

Exercise walk-through

8.4

Electric Fields of Charge Distributions

AP Physics C: Electricity and Magnetism

Fields of charge distributions

Split a spread-out charge into pieces dq and **integrate**:

Continuous superposition

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

Densities

$$: \lambda = \frac{dq}{d\ell}, \sigma = \frac{dq}{dA}, \rho = \frac{dq}{dV}.$$

Use **symmetry**

to cancel components.

Two standard results

$$\text{ring (axis)} : E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$\text{infinite line} : E = \frac{2k\lambda}{r}$$

Point $\sim 1/r^2$, line $\sim 1/r$, sheet $\sim$ uniform.

Choose the density that matches the geometry

linear λ

$dq = \lambda dl$ – a rod, a ring, an arc

surface σ

$dq = \sigma dA$ – a disc, a sheet, a shell

volume ρ

$dq = \rho dV$ – a solid sphere or cylinder

then integrate

$\vec{E} = \int \frac{k dq}{r^2} \hat{r}$, and use **symmetry**
that cancel

to kill the components

Picking λ , σ or ρ correctly is the **first marked step** of every distribution FRQ. Write dq before you write the integral.

Charge densities, and the symmetry to match

Linear charge density **Linear charge density** λ in C/m – for a rod or a wire.

Surface charge density **Surface charge density** σ in C/m² – for a sheet or a conductor's surface.

Volume charge density **Volume charge density** ρ in C/m³ – for a solid insulating ball.

Match the symmetry Gauss's law is only useful with symmetry: a Gaussian cylinder for **cylindrical symmetry**, a sphere for **spherical symmetry**, a pillbox for **planar symmetry**.

Choose the surface so that $\vec{E}$ is constant and either parallel or perpendicular to it everywhere. Then the integral becomes multiplication.

Exercises 2.1-2.3

2.1 Write the charge dq on a length dl of a rod with linear charge density λ . [1]

2.2 State the SI units of surface charge density σ . [1]

2.3 State what the field of any finite charge distribution looks like from very far away. [1]

Exercises 2.1-2.3

2.1 Write the charge dq on a length dl of a rod with linear charge density λ . [1]

2.1

- $dq = \lambda dl$
-

2.2 State the SI units of surface charge density σ . [1]

2.2

- C m^{-2}
-

2.3 State what the field of any finite charge distribution looks like from very far away. [1]

2.3

- it approaches the field of a point charge

Exercises 3.1-3.2

3.1 The linear charge density λ is defined as

[1]

☐

A charge per unit volume

B charge per unit area

C charge per unit length

D charge $\times$ length

3.2 To find the field of a continuous distribution you

[1]

☐

A add two point charges

B integrate $d\vec{E}$ over all the charge elements dq

C use $V = IR$

D ignore symmetry

Exercises 3.1-3.2

3.1 The linear charge density λ is defined as

[1]

- ☐ **A** charge per unit volume
B charge per unit area
C charge per unit length
D charge $\times$ length

3.1 C

- charge per unit length.

3.2 To find the field of a continuous distribution you

[1]

- ☐ **A** add two point charges
B integrate $d\vec{E}$ over all the charge elements dq
C use $V = IR$
D ignore symmetry

3.2 B

- integrate $d\vec{E}$ over all the charge elements dq .

Exercise 3.3 (a)

3.3 A ring of radius a carries total charge Q . On its axis at distance x the field is

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}.$$

(a) State the field at the centre of the ring ($x = 0$).

[1]

Exercise 3.3 (a)

3.3 A ring of radius a carries total charge Q . On its axis at distance x the field is

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}.$$

(a) State the field at the centre of the ring ($x = 0$).

[1]

(a)

- $E = 0$ (the contributions cancel by symmetry)

Exercise 3.3 (b)

3.3 A ring of radius a carries total charge Q . On its axis at distance x the field is

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}.$$

(b) State what E approaches when $x \gg a$, and explain.

[2]

Exercise 3.3 (b)

3.3 A ring of radius a carries total charge Q . On its axis at distance x the field is

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}.$$

(b) State what E approaches when $x \gg a$, and explain. [2]

(b)

- $E \rightarrow \frac{kQ}{x^2}$ – far away the ring looks like a point charge Q