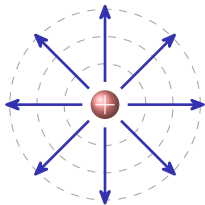


Electric Potential

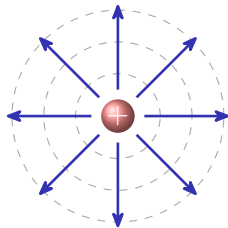
AP Physics C: E&M ■ Unit 9

AP Physics C: E&M



What we will cover

- 1 Electric potential energy
- 2 Electric potential (voltage)
- 3 Potential and field
- 4 Conservation of electric energy





Electric potential energy

Electric potential energy is the work to assemble charges from infinity:

A pair, and a system

$$U = k \frac{q_1 q_2}{r}, \quad U_{\text{tot}} = k \sum_{\text{pairs}} \frac{q_i q_j}{r_{ij}}$$

Like charges: $U > 0$ (fly apart). Unlike: $U < 0$ (a well). Note r , not r^2 ; $W = -\Delta U$.



Power lines

The electric force is conservative

conservative force

the work it does depends only on the **start and end points**, never the path taken

so

$W_{\text{elec}} = -\Delta U_E$ – a potential energy exists at all only because of this

a group of charges

add the **superposition** of every **pair**, counting each pair once: $U_{\text{tot}} = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$

signs matter

U is a **scalar** with a sign – carry the sign of each q into the sum, do not take magnitudes

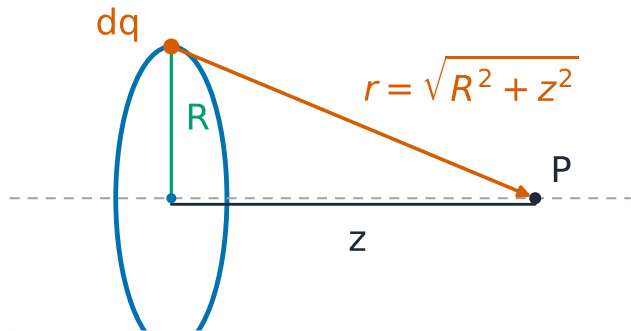
Three charges give **three** pairs, four give six. Count the pairs before you write the sum and you will not lose a term.

Continuous charge distributions

For a **continuous charge distribution**, cut it into small pieces dq , treat each piece as a point charge, and **integrate** :

AP expects you to handle four shapes with calculus: a thin ring (at a point on its axis), an arc (at its centre), a finite line or wire (collinear, or on its perpendicular bisector).

Continuous charge distributions, in detail



$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + z^2}} \quad (\text{all } dq \text{ share the same } r)$$

A ring of charge: every element dq is the same distance from an axis point

Potential is easier than field, and here is why

V is a scalar

no components, no cancellation – just add $\frac{k dq}{r}$ over the distribution

a ring or arc

every element sits the **same** distance r from the centre, so $V = \frac{kQ}{R}$ whatever the arc's shape

say it

“ r is the same for every element” is the **marked step** on the FRQ – write the sentence, not just the answer

potential difference

$\Delta V = \Delta U_E / q$: the energy change per unit charge moved. A battery is a device that maintains one

When a question asks for a *field*, you must handle direction. When it asks for a *potential*, you never do – so reach for V whenever the question lets you.



Electric potential (voltage)

Electric potential is energy per unit charge – a **scalar**:

Point charge, sum, integral

$$V = \frac{U}{q} = k \frac{q}{r}$$

$$V = k \sum_i \frac{q_i}{r_i} = k \int \frac{dq}{r}$$

Adding potentials is **easy** – just add numbers with signs, no vectors.

Worked example

+3 nC and -3 nC, each 2 m away:

$$V = k \left(\frac{+3}{2} + \frac{-3}{2} \right) \times 10^{-9} = 0$$

Yet $\vec{E} \neq 0$ there – V and $\vec{E}$ tell different stories.

Potential and field

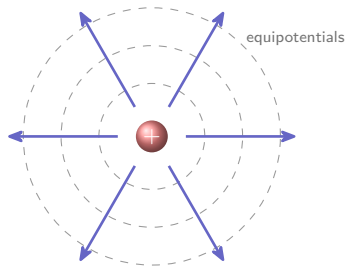
Field and potential are linked by calculus:

Integrate, or take the gradient

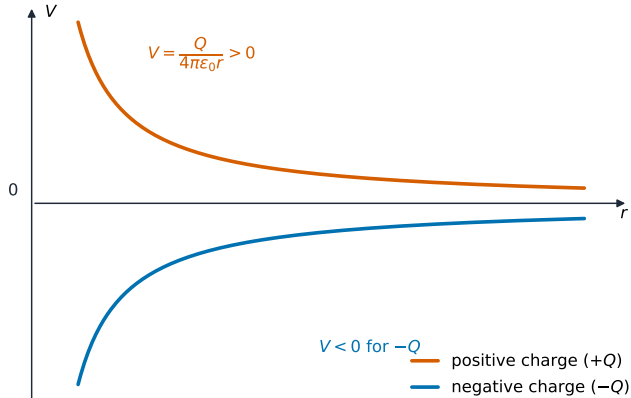
$$V = - \int \vec{E} \cdot d\vec{l}, \quad \vec{E} = -\nabla V$$

In 1-D, $E_x = -\frac{dV}{dx}$: the field points **downhill** in V .

Equipotentials are **always** perpendicular to field lines; no work moving along one.

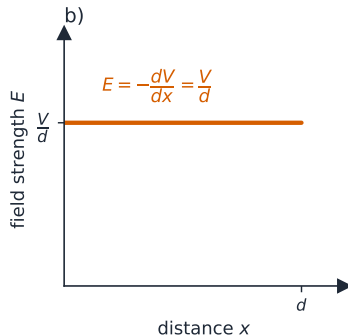
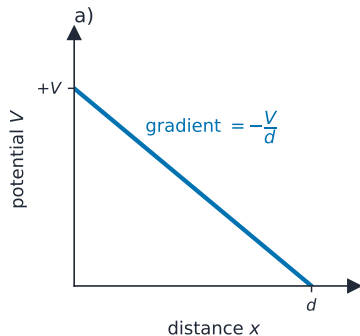


The potential near a point charge varies as – one



The potential near a point charge varies as $1/r$

The potential near a point charge varies as – two

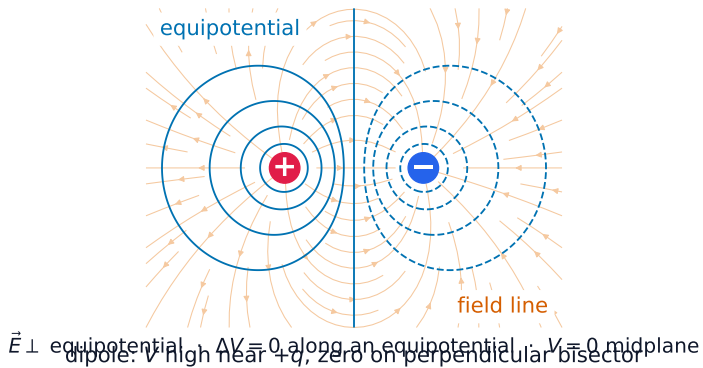


In a uniform field the potential falls steadily with distance

Equipotential maps

- Isolines are **perpendicular** to field lines everywhere they cross.
- $\vec{E}$ points from high V to low V – never along an isoline. Moving a charge along an isoline takes no work.
- Closely spaced isolines mean a strong field: $E \approx -\Delta V/\Delta x$.
- You can sketch the field map from an isoline map, and the isoline map from a field map.

Equipotential maps, in detail



Moving a charge along an isoline takes no work.

Reading the field off an equipotential map

the relation

$E = -\frac{dV}{dx}$: the field points **down** the steepest slope of V

estimating

isolines every 10 V, spaced 2 cm: $E \approx \frac{10}{0.02} = 500 \text{ V/m}$

direction

perpendicular to the isolines, pointing from **high** V to **low** V

crowding

lines close together mean a **strong** field; widely spaced means weak

An **equipotential surface** costs no work to move along, because $\Delta V = 0$ everywhere on it. That is why $\vec{E}$ must be perpendicular to it.



Conservation of electric energy

Moving a charge through a **potential difference** does work:

Work and kinetic energy

$$W = q\Delta V, \quad \Delta K = q\Delta V$$

- a + charge speeds up moving to lower V
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ (an energy)

Worked example

An electron through 200 V:

$$\Delta K = 200 \text{ eV} = 3.2 \times 10^{-17} \text{ J}$$

$$v = \sqrt{\frac{2\Delta K}{m}} \approx 8.4 \times 10^6 \text{ m/s}$$

Which sign speeds up which charge

positive charge

speeds up moving to **lower** potential – it falls down the potential hill

negative charge

speeds up moving to **higher** potential – it falls *up* the hill

why

$\Delta U_E = q \Delta V$: flip the sign of q and you flip which direction loses energy

the check

if only the electric force acts, $\Delta K = -\Delta U_E$ – kinetic energy rises exactly as potential energy falls

Watch **both** signs together, the charge's and the potential change's. Getting one right and the other wrong gives a plausible number pointing the wrong way.

Worked example – a proton accelerated through a potential drop

A proton ($q = 1.6 \times 10^{-19} \text{ C}$, $m = 1.67 \times 10^{-27} \text{ kg}$) starts at rest and falls through 500 V. Find its final speed.

- $\Delta K = q\Delta V = (1.6 \times 10^{-19})(500) = 8.0 \times 10^{-17} \text{ J}$
- $\frac{1}{2}mv^2 = 8.0 \times 10^{-17}$
- $v = \sqrt{\frac{2(8.0 \times 10^{-17})}{1.67 \times 10^{-27}}} = 3.1 \times 10^5 \text{ m/s}$

Only the **end-point potentials** enter. Choose energy over kinematics whenever the field is non-uniform or the path curves – there is no constant a to put in a SUVAT equation.

Exam tips

- Relate field and potential by $V = - \int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$ (in 1-D, $E_x = -\frac{dV}{dx}$).
- Potential is a **scalar** – add contributions with signs, no vector components needed.
- Potential energy of a pair is $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$; use energy conservation for a charge's speed.
- Know that no work is done moving along an **equipotential**, which is perpendicular to $\vec{E}$.
- Choose the zero of potential (usually infinity) and state it.



Unit 9 – key takeaways

1

Energy

$U = kq_1q_2/r$, sum over distinct pairs; $W = -\Delta U$

2

Potential

$V = kq/r$, a scalar; $V = k \int dq/r$ for distributions

3

Field link

$V = - \int \vec{E} \cdot d\vec{l}$, $\vec{E} = -\nabla V$; equipotentials $\perp$ field

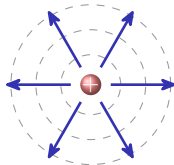
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Conservation

$\Delta K = q \Delta V$; energy in electron volts

Check yourself

- 1 Is electric potential a scalar or a vector?
- 2 How do you get the field from the potential?
- 3 How much work moves a charge along an equipotential?
- 4 An electron falls through 50 V – how much energy in eV?



Answers 1. a scalar 2. $\vec{E} = -\nabla V$ 3. zero 4. 50 eV