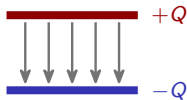


Conductors and Capacitors

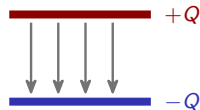
AP Physics C: E&M ■ Unit 10

AP Physics C: E&M



What we will cover

- 1 Conductors in electrostatic equilibrium
- 2 Redistribution of charge
- 3 Capacitors
- 4 Dielectrics





Conductors in electrostatic equilibrium

With charges at rest in a **conductor** :

- the field is **zero** everywhere inside
- all excess charge sits on the **outer surface**
- outside, the field is **perpendicular** to the surface
- the whole conductor is one **equipotential**

A metal shell keeps its inside field zero – a **Faraday cage** .



Capacitors

What a conductor in equilibrium must look like

inside

$\vec{E} = 0$ everywhere in the bulk – if it were not, the free electrons would still be moving

the surface

all **excess charge** sits on it. Negative net charge means extra surface electrons; positive means a shortage

the field just outside

perpendicular to the surface; a parallel component would push charge along it

sharp points

charge density is largest where the surface curves sharply, so the outside field is **strongest at points and edges**

electrostatic shielding

in an external field the conductor **polarizes** until its own charges cancel the field inside – a cavity is shielded

The whole conductor is one **equipotential surface**, surface included. Every conductor result on this page follows from $\vec{E} = 0$ inside.



Redistribution of charge

Touch two conductors and charge flows until they reach a **common potential**:

- **conservation of charge** fixes the total
- a bigger conductor holds more at the same V
- an external field induces opposite charges on the two sides

Worked example – identical spheres

12 nC touched to a neutral twin:

each ends with 6 nC

Touch a much larger sphere instead and most charge flows onto it.

Worked example – two spheres touch and separate

A sphere of radius R carrying $+6.0 \mu\text{C}$ touches a distant sphere of radius $2R$, then they separate. Find each charge.

- in contact they are one conductor, so their potentials are **equal**: $\frac{kq_1}{R} = \frac{kq_2}{2R}$
- so $q_2 = 2q_1$, and $q_1 + q_2 = 6.0 \mu\text{C}$
- $q_1 = 2.0 \mu\text{C}$ (small), $q_2 = 4.0 \mu\text{C}$ (large)

Charge splits in proportion to **radius**, not equally.

Ground is the limiting case: an idealised reference at $V = 0$ that can absorb or supply any amount of charge, so grounding near an external charge leaves an **induced charge** behind.



Capacitors

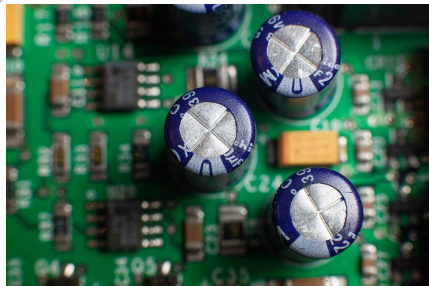
A **capacitor** stores charge $\pm Q$ on two plates:

Capacitance and stored energy

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

Energy lives in the field: $u = \frac{1}{2} \epsilon_0 E^2$. Combine **opposite** to resistors – parallel adds.



Pcb capacitors

The parallel-plate derivation, in three steps

1. field

Gauss's law plus **superposition** give $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$ between the plates, and zero outside

2. potential

the field is uniform, so $\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$

3. capacitance

$$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

the energy

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$$

That $\frac{1}{2}$ is the **triangle** under the Q - ΔV line: the first charge moved across costs almost nothing, the last costs the full ΔV . The same $E \rightarrow \Delta V \rightarrow C$ chain does the cylinder and the sphere.

Capacitance is geometry, and nothing else

the definition

$$C = \frac{Q}{\Delta V} - \text{but } C \text{ depends on neither}$$

what sets it

the plate area, the gap, the shape, and the material filling the gap

double Q

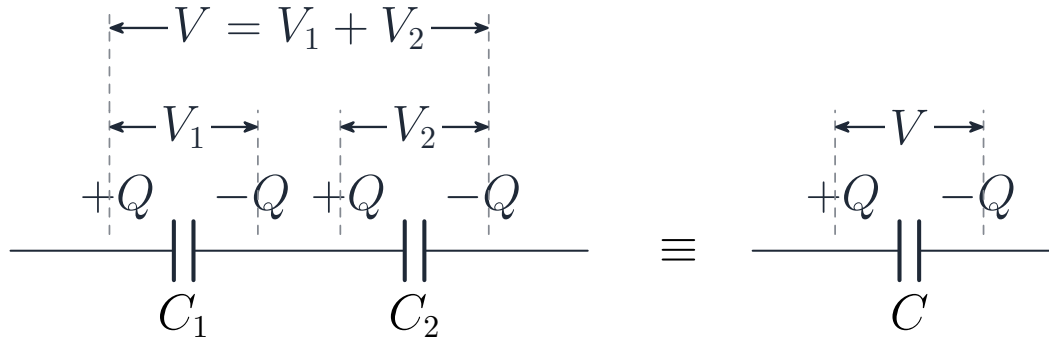
ΔV doubles with it, and C is unchanged

the units

1 F = 1 C/V; real capacitors are pF to μF because a farad is enormous

If your answer makes C depend on Q or on ΔV , you have made an algebra error. Geometry only.

Capacitors in series carry the same charge – one



Capacitors in series carry the same charge, and their p.d.s add

Inside the plates, a charge is a projectile

the field

uniform between the plates, so the force qE and the acceleration are **constant**

across the gap

velocity along the plates is unchanged; the transverse motion accelerates – exactly **projectile motion**

time in the field

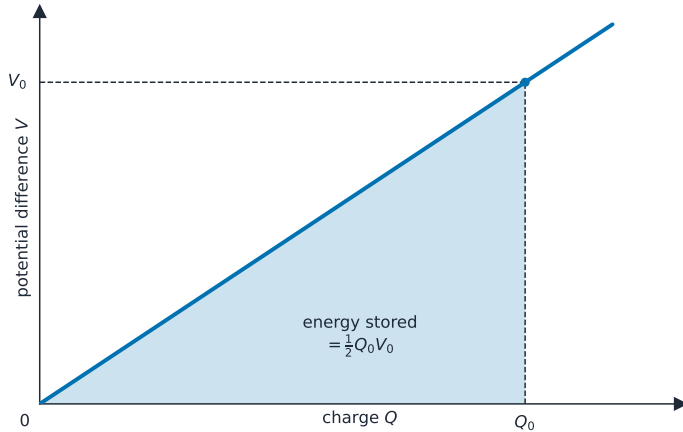
$t = L/v_0$, set entirely by the horizontal motion

deflection

$$y = \frac{1}{2}at^2 = \frac{qEL^2}{2mv_0^2}$$

An electron entering midway at $v_0 = 2.0 \times 10^7$ m/s through $E = 1.0 \times 10^3$ N/C is deflected by exactly this formula. **Split the motion into two perpendicular directions** and the mechanics is already done.

Capacitors in series carry the same charge – two



The energy stored in a capacitor is the area under its charge-voltage line



Dielectrics

A **dielectric** is an insulator that **polarizes**, building an internal field that opposes the applied one.

Sliding it in

$$C = \kappa C_0$$

the **dielectric constant** κ – and the field between the plates drops.

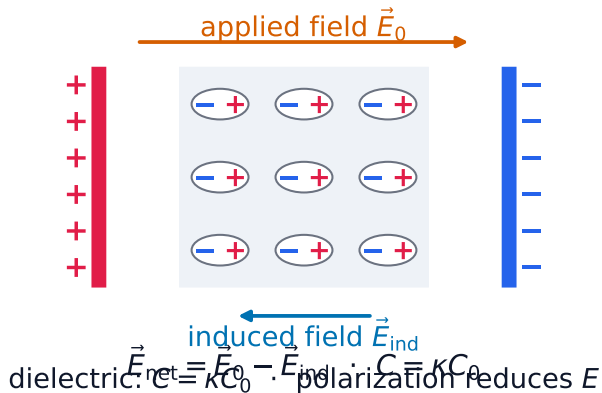
Constant charge: V and U
drop

Constant voltage: Q and U
rise

$\kappa = 3$:

$$C_0 = 5 \rightarrow 15 \mu\text{F}.$$

A polarized dielectric creates an internal field



A polarized dielectric creates an internal field opposing the applied field

Worked example – insert a dielectric with the battery removed

A 100 pF capacitor is charged to 12 V, then filled with a $\kappa = 3$ dielectric. Find the new ΔV and U .

- disconnected $\Rightarrow Q$ is trapped:
 $Q = C_0 \Delta V_0 = 1.2 \text{ nC}$
- $C = \kappa C_0 = 300 \text{ pF}$, so $\Delta V = Q/C = 4.0 \text{ V}$
- $U = \frac{Q^2}{2C}$ falls to a third: $7.2 \rightarrow 2.4 \text{ nJ}$

Write down which quantity is held fixed first – Q if the battery is removed, ΔV if it stays connected – then let $C = \kappa C_0$ drive everything else. Get that wrong and every later line is wrong.

What a dielectric is, and what it does

dielectric

an **insulating** material: its charges cannot travel, but each molecule stretches or turns in a field

the effect

the material **polarizes** and sets up its own opposing field, weakening the net field in the gap

so

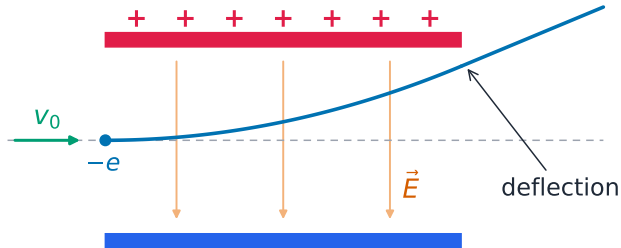
$C = \kappa C_0$, where κ is the **dielectric constant** – the ratio of the material's permittivity to the **permittivity of free space** ϵ_0

κ is always > 1

a dielectric can only *raise* capacitance, never lower it

It also lets the plates sit closer without arcing, which raises C a second time through the smaller d .

Capacitors, continued



$a_y = qE/m$ · $a_x = 0$ (same kinematics as a projectile under gravity)
 between plates: $a = qE/m$ constant · parabolic path

A charged particle between the plates follows a parabola, like a projectile

Battery connected or removed: the two columns

held fixed

battery **connected**: ΔV fixed | battery **removed**: Q fixed

charge Q

rises by κ | unchanged

voltage ΔV

unchanged | falls by κ

field E

unchanged (still $\Delta V/d$) | falls by κ

energy U

rises by κ (battery does work) | falls by κ

Both columns raise C by κ . Everything else in the two columns moves the **opposite** way – which is why naming the fixed quantity is step one.

Exam tips

- In electrostatic equilibrium a conductor has $\vec{E} = 0$ **inside and all excess charge on the surface**.
- Apply **Gauss's law** $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ with a symmetry-matched Gaussian surface (sphere, cylinder, pillbox).
- The whole conductor is one **equipotential**, and the surface field is perpendicular to it.
- For a capacitor use $C = \frac{Q}{V}$, energy $U = \frac{1}{2} CV^2$, and how a **dielectric** raises C .
- Pick the Gaussian surface so $\vec{E}$ is constant and parallel (or zero) on each part.



Unit 10 – key takeaways

1

Conductors

field zero inside; charge on the surface; one equipotential

2

Redistribution

touching conductors reach a common potential

3

Capacitors

$$C = Q/V = \epsilon_0 A/d; \quad U = \frac{1}{2} CV^2$$

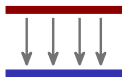
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Dielectrics

$C = \kappa C_0$; effect depends on what is held fixed

Check yourself

- 1 What is the electric field inside a conductor in equilibrium?
- 2 12 nC touched to an identical neutral sphere – charge on each?
- 3 Double a capacitor's voltage – what happens to its stored energy?
- 4 Insert a dielectric ($\kappa = 3$) – what happens to the capacitance?



Answers 1. zero 2. 6 nC each 3. it quadruples 4. it triples