

Magnetic Fields & Electromagnetism

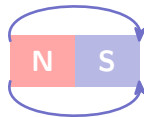
AP Physics C: E&M ■ Unit 12

AP Physics C: E&M



What we will cover

- 1 Magnetic fields
- 2 Force on a moving charge
- 3 Fields of current-carrying wires
- 4 Ampère's law





Magnetic fields

A **magnetic field** $\vec{B}$ has field lines in **closed loops** – there are **no monopoles**. Materials differ by how their **atomic dipoles** align:

- **ferromagnetic** – an external field aligns whole **magnetic domains**, and the alignment **survives** the field being removed – a permanent magnet; align strongly, stay (iron)
- **paramagnetic** – align weakly
- **diamagnetic** – weakly oppose



Aurora magnetism

The magnetic field, and why there are no monopoles

magnetic field $\vec{B}$

a **vector field** surrounding magnets, moving charges and currents; it sets the force on any *moving* charge

field lines

run out of the north pole and into the south *outside* the magnet, and continue through it – they always **close on themselves**

Gauss's law for B

$\oint \vec{B} \cdot d\vec{A} = 0$ through *any* closed surface – as many lines leave as enter

what that means

no isolated **magnetic monopole** has ever been found; cut a magnet in half and you get two magnets

a compass

a small **magnetic dipole** that lines up **tangent** to the local field

Compare with electric flux, which equals $Q_{\text{enc}}/\epsilon_0$ and is non-zero. The **zero** on the right is the whole difference between the two fields.



Force on a moving charge

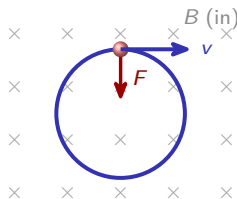
A cross product

$$\vec{F} = q \vec{v} \times \vec{B}$$

Size $qvB\sin\theta$; $\perp$ to $\vec{v}$, so it does **no work**. A wire feels $\vec{F} = I\vec{L} \times \vec{B}$.

Perpendicular entry $\Rightarrow$ a circle:

$$qvB = \frac{mv^2}{r}, \text{ so } r = \frac{mv}{qB}.$$



How a material answers a magnetic field

ferromagnetic

iron, nickel, cobalt: domains align *with* the field and stay aligned – a strong response that can be made permanent

paramagnetic

a weak alignment *with* the field, lost as soon as the field is removed

diamagnetic

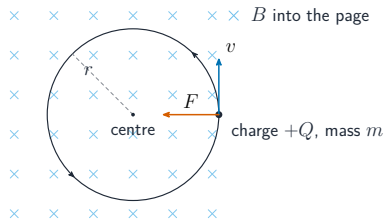
present in **all** materials: the electronic structure produces a usually weak alignment *opposite* to the field

permeability

the measure of that response. Free space has the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$

Diamagnetism is universal but weak, so it only shows when nothing stronger is present. The other two are extra effects layered on top of it.

A moving charge both makes and feels a field



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

A moving charge does **two** things: it *creates* a field, perpendicular to its velocity and circling it, and it *feels* a force in an external one. The force is perpendicular to both $\vec{v}$ and $\vec{B}$ – fingers along $\vec{v}$, curl toward $\vec{B}$, thumb gives the force on a **positive** charge – so it does no work and the path is a circle.

The velocity selector

the setup

an electric and a magnetic field, crossed, in the same region

they act independently the two forces add as **vectors**: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

balance them

$qE = qvB$, so only particles with $v = E/B$ pass straight through

note

the selected speed depends on **neither** q nor m – so it filters by speed alone

Put a **velocity selector** in front of a mass spectrometer and the second stage's radius $r = mv/qB$ then measures m/q cleanly, because v is already known.

Worked example – a proton circling in a field

A proton ($q = 1.6 \times 10^{-19} \text{ C}$, $m = 1.67 \times 10^{-27} \text{ kg}$) enters a 0.50 T field at $2.0 \times 10^5 \text{ m/s}$, perpendicular to it. Find r and T .

- the magnetic force is the **centripetal**

force : $qvB = \frac{mv^2}{r}$

- $r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27})(2.0 \times 10^5)}{(1.6 \times 10^{-19})(0.50)} = 4.2 \times 10^{-3} \text{ m}$

- $T = \frac{2\pi m}{qB} = 1.3 \times 10^{-7} \text{ s}$

T contains **no** v . Faster particles ride larger circles in exactly the same time – which is the principle the cyclotron is built on.



Fields of current-carrying wires

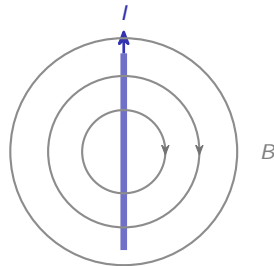
Moving charge **makes** magnetic field:

Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

Integrate for standard results:

- long straight wire: $B = \frac{\mu_0 I}{2\pi r}$
- loop centre: $B = \frac{\mu_0 I}{2R}$



Biot–Savart, and the arc it makes easy

Biot–Savart law -

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} - \text{the field from one current element}$$

a straight wire

gives **concentric circles** around it, $B = \frac{\mu_0 I}{2\pi r}$, direction from the right-hand rule

at a loop's centre

every element $d\vec{l}$ is **perpendicular** to $\hat{r}$ and equally distant R
– say that sentence, it is the marked step – giving $B = \frac{\mu_0 I}{2R}$

an arc

a fraction of that circle contributes the same fraction: $B = \frac{\mu_0 I \theta}{4\pi R}$ for angle θ

For several wires, **superpose** : work out each field's magnitude and direction separately, then add as vectors.

The field of a current, and one measurement

Biot–Savart law

The **Biot–Savart law** gives the field of a current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$ – integrate it for any geometry.

Magnetic permeability

μ_0 is the **magnetic permeability** of free space, the constant that fixes the strength of the magnetic interaction.

Superposition

Superposition does the rest: the field of several wires is the vector sum of their separate fields.

Hall effect

The **Hall effect**: a magnetic field pushes the carriers to one side of a conductor, producing a transverse voltage – which reveals the *sign* of the charge carriers.

Magnetic field lines form closed loops – there is no magnetic charge for them to start on.



Ampère's law

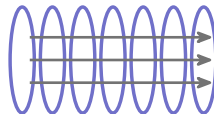
For enough symmetry, Ampère's law beats integrating Biot-Savart:

The loop integral

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Choose an **Amperian loop** where B is constant and parallel, so the integral becomes $B \times \text{length}$.

Inside a **solenoid** : $B = \mu_0 nI$, uniform.



$$B = \mu_0 nI$$

Worked example – inside a current-carrying cylinder

A solid cylinder of radius R carries current I spread uniformly over its cross-section.

Find B at radius $r < R$.

- loop: a circle of radius r , concentric – B is constant on it and parallel to it by symmetry

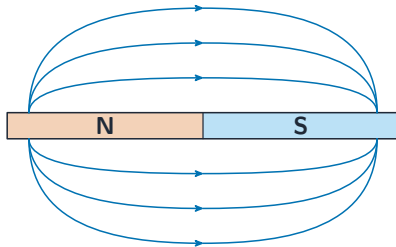
- enclosed current scales with area:

$$I_{\text{enc}} = I \frac{\pi r^2}{\pi R^2} = I \frac{r^2}{R^2}$$

- $B(2\pi r) = \mu_0 I \frac{r^2}{R^2}$, so $B = \frac{\mu_0 I r}{2\pi R^2}$

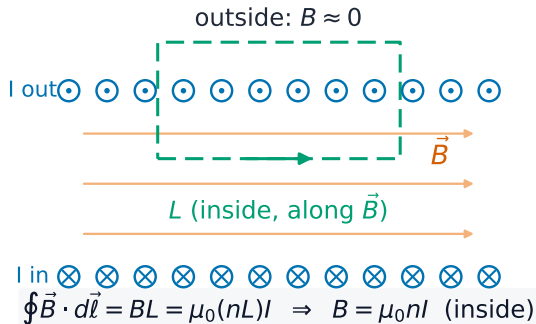
Inside, B grows **linearly** with r ; outside it falls as $1/r$, so the field peaks at the surface. Check both limits agree at $r = R$.

Closed loops, and what that means



Field lines run N to S outside the magnet and close on themselves inside it. That is [Gauss's law for magnetism](#) – the second of **Maxwell's equations** : the net flux through *any* closed surface is zero, as many lines leaving as entering. It is the same statement as “no isolated **magnetic monopole** has ever been found”.

Ampère's Law, continued



A rectangular Amperian loop derives the uniform field inside a solenoid

Every Ampère's-law answer earns its marks in the setup

1. name the loop

say which **Ampèrian loop** you chose and where it runs

2. justify it

why B is **constant** on it and **parallel** to it – by symmetry, in words

3. count I_{enc}

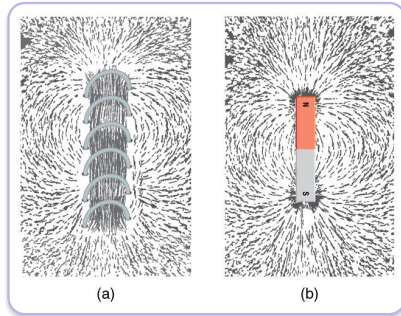
only current *through* the loop; for a spread-out current use the **current density** and the enclosed area

4. solve

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$, giving $B(2\pi r) = \mu_0 I_{\text{enc}}$ for a circular loop

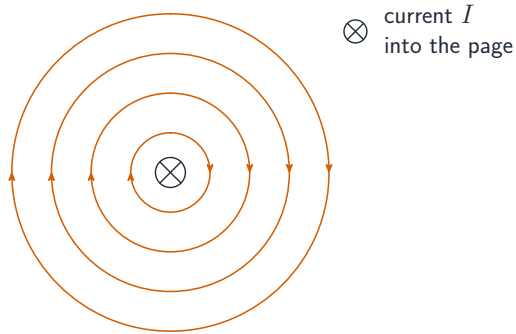
For symmetric distributions this beats Biot–Savart by a long way – but only when the symmetry argument is **stated**, not assumed.

Magnetic Fields of Current-Carrying Wires



A current-carrying coil (solenoid) makes a magnetic field shaped just like a bar magnet's

Concentric circular field lines surround a straight



circular field lines (right-hand grip rule)

Concentric circular field lines surround a straight current-carrying wire

Exam tips

- The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ is **perpendicular** to velocity – use the right-hand rule and note it does **no work**.
- A charge in a uniform field moves in a circle with $r = \frac{mv}{qB}$.
- Find the field of currents with **Biot–Savart** $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$ or **Ampère's law** $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ when there is symmetry.
- Force on a wire is $\vec{F} = I\vec{L} \times \vec{B}$; keep the cross-product direction straight.
- Distinguish the force **on** a charge from the field the current **creates**.



Unit 12 – key takeaways

1

Fields

lines in closed loops; no magnetic monopoles

2

Force

$\vec{F} = q\vec{v} \times \vec{B}$, does no work; circle
 $r = mv/qB$

3

Wire fields

Biot-Savart; long wire $\mu_0 I / 2\pi r$

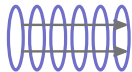
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Ampère

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$; solenoid $\mu_0 nI$

Check yourself

- 1 Why does the magnetic force on a moving charge do no work?
- 2 Which law gives the field of a symmetric current quickly?
- 3 Double the distance from a straight wire – what happens to B ?
- 4 Is the field inside an ideal solenoid uniform?



Answers 1. it is perpendicular to $\vec{v}$ 2. Ampère's law 3. it halves 4. yes