on the correct (left) side of the sphere.

Solution 1

(e)(ii)

Mark scheme

- Because Sphere 3 is a conductor, its free (mobile) negative charges are attracted toward the positive charges on Sphere 2 and migrate to the side of Sphere 3 nearest Sphere 2 (the right side), leaving a net excess of positive charge concentrated on the far side of Sphere 3, away from Sphere 2 (the left side). 1 point for a statement that correctly indicates this charge rearrangement on Sphere 3 due to the electric force from Sphere 2's charges.

Solution 1

(e)(iii) Mark scheme

- $\theta_2 < \theta_1$ (1 pt, for selecting $\theta_2 < \theta_1$ with an attempt at a relevant justification). Justification (1 pt, for indicating either of the following, consistent with the charge location drawn in part (e)(i)): on Sphere 3 the excess like (positive) charges concentrate on the side farther from Sphere 2, so the average separation between the concentrated charges on the two spheres is greater than it would be for point charges at the spheres' centers — equivalently, the electrostatic repulsive force between the spheres is smaller than in the original point-charge case. A smaller repulsive force at the same horizontal separation d_1 requires a smaller angle from vertical to balance gravity, so $\theta_2 < \theta_1$.

Question 1

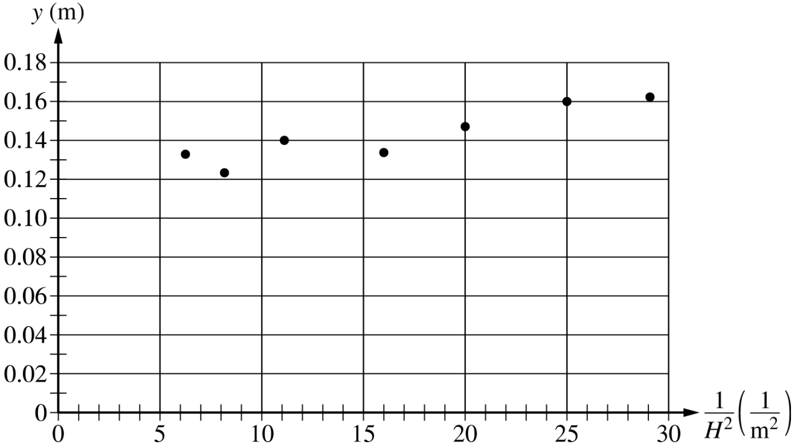
2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.] Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Question 1

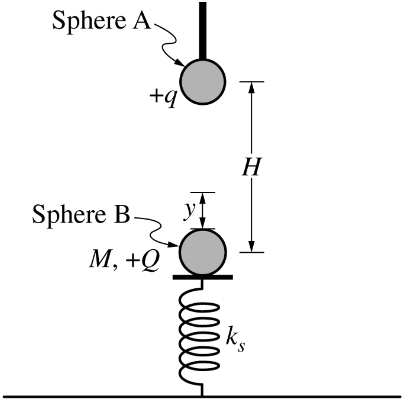
Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.
(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. [Diagram: a single dot with a short dashed horizontal line and a short dashed vertical line crossing through it, indicating axes for drawing force vectors. No forces are pre-drawn; the student draws them.]

Question 1



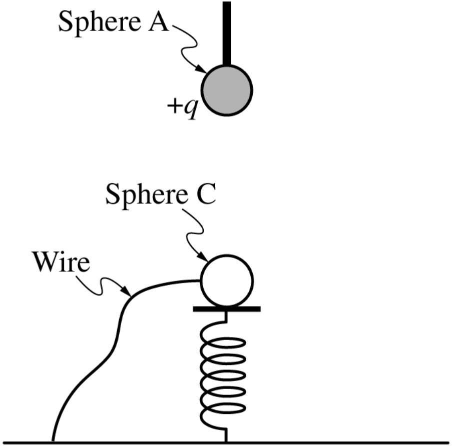
y (m)
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
$\frac{1}{H^2}\left(\frac{1}{m^2}\right)$

Question 1



Note: Figure not drawn to scale.

Question 1



Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(b) Derive the relationship between y and H to show that

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}.$$

Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]

Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(c)(i) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph. [Graph: y-axis labeled "y (m)" ranging from 0 to 0.18 in increments of 0.02 (gridlines at 0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18); x-axis labeled "$\frac{1}{H^2}\left(\frac{1}{\text{m}^2}\right)$" ranging from 0 to 30 in increments of 5. Data points (approximate): (0, 0.10), (7, 0.125), (10, 0.12), (13, 0.135), (17, 0.14), (20, 0.135), (23, 0.16), (27, 0.165), (29, 0.165).]

Draw the best-fit line for the data.

(c)(ii) Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.

(c)(iii) Using the best-fit line, calculate an experimental value for the mass of Sphere B.

Question 1

2023 Set 2 ▪ Q1

[Figure: A vertical setup. At the top, Sphere A (charge $+q$) hangs from a fixed support on a thin rod, positioned directly above Sphere B. A vertical distance H separates the centers of Sphere A and Sphere B. Sphere B (mass M, charge $+Q$) rests on an insulating platform of negligible mass, which sits on top of a vertical ideal spring with spring constant k_s. The spring rests on a fixed base. Note: Figure not drawn to scale.]
Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

Question 1

(d)(i) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.

Is y_2 greater than, less than, or equal to y_1?

______ $y_2 > y_1$ ______ $y_2 < y_1$ ______ $y_2 = y_1$

Justify your answer.

(d)(ii) [Figure: Sphere A (charge $+q$) hangs above Sphere C. Sphere C rests on the platform above the spring, with a wire connected from Sphere C down to the ground/base.]

Question 1

Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."

Solution 1

(a)

Mark scheme

- Free-body diagram on Sphere B (dot), three vectors all starting at the dot: electrostatic force F_E pointing DOWNWARD (repulsion from Sphere A), normal force F_N pointing UPWARD (platform/spring pushing up on B; label F_S is also acceptable), and gravitational force F_g pointing DOWNWARD. 1 point for correctly drawing and labeling F_E downward. 1 point for drawing F_N upward AND F_g downward together. Scoring note: a maximum of 1 point may be earned if extraneous forces are included.

Solution 1

(b) Mark scheme

- Derive $y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$ from equilibrium of Sphere B. Start from the equilibrium equation consistent with the part (a) diagram: $F_N - F_E - F_g = 0$ (1 point). Substitute the spring/normal force $F_N = k_s y$ (1 point). Substitute the electrostatic (Coulomb) force $F_E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{H^2}$ (1 point). Substitute the gravitational force $F_g = Mg$ (1 point). Full derivation: $F_N = F_E + F_g \Rightarrow k_s y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{H^2} + Mg \Rightarrow y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$, matching the given result.

Solution 1

(c)(i)

Mark scheme

- Draw a straight line (best-fit/trend line) through the plotted y vs $1/H^2$ data that reasonably approximates the trend of the scattered points (data run roughly from $y \approx 0.10–0.12$ m at $1/H^2 = 0$ up to $y \approx 0.16–0.165$ m at $1/H^2 = 30\ \mathrm{m}^{-2}$). The line need not pass through the origin or any specific data point; it must simply approximate the overall upward trend of the data (e.g. rising from about (0, 0.12) to about (30, 0.16)).

Solution 1

(c)(ii) Mark scheme

- Calculate an experimental value of ε_0 from the graph, given $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m. (1) Use two points ON the student's trend line to compute the slope: slope $= \dfrac{\Delta y}{\Delta(1/H^2)}$ (e.g. using (0, 0.12 m) and (20, 0.15 m): slope $= \dfrac{0.15 - 0.12}{20 - 0} = 0.0015\ \mathrm{m}^3$). (2) Relate the slope to ε_0 from $y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{k_s H^2} + \dfrac{Mg}{k_s}$: slope $= \dfrac{Qq}{4\pi\varepsilon_0 k_s}$. (3) Substitute the slope and given quantities: $\varepsilon_0 = \dfrac{Qq}{4\pi k_s(\text{slope})} = \dfrac{(2 \times 10^{-6}\,\mathrm{C})^2}{4\pi(25\,\mathrm{N/m})(0.0015\,\mathrm{m}^3)} \approx 8.5 \times 10^{-12}\ \mathrm{C}^2/(\mathrm{N} \cdot \mathrm{m}^2)$. A

Solution 1

numerical answer without units may earn the third point; exact value depends on the student's own trend line and chosen points.

Solution 1

(c)(iii) Mark scheme

- Calculate an experimental value of the mass M of Sphere B from the graph's y-intercept. (1) Identify that the y-intercept equals $\frac{Mg}{k_s}$. (2) Substitute the graph's y-intercept (approximately 0.12 m) and given quantities: $M = \frac{(\text{y-intercept})\, k_s}{g} = \frac{(0.12\,\mathrm{m})(25\,\mathrm{N/m})}{9.8\,\mathrm{m/s^2}} \approx 0.30\ \mathrm{kg}$. A numerical answer without units may earn the second point; exact value depends on the student's own trend line's intercept.

Solution 1

(d)(i) Mark scheme

- Compare y_2 (conducting Sphere C, modified experiment) to y_1 (nonconducting Sphere B, original experiment) at the same separation H_1: the correct answer is $y_2 < y_1$ (1 point for selecting this with an attempted relevant justification). Justification (1 point): because Sphere C is conducting, its charges are free to redistribute. Electrons in Sphere C are attracted toward the excess positive charge on Sphere A and shift toward A, leaving the far side of C more positively charged. This charge rearrangement (polarization) results in LESS net electric repulsion between the spheres than in the nonconducting case, so the spring compresses less, giving $y_2 < y_1$.

Solution 1

(d)(ii) Mark scheme

- Draw an arrow indicating the platform moves UPWARD immediately after Sphere C is grounded (1 point; the SG's example response shows a single upward-pointing arrow at the spring/platform). Reasoning: grounding forces Sphere C's electric potential to zero; since C sits in the positive potential produced by nearby charged Sphere A, reaching zero net potential requires C's own net charge to decrease (effectively negative charge flows onto C from ground, or positive charge flows off to ground). This reduces the electrostatic repulsion between A and C (and can even become attractive), decreasing the downward force on C, so the spring relaxes and the platform rises.

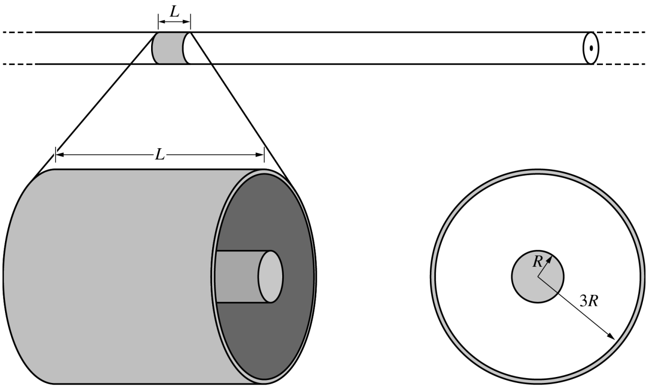
Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view. [Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

Question 1

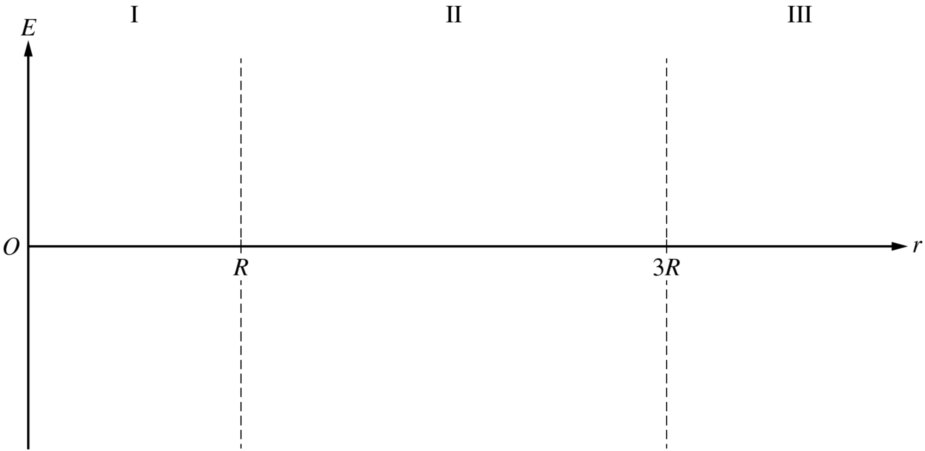


Cutout View

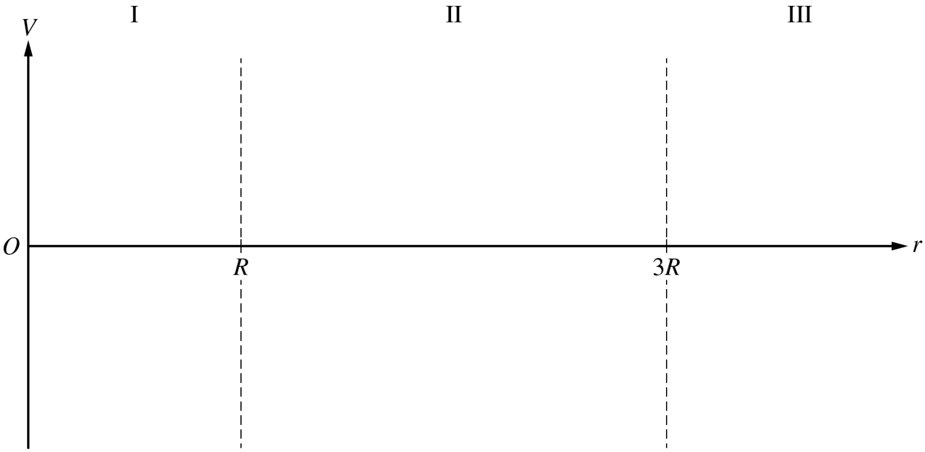
Cross-Section View

Note: Figures not drawn to scale.

Question 1



Question 1



Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(c) The magnitude of the electric field at $r = R$ is 12 N/C. Calculate the value of the electric field at $r = 2R$.

Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

Question 1

2022 Set 2 ▪ Q1

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

[Cutout View: a long horizontal cylinder (the inner nonconducting cylinder) enclosed by a concentric thin outer cylindrical shell; a segment of length L is marked at the left end of the assembly.] [Cross-Section View: two concentric circles; the inner circle has radius R, the outer circle has radius $3R$.] [Note: Figures not drawn to scale.]

(e)(i) On the following axes that include regions **I**, **II**, and **III**, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.

Question 1

[Graph axes: vertical axis E, horizontal axis r; three regions labeled I, II, III are marked above the axis, with vertical dashed reference lines at $r = R$ (boundary between I and II) and $r = 3R$ (boundary between II and III); origin labeled O.]

(e)(ii) On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.

[Graph axes: vertical axis V, horizontal axis r; three regions labeled I, II, III are marked above the axis, with vertical dashed reference lines at $r = R$ (boundary between I and II) and $r = 3R$ (boundary between II and III); origin labeled O.]

Solution 1

(a)

Mark scheme

- By charge conservation on the shell: $q_{net} = q_{inner} + q_{outer}$, so $q_{outer} = q_{net} - q_{inner} = (+4Q) - (-Q) = +5Q$. The outer surface of the shell carries charge $+5Q$ within length L. (A correct final value may earn the point even with no work shown.)

Solution 1

(b) Mark scheme

- Apply Gauss's law on a coaxial cylindrical Gaussian surface of radius $r < R$, length L: $\dfrac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA = E(2\pi rL)$. The enclosed charge scales with volume: $q_{enc} = \rho V = \left(\dfrac{Q}{\pi R^2 L}\right)(\pi r^2 L) = Q\dfrac{r^2}{R^2}$. Substituting: $E = q_{enc}\dfrac{1}{\varepsilon_0(2\pi rL)} = \left(Q\dfrac{r^2}{R^2}\right)\dfrac{1}{\varepsilon_0(2\pi rL)}$, giving $E = \dfrac{Qr}{2\pi\varepsilon_0 R^2 L}$ for $r < R$. Points: (1) using Gauss's law with the area or the enclosed charge substituted, (2) correct $q_{enc}(r)$, (3) correct area $2\pi rL$.

Solution 1

(c)

Mark scheme

- Outside the nonconducting cylinder (in the gap before the shell), $E \propto 1/r$. Since $E(R) = 12$ N/C and $2R$ is twice the distance, $E_{new} = E_{old}/2 = 12/2 = 6$ N/C. Points: (1) recognizing the reciprocal $E \propto 1/r$ dependence, (2) correct value with units, $E(2R) = 6$ N/C.

Solution 1

(d) Mark scheme

- Use $V_f - V_i = -\int_{s_i}^{s_f} E\,dr$ from $r = R$ to $r = 3R$: $\Delta V = -\int_R^{3R} E\,dr$. Substitute $E(r) = \dfrac{Q}{2\pi\varepsilon_0 rL}$ (from parts b/c): $\Delta V = -\dfrac{Q}{2\pi\varepsilon_0 L}\int_R^{3R} \dfrac{1}{r}\,dr = -\dfrac{Q}{2\pi\varepsilon_0 L}[\ln r]_R^{3R} = -\dfrac{Q}{2\pi\varepsilon_0 L}[\ln(3R) - \ln(R)] = -\dfrac{Q}{2\pi\varepsilon_0 L}\ln(3)$. So the magnitude of the potential difference is $|\Delta V| = \dfrac{Q\ln 3}{2\pi\varepsilon_0 L}$. Points: (1) correct integral setup with limits or evaluation attempted, (2) correct $E(r)$ substituted (consistent with part c), (3) correctly integrating with the correct limits.

Solution 1

(e)(i) Mark scheme

- Sketch of E vs r: in Region I ($0 < r < R$), E is positive and increases linearly from 0 at the axis to a maximum at $r = R$ (uniformly charged nonconducting cylinder, $E \propto r$). In Region II ($R < r < 3R$), E is positive, continuous with the value at R, and decreases along a concave-up curve (the $1/r$ falloff from parts b-d). At $r = 3R$ there is a discontinuous JUMP UP in magnitude (crossing the shell's surface charge). In Region III ($r > 3R$), E continues to decrease from this higher value. Points: (1) E positive with a linear increase from 0 to R; (2) a positive, concave-up decreasing graph for $r > R$; (3) a discontinuous increase in magnitude at $r = 3R$.

Solution 1

(e)(ii) Mark scheme

- Sketch of V vs r: V decreases monotonically across all three regions (never increases). In Region I it is concave down and decreasing; in Regions II and III it is concave up and decreasing; the curve is CONTINUOUS across $r = R$ and $r = 3R$ (no jump, unlike the E-graph). The vertical-axis intercept and where the curve crosses the horizontal axis are both irrelevant to scoring (the curve may sit entirely below the axis). A response with all three segments reflected across the horizontal axis caps at 2 of the 3 points. Points: (1) concave down and decreasing in Region I; (2) concave up and decreasing in Regions II and III; (3) continuous across all three regions.

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance *d*, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation *d* between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

Question 2

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.
[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\text{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(a)(i) Draw a line that represents the best fit to the points shown.

(a)(ii) Use the graph to calculate the charge Q.

Question 2

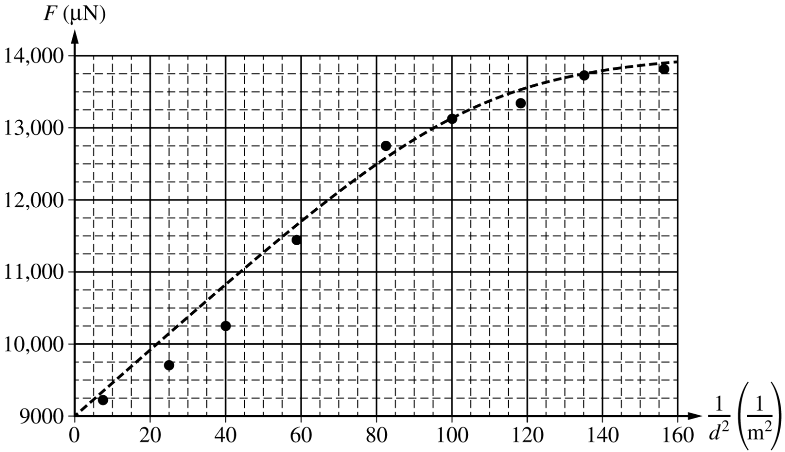
(a)(iii) [Second, extended graph: vertical axis F (μN) ranging from 9000 to 14,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\mathrm{m}^2}\right)$ ranging from 0 to 160 in increments of 20; a dashed best-fit line/curve is drawn through data points from the origin region up through about $(140, 13,700)$; plotted data points (approximate): $(10, 9100)$, $(30, 9700)$, $(50, 10,000)$, $(65, 11,500)$, $(90, 12,700)$, $(105, 13,000)$, $(120, 13,300)$, $(140, 13,700)$, $(155, 13,800)$.]

On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

(a)(iv) Determine the distance between the centers of the spheres for the data point indicated above.

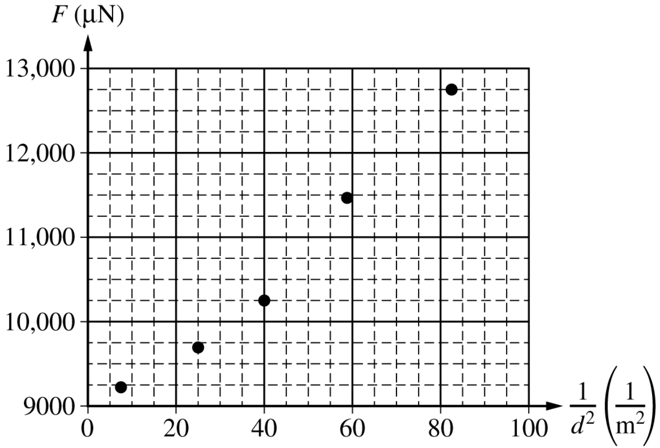
(a)(v) What physical quantity does the vertical intercept represent? Justify your answer.

Question 2



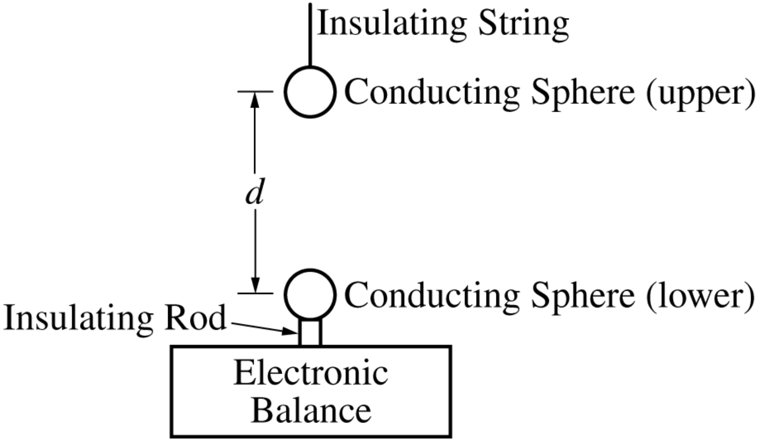
F (μN)
14,000
13,000
12,000
11,000
10,000
9000
0
20
40
60
80
100
120
140
160
1/d² (1/m²)

Question 2



F (μN)
13,000
12,000
11,000
10,000
9000
0
20
40
60
80
100
$\frac{1}{d^2}\left(\frac{1}{m^2}\right)$

Question 2


Insulating String
Conducting Sphere (upper)
d
Conducting Sphere (lower)
Insulating Rod
Electronic
Balance

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance d, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation d between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and

Question 2

the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\text{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(b) The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

_______ Yes _______ No

Question 2

Justify your answer.

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance d, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation d between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and

Question 2

the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\mathrm{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(c)(i) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

On the circles representing the spheres below, use a single "$+$" sign on each sphere to represent the locations of highest concentration of the excess positive charges.

[Two blank circles are shown, stacked vertically, representing the upper and lower spheres, for the student to mark.]

(c)(ii) Explain how this rearrangement could be responsible for the discrepancy.

Question 2

2021 ▪ Q2

[Apparatus diagram: A fixed support holds an insulating string from which a Conducting Sphere (upper) hangs, labeled "Insulating String" at top. Directly below it, separated by a distance d, is a Conducting Sphere (lower) mounted on an "Insulating Rod," which rests on an "Electronic Balance" (drawn as a rectangular box). The two spheres are identical conducting spheres, vertically aligned with separation d between their centers.]

Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and

Question 2

the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.

[Graph: vertical axis F (μN) ranging from 9000 to 13,000 in increments of 1000; horizontal axis $\frac{1}{d^2}\left(\frac{1}{\mathrm{m}^2}\right)$ ranging from 0 to 100 in increments of 20; plotted data points (approximate): $(5, 9200)$, $(30, 9700)$, $(45, 10, 300)$, $(65, 11, 400)$, $(90, 12, 900)$; no line of best fit drawn yet.]

(d)(i) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

On the circles representing the spheres below, use a single "$+$" sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single "$-$" sign on the top sphere to represent the location of the highest concentration of the excess negative charges.

Question 2

[Two blank circles are shown, stacked vertically, representing the upper (top) and lower (bottom) spheres, for the student to mark.]

(d)(ii) For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Draw a single straight best-fit line through the plotted data points (approximately through the trend from the lower-left points near $(10, 9100)$ up through the upper-right points near $(140, 13700)$), balancing points above and below the line rather than connecting the dots.

Solution 2

(a)(ii) Mark scheme

- Using two points on the best-fit line, the slope is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{(12{,}000 - 10{,}500)\ \mu\text{N}}{(72 - 40)\ \text{m}^{-2}} = 4.7 \times 10^{-5}\ \text{N} \cdot \text{m}^2$. Since $F = \dfrac{kQ^2}{r^2} = kQ^2 \cdot \dfrac{1}{d^2}$, the slope of F vs $1/d^2$ equals kQ^2. Solving for the charge: $Q = \sqrt{\dfrac{\text{slope}}{k}} = \sqrt{\dfrac{4.7 \times 10^{-5}\ \text{N} \cdot \text{m}^2}{9 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2}} = 7.2 \times 10^{-8}\ \text{C}$. Points: (1) correctly calculating the slope from two points on the best-fit line, (2) correctly relating the slope to the charge (slope $= kQ^2$), (3) correctly calculating $Q = 7.2 \times 10^{-8}$ C consistent with that slope.

Solution 2

(a)(iii)

Mark scheme

- Circle the data point corresponding to $1/d^2 \approx 82.5\ \mathrm{m}^{-2}$ (the point near $F \approx 12{,}900\ \mu\mathrm{N}$ that is used in part (a)(iv) to determine the separation distance d).

(a)(iv)

Mark scheme

- From the graph, the circled data point corresponds to $1/d^2 = 82.5\ \mathrm{m}^{-2}$, so $d = \sqrt{1/82.5\ \mathrm{m}^{-2}} = 0.11\ \mathrm{m}$.

Solution 2

(a)(v) Mark scheme

- The vertical (F) intercept represents the weight of the (lower) conducting sphere and insulating rod, i.e. the balance/scale reading with no electric force present. Justification: When the spheres are infinitely far apart ($1/d^2 \to 0$), there is no electric force between them, so the only force recorded by the balance is the weight of the lower sphere and insulating rod — equivalently, the scale is zeroed before the rod and lower sphere are placed on it, so the nonzero y-intercept is their weight. Points: (1) indicating the intercept is the weight of the sphere/rod, (2) a correct justification.

Solution 2

(b)

Mark scheme

- Yes. Justification: Because the later force measurements fall below the values predicted by extrapolating the best-fit line, a decrease in charge (charge slowly leaking off the top sphere over the course of the experiment, so Q — and hence $F \propto Q^2$ — decreases) could explain the discrepancy seen in the data.

Solution 2

(c)(i) Mark scheme

- On each sphere, mark a single '+' at the point farthest from the other sphere (e.g. at the top of the upper sphere and the bottom of the lower sphere). Since both spheres carry excess positive charge, the mutual repulsion between the two spheres' charge distributions pushes each sphere's own excess positive charge toward the side away from the other sphere. Points: (1) correctly indicating the position of the excess positive charges (concentrated away from the other sphere on each), (2) correctly indicating the net charge on each sphere is positive (a single '+' marked, not a mix of signs).

Solution 2

(c)(ii)

Mark scheme

- On a conductor, excess charges are free to move and repel each other; the excess (same-sign) charges on each sphere rearrange until they are as far apart as possible from the charge on the other sphere. This polarization/redistribution increases the effective separation between the charge concentrations (beyond simply using the center-to-center distance d), which reduces the actual force below the $F = kQ^2/d^2$ prediction — explaining why the closer-spacing data points fall below the best-fit line.

Solution 2

(d)(i)

Mark scheme

- Mark a single '$-$' on the top sphere at the point closest to the bottom sphere, and a single '$+$' on the bottom sphere at the point closest to the top sphere. Because the spheres now carry opposite charges, they attract; unlike charges concentrate on the sides nearest each other.

Solution 2

(d)(ii)

Mark scheme

- Less than. Justification: The oppositely charged spheres attract each other, and this attractive force pulls the lower sphere upward (toward the upper sphere), decreasing the upward normal force the scale must exert to support it; thus the force (scale) reading with opposite charges is less than the reading obtained with spheres of the same (like) charge, which repelled and increased the reading. Points: (1) selecting 'Less than' and attempting a relevant justification, (2) a fully correct justification.

Question 1

2018 ▪ Q1

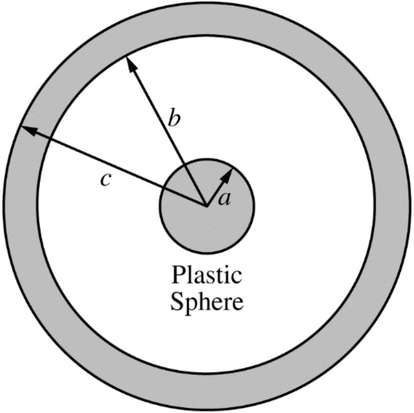
A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

Question 1

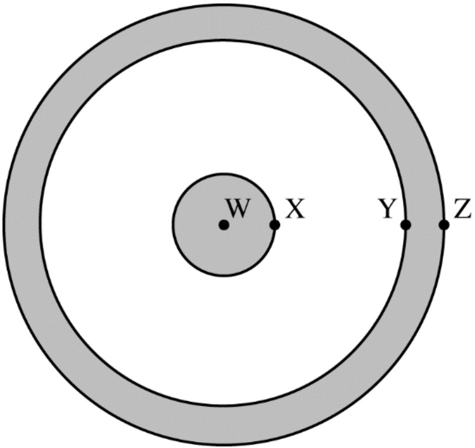
(a)(i) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following region.
$r < a$
(a)(ii) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following region.
$a < r < b$

Question 1



Conducting Spherical Shell

Question 1



Question 1

Question 1

2018 ▪ Q1

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

(b)(i) Use Gauss's law to derive an expression for the magnitude of the electric field in the following region.

$r < a$

Question 1

(b)(ii) Use Gauss's law to derive an expression for the magnitude of the electric field in the following region.

$a < r < b$

Question 1

2018 ▪ Q1

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

(c)(i) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

Determine the charge on the inner surface of the conducting shell.

Question 1

Justify your answer.

(c)(ii) Determine the charge on the outer surface of the conducting shell.

Question 1

2018 ▪ Q1

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

[Figure: A cross-sectional diagram showing a solid gray "Plastic Sphere" of radius a at the center, surrounded by a gray shaded ring labeled "Conducting Spherical Shell" with inner radius b and outer radius c. Arrows from the center label the radius a to the edge of the plastic sphere, radius b to the inner edge of the shell, and radius c to the outer edge of the shell.]

(d)(i) On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

Question 1

[Graph: Axes with vertical axis labeled E and horizontal axis labeled r, origin at O. Horizontal axis shows tick marks at a, b, c (with b and c close together near the right side, and a dashed segment continuing past c). No data plotted — blank axes for the student to sketch on.]

(d)(ii) The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

______ W ______ X ______ Y ______ Z

[Figure: A repeat of the cross-sectional diagram (plastic sphere inside conducting spherical shell), with four labeled points marked: W at the center of the sphere, X on

Question 1

the surface of the sphere (just outside W), Y on the inner surface of the shell, and Z on the outer surface of the shell.]

Solution 1

(a)(i)

Mark scheme

- Set up the enclosed-charge integral using $\rho(r) = \beta r$: $q_{enc} = \int \rho \, dV = \int \beta r \, dV$. Substitute the spherical shell volume element $dV = 4\pi r'^2 \, dr'$: $q_{enc} = \int_0^r (4\pi r'^2)(\beta r') \, dr' = 4\beta\pi \int_0^r r'^3 \, dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_0^r$. Final answer: $q_{enc} = \beta\pi r^4$. Points: 1 for setting up the integral with $\rho = \beta r$, 1 for correctly substituting dV, 1 for the correct final expression.

Solution 1

(a)(ii)

Mark scheme

- For $a < r < b$ the Gaussian sphere encloses the entire plastic sphere (charge stops at $r = a$), so the enclosed charge is just the part (a)(i) result evaluated at $r = a$: $q_{enc} = \beta\pi a^4$.

Solution 1

(b)(i)

Mark scheme

- Apply Gauss's law with spherical symmetry: $\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = E(4\pi r^2)$ (1 point). Substitute $q_{enc} = \beta\pi r^4$ from part (a)(i): $\frac{\beta\pi r^4}{\varepsilon_0} = E(4\pi r^2)$, giving $E = \dfrac{\beta r^2}{4\varepsilon_0}$ (1 point).

Solution 1

(b)(ii)

Mark scheme

- Substitute $q_{enc} = \beta\pi a^4$ from part (a)(ii) into Gauss's law: $\dfrac{\beta\pi a^4}{\varepsilon_0} = E(4\pi r^2)$, giving $E = \dfrac{\beta a^4}{4\varepsilon_0 r^2}$.

Solution 1

(c)(i)

Mark scheme

- $q_{inner} = -\beta\pi a^4$ (opposite sign of, and consistent with, the part (a)(ii)/(b)(ii) sphere charge $\beta\pi a^4$) (1 point). Justification: since $E = 0$ everywhere outside the shell, $\frac{q_{enc}}{\varepsilon_0} = 0$ for a Gaussian surface enclosing the whole shell, so $q_{enc} = 0 = q_{sphere} + q_{inner}$, giving $q_{inner} = -q_{sphere} = -\beta\pi a^4$ (1 point).

Solution 1

(c)(ii)

Mark scheme

- $q_{outer} = 0$. Since $E = 0$ outside the shell, the shell's total charge $Q = q_{inner} + q_{outer}$ must equal $-q_{sphere}$ (to cancel the sphere's field beyond the shell). Because $q_{inner} = -q_{sphere}$ already accounts for all of that required charge, the outer surface carries none: $q_{outer} = 0$.

Solution 1

(d)(i) Mark scheme

- Sketch: for $0 \leq r < a$, $E = \dfrac{\beta r^2}{4\varepsilon_0}$ is a concave-up curve starting at $E = 0$ at the origin and increasing up to $r = a$ (1 point). For $a < r < b$, $E = \dfrac{\beta a^4}{4\varepsilon_0 r^2}$ is a concave-up curve ($\propto 1/r^2$) that continues from the $r = a$ value and decreases to a nonzero value at $r = b$ (1 point). For $r > b$ (inside the conducting shell material, where $E = 0$ always, and beyond the shell, given $E = 0$ there), the curve sits at $E = 0$ (1 point). A discontinuity in the curve exactly at $r = b$ is not required due to scale.

Solution 1

(d)(ii)

Mark scheme

- Ranking (1 = largest potential): W = 1, X = 2, Y = 3, Z = 3 (Y and Z tied). Reasoning: potential decreases moving outward from the center in this configuration, so W (center) $>$ X (sphere surface) $>$ shell surfaces; the conducting shell is an equipotential (E=0 inside the conductor), so Y (inner shell surface) and Z (outer shell surface) are at the same potential and share the ranking 3. 1 point for any ranking with Y and Z tied, 1 point for the order W $>$ X $>$ Y & Z.