

On this exam the marks concentrate in the setup: the symmetry argument that justifies a law, the choice of dq , the limits of the integral. Each solution below writes the justification first and evaluates last.

Electric field and Gauss's law

I A solid insulating sphere of radius R carries a uniform charge density ρ . Find the electric field inside and outside.

Inside: $E = \rho r / 3\epsilon_0$. Outside: $E = \rho R^3 / 3\epsilon_0 r^2$.

Commentary: The distribution is spherically symmetric, so E is radial and constant in magnitude on any concentric sphere – that statement is what makes Gauss's law usable and it is a scored line. Take a Gaussian sphere of radius r . Inside ($r < R$) the enclosed charge is $\rho(4/3)\pi r^3$, so $E(4\pi r^2) = \rho(4/3)\pi r^3 / \epsilon_0$ and $E = \rho r / 3\epsilon_0$, growing linearly with r . Outside ($r > R$) the whole charge $\rho(4/3)\pi R^3$ is enclosed, giving $E = \rho R^3 / 3\epsilon_0 r^2$, which falls as $1/r^2$. Check continuity at $r = R$: both expressions give $\rho R / 3\epsilon_0$, so the field is continuous – always perform that check.

I A thin rod of length L carries uniform linear charge density λ . Find the potential at a point on the axis, a distance d from the near end.

$V = (\lambda / 4\pi\epsilon_0) \ln((d + L)/d)$.

Commentary: Potential is a scalar, so no components are needed – say so, since choosing potential over field is the decision being tested. Take an element dx at distance x from the point, carrying $dq = \lambda dx$. Its contribution is $dV = \lambda dx / (4\pi\epsilon_0 x)$. Integrate from $x = d$ to $x = d + L$: $V = (\lambda / 4\pi\epsilon_0) [\ln x]$ from d to $d+L = (\lambda / 4\pi\epsilon_0) \ln((d+L)/d)$. Check the far limit: for d much greater than L the logarithm approaches L/d , giving $V \approx \lambda L / 4\pi\epsilon_0 d$ – the potential of a point charge λL , as it must be.

Capacitance and circuits

I A parallel-plate capacitor with plate area A and separation d is charged to voltage V and disconnected. A dielectric of constant κ is then inserted. Find the new voltage and the change in stored energy.

$V' = V/\kappa$; the energy falls to U/κ .

Commentary: Disconnected means the CHARGE is fixed – identify that first, because the whole answer depends on it. The capacitance rises to κC , and since $Q = CV$ is fixed, $V' = Q/\kappa C = V/\kappa$. The energy $U = Q^2/2C$ becomes $Q^2/2\kappa C = U/\kappa$, so it falls. The energy went into pulling the dielectric in: the field polarises it and the resulting attraction does work on it. If instead the battery had stayed connected, V would be fixed and the energy would RISE to κU – the exam asks for both cases and the distinction is the point.

I In an RC circuit with $\mathcal{E} = 12$ V, $R = 2.0$ k Ω and $C = 100$ μ F, initially uncharged, find the current at $t = 0$, the final charge, and the time to reach half the final charge.

6.0 mA; 1.2 mC; 0.139 s.

Commentary: At $t = 0$ an uncharged capacitor has no voltage across it, so it behaves as a plain wire and the full e.m.f. is across the resistor: $I = 12/2000 = 6.0 \text{ mA}$. After a long time no current flows, so the full e.m.f. is across the capacitor: $Q = CV = 100 \times 10^{-6} \times 12 = 1.2 \text{ mC}$. For the half-charge time, $q = Q(1 - e^{-t/RC})$ with $q = Q/2$ gives $e^{-t/RC} = 1/2$, so $t = RC \ln 2 = 0.20 \times 0.693 = 0.139 \text{ s}$. Reasoning from the two limiting behaviours answers the first two parts with no exponential at all.

Magnetism and induction

I A long solenoid of n turns per metre carries current I . Use Ampère's law to find the field inside.

$B = \mu_0 n I$, uniform and parallel to the axis.

Commentary: Justify the loop before drawing it: the field outside an ideal solenoid is negligible and inside it is parallel to the axis, so choose a rectangular loop with one side of length ℓ inside along the axis and the opposite side outside. Only the inside side contributes to the line integral, giving $B\ell$. The enclosed current is $n\ell I$. Ampère's law gives $B\ell = \mu_0 n\ell I$, so $B = \mu_0 n I$ – independent of position within the solenoid and of its radius. Naming which sides contribute nothing, and why, is where the credit sits.

I A conducting bar of length L slides at constant speed v on frictionless rails in a uniform field B perpendicular to the plane, with a resistor R closing the circuit. Find the current, the force needed to maintain v , and the power dissipated.

$I = BLv/R$; $F = B^2 L^2 v/R$; $P = B^2 L^2 v^2/R$.

Commentary: The flux through the circuit changes because the area changes, so $\varepsilon = d\Phi/dt = BLv$, and the current is $I = BLv/R$. That current in the field feels a force $F = BIL = B^2 L^2 v/R$, directed to OPPOSE the motion by Lenz's law, so an equal applied force is needed to keep v constant. The mechanical power delivered is $Fv = B^2 L^2 v^2/R$, and the electrical power dissipated is $I^2 R = (BLv/R)^2 R = B^2 L^2 v^2/R$. They agree – that energy balance is a scored consistency argument, not just a check.