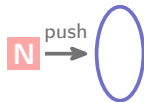


Electromagnetic Induction

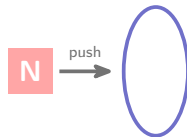
AP Physics C: E&M ■ Unit 13

AP Physics C: E&M



What we will cover

- 1 Magnetic flux
- 2 Faraday's law and Lenz's law
- 3 Induced currents and forces
- 4 Inductance
- 5 LR and LC circuits





Magnetic flux and Faraday's law

Flux, then Faraday

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta$$

$$\varepsilon = - \frac{d\Phi_B}{dt}$$

The minus sign is **Lenz's law** : the induced current **opposes the change** – energy conservation.

Motional emf: $\varepsilon = BLv$; a coil of N turns: $\times N$.



Motor rotor

Flux, and the three ways it changes

Magnetic flux measures how much field passes through a surface. For a uniform field through a flat loop it is a dot product, so the **area vector** – normal to the surface – sets the sign:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$$

- B changes – a stronger or weaker field
- A changes – the loop grows, shrinks or leaves the field
- θ changes – the loop rotates

Any one of the three induces an e.m.f.
A generator uses the third.

The sliding rod on rails

$$L = 0.20 \text{ m at } 3.0 \text{ m/s across } 0.50 \text{ T}$$

$$\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30 \text{ V}$$

The rod sliding on conducting rails, closed by a resistor, is the setup every question is built from: the growing loop area changes the flux.

Once a current flows, the field pushes back on it ($\vec{F} = \int I d\vec{l} \times \vec{B}$), and by Lenz's law that force always **resists the motion** that caused it. Push harder, get more current, feel more drag – the mechanical work is what becomes electrical energy.

Faraday's law is also the third of **Maxwell's equations** : a changing magnetic flux creates a **circulating electric field**.



Induced currents and forces

The **induced current** (Lenz's law) always creates a **retarding force** that resists the motion causing it.

- mechanical work → electrical energy → heat
- **eddy currents** in solid metal damp motion (magnetic braking)



Substation transformer

The induced current fights the change that made it

the force

once an induced current flows, the external field pushes on it: $\vec{F} = \int I d\vec{l} \times \vec{B}$

Lenz's law

that force always **resists the motion** that is changing the flux

why

energy conservation – if it helped the motion you would get free energy

eddy currents

loops induced in a solid conductor; the same opposing force is what brakes a magnet falling down a copper tube

The FRQ walks one chain: **flux** → **emf** → **current** → **force** → **Newton's second law**. Write each link separately and check the direction at every step.



Inductance

An **inductor** resists a **change** in current, creating a **back emf** :

Back emf and stored energy

$$\varepsilon = -L \frac{dI}{dt}, \quad U = \frac{1}{2} LI^2$$

Energy density $u = \frac{B^2}{2\mu_0}$ – the magnetic mirror of $\frac{1}{2}\varepsilon_0 E^2$.

Worked example

2 H with I rising at 3 A/s:

$$\varepsilon = L \frac{dI}{dt} = 2 \times 3 = 6 \text{ V}$$

At a steady 4 A: $U = 16 \text{ J}$, and the back emf vanishes.

The inductor, and the solenoid that makes one

inductance

L relates flux to current: $\varepsilon = -L \frac{dI}{dt}$ – it opposes a *change* in current, not current itself

inductor

a component built to have large L – usually a **solenoid**

the geometry

$L = \frac{\mu_0 N^2 A}{\ell}$: turns **squared**, so doubling N quadruples L

stored energy

$U_L = \frac{1}{2} L I^2$ – held in the **magnetic field**, the mirror of $\frac{1}{2} C (\Delta V)^2$ in a capacitor

An inductor is to current what mass is to velocity: it resists change. That analogy carries all the way through the RL and LC results.



LR and LC circuits

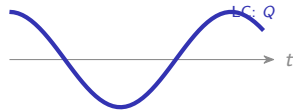
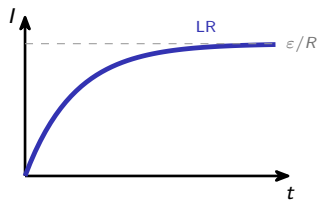
LR: smooth growth

$$\tau = \frac{L}{R}, \quad I = \frac{\varepsilon}{R} \left(1 - e^{-t/\tau} \right)$$

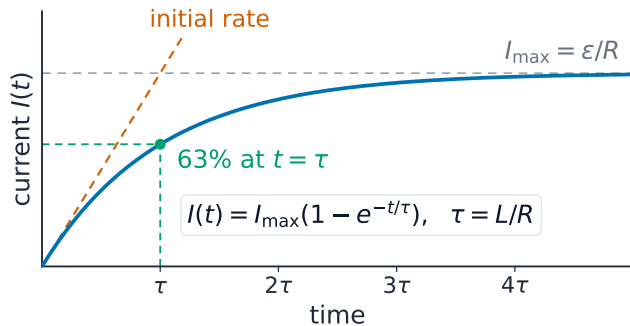
An inductor blocks current at first, passes it freely later – **opposite** to a capacitor.

LC: oscillation

$$\omega = \frac{1}{\sqrt{LC}}$$



The current in an RL circuit rises exponentially



$$\text{RL rise: } I = I_0(1 - e^{-t/\tau}) \cdot \tau = L/R$$

The current in an RL circuit rises exponentially, reaching 63% at one time constant

The two moments an RL question always asks about

just after closing ($t = 0$)

current cannot jump, so it is still **zero**; the inductor self-induces an emf equal and opposite to the applied one, and behaves like an **open circuit**

long after ($t \rightarrow \infty$)

$dl/dt \rightarrow 0$, so the inductor's emf vanishes and it behaves like a **plain wire**: $I = \mathcal{E}/R$

the capacitor is the mirror

a capacitor is a *wire* at $t = 0$ and an *open circuit* at $t \rightarrow \infty$ – exactly the other way round

in between

the exponential, governed by τ

Almost every circuit FRQ asks for these two limits before it asks for anything else. They need **no calculus at all** – just the two rules above.

The RL circuit, and its time constant

the loop rule

$$\varepsilon - IR - L \frac{dI}{dt} = 0 - \text{a differential equation, not an algebraic one}$$

the solution

$$I(t) = \frac{\varepsilon}{R} (1 - e^{-t/\tau}), \text{ with } \tau = \frac{L}{R}$$

time constant

after one τ a rising current reaches about 63% of its final value;
a decaying one falls to 37%

bigger L , smaller R

both make τ longer – the circuit takes longer to settle

$\tau = L/R$ has units of seconds. Checking that is the fastest way to catch an inverted expression.

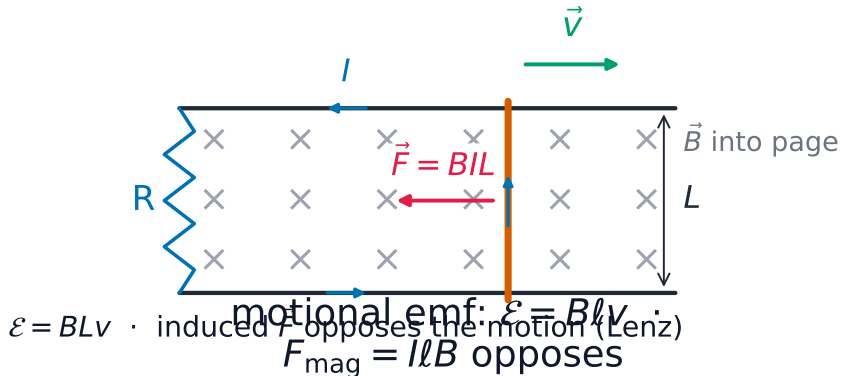
Worked example – the LC oscillator

$L = 2.0 \text{ H}$ and $C = 8.0 \mu\text{F}$. Find the angular frequency and the period.

- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0(8.0 \times 10^{-6})}}$
- $= 250 \text{ rad/s}$
- $T = \frac{2\pi}{\omega} = 0.025 \text{ s}$

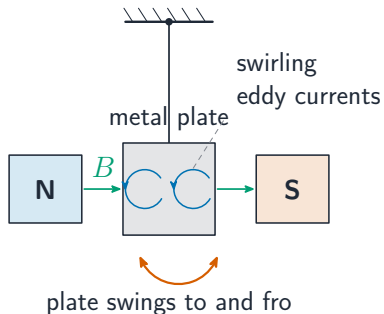
This is the same equation as a mass on a spring: **simple harmonic motion** with charge in place of displacement, L in place of mass and $1/C$ in place of k . **Conservation of energy** moves it between the capacitor's field and the inductor's.

Induced Currents and Magnetic Forces



A rod sliding on rails: the induced current feels a force opposing the motion

Induced eddy currents oppose motion



induced eddy currents oppose the motion $\Rightarrow$ swing is damped

Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field

Flux, the rules, and what falls out of them

Surface integral

Magnetic flux is a **surface integral**: $\Phi_B = \int \vec{B} \cdot d\vec{A}$ – and only the component through the surface counts.

Right-hand rule

The **right-hand rule** fixes every direction here: the field of a current, the force on a moving charge, and the sense of an induced current.

Permeability

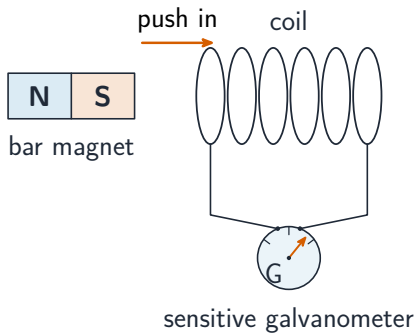
μ_0 , the **magnetic permeability** of free space, appears in Ampère's law and again in the speed of light.

Electromagnetic waves

Maxwell's equations give $c = 1/\sqrt{\mu_0\epsilon_0}$ – so **electromagnetic waves** travel at a speed built from two electrostatic and magnetostatic constants.

Lenz's law is conservation of energy in disguise: the induced current opposes the change that made it, or you would get energy for nothing.

Electromagnetic Induction



Moving a magnet into a coil induces an e.m.f. that drives a current

Exam tips

- Compute **flux** $\Phi_B = \int \vec{B} \cdot d\vec{A}$ and get the induced EMF from **Faraday's law**
 $\mathcal{E} = -\frac{d\Phi_B}{dt}$.
- Use **Lenz's law** (the minus sign) to fix the direction: the induced current opposes the change in flux.
- Flux changes three ways – changing B , changing area, or changing angle – identify which and differentiate.
- For a rod of length L moving at speed v , the motional EMF is BLv .
- An inductor stores energy $\frac{1}{2}LI^2$ and resists **changes** in current (RL time constant $\tau = L/R$).



Unit 13 – key takeaways

1

Faraday

$\varepsilon = -d\Phi_B/dt$; Lenz opposes the change

2

Induced force

always retarding; eddy currents damp motion

3

Inductance

$\varepsilon = -L di/dt$; stores $U = \frac{1}{2}LI^2$

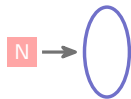
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Circuits

LR grows with $\tau = L/R$; LC oscillates at $1/\sqrt{LC}$

Check yourself

- 1 Write Faraday's law for the induced emf.
- 2 Which law fixes the direction of the induced current?
- 3 At the first instant, how does an inductor behave?
- 4 What sets the oscillation frequency of an LC circuit?



Answers 1. $\varepsilon = -d\Phi_B/dt$ 2. Lenz's law 3. it blocks current (open) 4. $\omega = 1/\sqrt{LC}$