Course Companion

AP Physics 2

Every topic handout in one volume

全部专题讲义合集

exercises.lol

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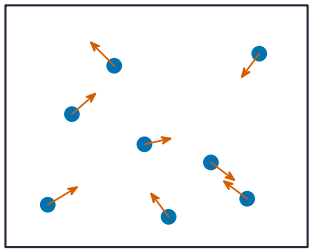
Thermodynamics

AP Physics 2

Kinetic Theory of Temperature and Pressure

Pump up a bicycle tyre and the pump barrel gets warm. Leave a can of soda in a hot car and it can burst. Both come from the same picture: a gas is a huge crowd of tiny particles, moving fast in every direction, and what we measure as pressure and temperature is the average of all their collisions.

Thermodynamics 热力学 studies heat and energy in large collections of particles. The **kinetic theory** 分子运动论 explains gas behavior from the random motion of its molecules:

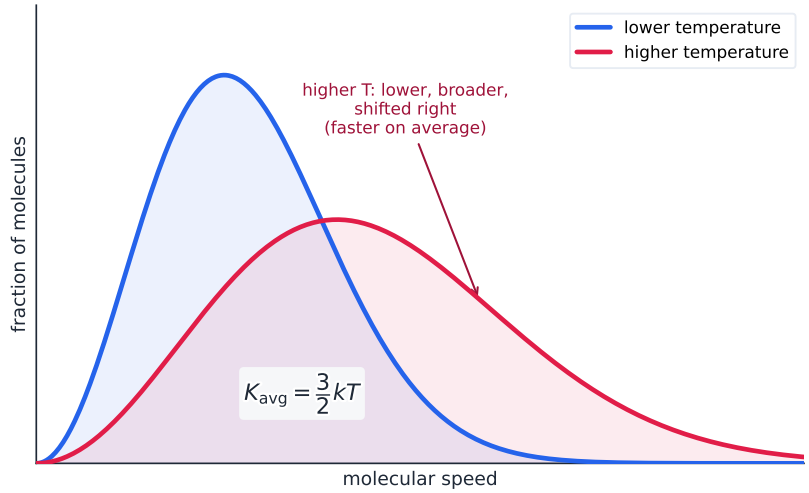


1. many identical molecules, in continuous **random motion**
2. their own volume is **tiny** next to the container
3. straight lines between hits - intermolecular forces **ignored**
4. collisions are **elastic**: no kinetic energy is lost

poor at very high pressure (volume matters) or very low temperature (forces matter)

The key assumptions of the kinetic theory of an ideal gas

- In the ideal-gas model, absolute **Temperature** 温度 is proportional to the molecules' **average translational kinetic energy**: $K_{\text{avg}} = \frac{3}{2}k_B T$ (with T in kelvin). Hotter means faster-moving molecules.
- **Pressure** 压强 comes from molecules colliding with the container walls – more frequent or harder collisions give more pressure.



The Maxwell-Boltzmann speed distribution shifts right and flattens at higher temperature

Worked example. Find the average kinetic energy of a gas molecule at 300 K, using $k_B = 1.38 \times 10^{-23}$ J/K:

$$K_{\text{avg}} = \tfrac{3}{2}k_BT = 1.5 \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21} \text{ J}.$$

It depends only on temperature, not on the type of gas – at the same T, light and heavy molecules share the same average kinetic energy (so the light ones move faster).

Pressure from collisions. When a molecule of mass m moving at speed v hits a wall head-on and bounces straight back, its momentum changes by $2mv$. By the impulse–momentum theorem, the wall gives it that impulse, and by Newton's third law the molecule gives the wall an equal and opposite push. Pressure is the **sum** of the perpendicular parts of all these pushes, per unit area of wall. More molecules, faster molecules, or a smaller box (so each molecule hits the walls more often) all mean more pressure. Pressure exists **throughout** the gas, not only at the walls: a small surface placed anywhere inside would be hit in the same way.

Root-mean-square speed. The speed that goes with the average kinetic energy is the **root-mean-square speed** 方均根速率:

$$\tfrac{1}{2}mv_{\text{rms}}^2 = \tfrac{3}{2}k_BT \;\Rightarrow\; v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}.$$

For nitrogen molecules ($m = 4.65\times10^{-26}$ kg) at 300 K, $v_{\text{rms}} = \sqrt{\dfrac{3(1.38 \times 10^{-23})(300)}{4.65 \times 10^{-26}}} =$

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517 m/s. Doubling the **kelvin** temperature multiplies v_{rms} by $\sqrt{2}$, not by 2.

Reading the Maxwell–Boltzmann graph. Not all molecules move at the same speed: the graph shows how many move at each speed. When the gas is heated, the peak moves to **higher** speed and becomes **lower and wider**. The area under the curve (the number of molecules) stays the same.

The Ideal Gas Law



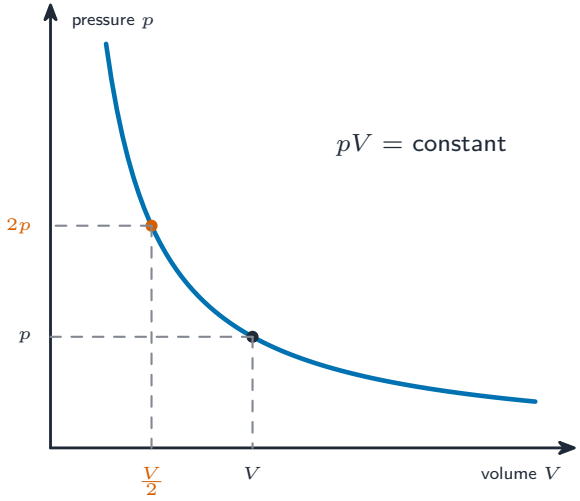
An iceberg: heat, density and phase changes connect thermodynamics to fluids and buoyancy

An **ideal gas** 理想气体 obeys

$$PV = nRT \qquad (\text{or } PV = Nk_BT),$$

linking pressure P, volume V, amount (n moles or N molecules), and absolute temperature T. Use it to predict how a gas responds when you change one quantity and hold others fixed (e.g. heating at constant volume raises pressure).

The ideal-gas model assumes four things: the molecules move in random directions; their own volume is negligible compared with the container; their collisions are elastic; and the only forces between them act during collisions. Real gases follow it well at low pressure and high temperature, when the molecules are far apart.



Boyle's law: at constant temperature, pressure times volume is constant

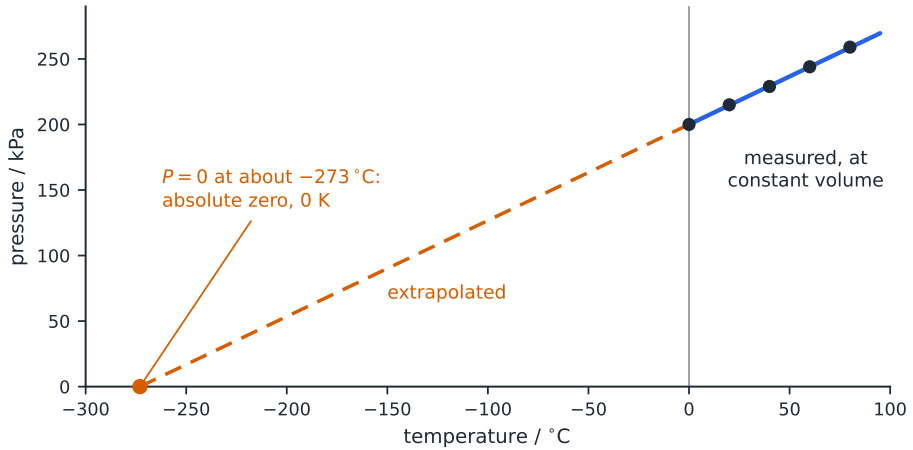
Worked example. A sealed rigid container of gas is at 1.0×10^5 Pa and 300 K. It is heated to 450 K. Because the volume and amount are fixed, P/T is constant:

$$P_2 = P_1 \frac{T_2}{T_1} = 1.0 \times 10^5 \times \frac{450}{300} = 1.5 \times 10^5 \text{ Pa}.$$

Always convert temperatures to **kelvin** before using a gas law – using Celsius here would give nonsense.

Finding absolute zero from a graph. Measure the pressure of a fixed amount of gas at **constant volume** at several temperatures, using an ice–water bath, room-temperature water and hot water. Only trials at the same volume belong on the graph. The points lie on a straight line; extend it back to $P = 0$.

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Extending the constant-volume line to zero pressure finds absolute zero, about −273 °C

The line reaches zero pressure at about −273 °C, whatever gas is used. That temperature is **absolute zero** 绝对零度, 0 K. So pressure and temperature are **directly** proportional at fixed volume and amount, $P \propto T$, measured in kelvin. "Cold air at the North Pole is at high pressure, so P and T are inversely related" is wrong: the air there sinks, so more air is piled above that spot, and the number of molecules per volume is not fixed.

A gas that may not be ideal. For an ideal gas at constant temperature, $\rho = \frac{m}{V} = \frac{PM}{RT}$, where M is the molar mass: density is **proportional** to pressure, a straight line through the origin. If a measured density–pressure graph **curves**, the gas does not behave ideally over that range.



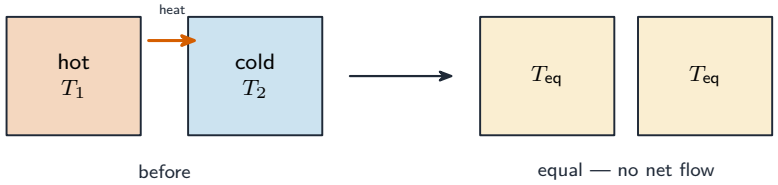
Heating the gas inside a balloon lowers its density, so the balloon rises – the ideal gas law in action

Thermal Energy Transfer and Equilibrium

Heat 热量 Q is energy transferred because of a temperature difference; it flows from hot to cold. Two objects in contact reach **thermal equilibrium** 热平衡 when they share the same temperature, and net heat flow stops. The three transfer methods are **conduction** 传导, **convection** 对流, and **radiation** 辐射.

Two systems are in **thermal contact** when energy can pass between them by these processes. **Heating** means energy entering a system this way; **cooling** means energy leaving it.

Why hot to cold? When fast particles of a hot object collide with slow particles of a cold one, energy is **most likely** to pass from the faster particle to the slower one. After many collisions, the most probable state, by far, is one where both objects have the same average kinetic energy: the same temperature. Then there is no **net** energy transfer, which is what thermal equilibrium means. Individual collisions still happen, but they transfer energy equally in both directions.



Heat flows from hot to cold until both share one temperature; then the net flow stops

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> **Worked example (which way does energy flow?).** Two samples of the same gas, with the same number of particles, are put in thermal contact. Sample 2 has $PV = 400$ J and sample 3 has $PV = 100$ J. Since $T = \dfrac{PV}{Nk_B}$ and N is the same, sample 2 is at the higher temperature, so energy flows from sample 2 to sample 3 until their temperatures are equal.

The First Law of Thermodynamics

The **first law** 热力学第一定律 is conservation of energy for a gas:

$$\Delta U = Q + W,$$

where ΔU is the change in **internal energy** 内能 (depending only on temperature for an ideal gas), Q is heat added **to** the gas, and W is work done **on** the gas. On a pressure–volume diagram, the work done **by** the gas is the area under the process curve. Watch signs: compressing a gas does positive work on it.

Internal energy is the total kinetic energy of the particles plus the potential energy of their arrangement. In an ideal gas, the particles do not attract each other, so there is **no** potential energy: the internal energy is just the kinetic energy of the molecules. For a monatomic ideal gas,

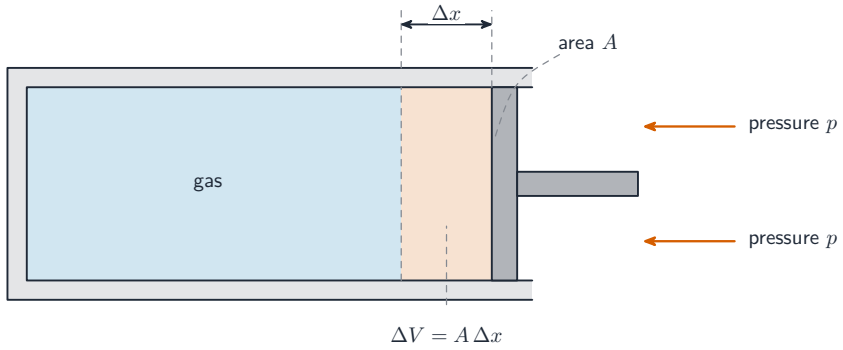
$$U = \tfrac{3}{2}nRT = \tfrac{3}{2}PV.$$

So U depends only on temperature. A system can gain internal energy without its center of mass moving at all: a hot brick on a table is not going anywhere.

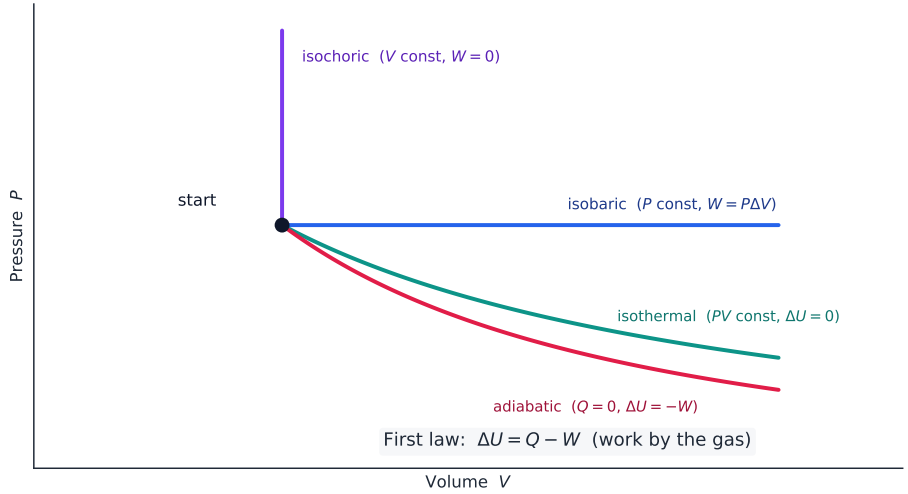
The work done **on** a gas by a constant (or average) outside pressure is

$$W = -P\,\Delta V.$$

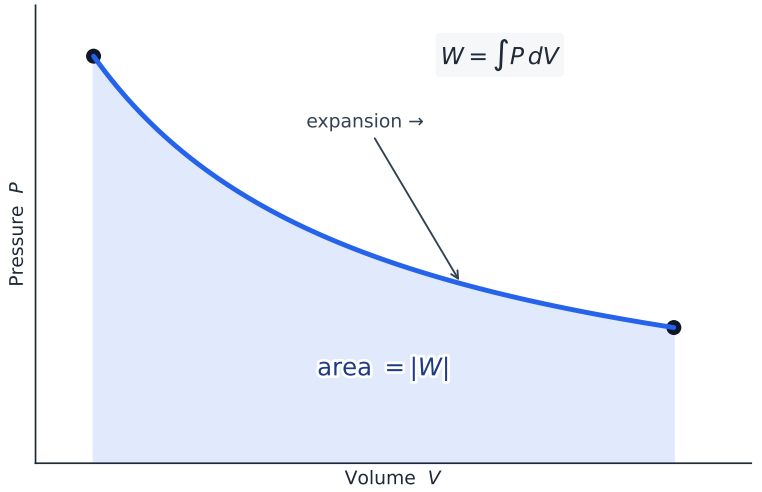
When the gas expands ($\Delta V > 0$), W is negative: the gas does work on its surroundings. An **isolated** system exchanges no energy with its surroundings, so its total energy is constant. A **closed** system exchanges energy but not matter, and the first law, $\Delta U = Q + W$, tracks that exchange.



A gas pushing a piston out by a small distance does work equal to p times the volume change



The four thermodynamic processes drawn from a common starting state on a PV diagram



The work done during a volume change equals the area under the P-V curve

The four special processes.

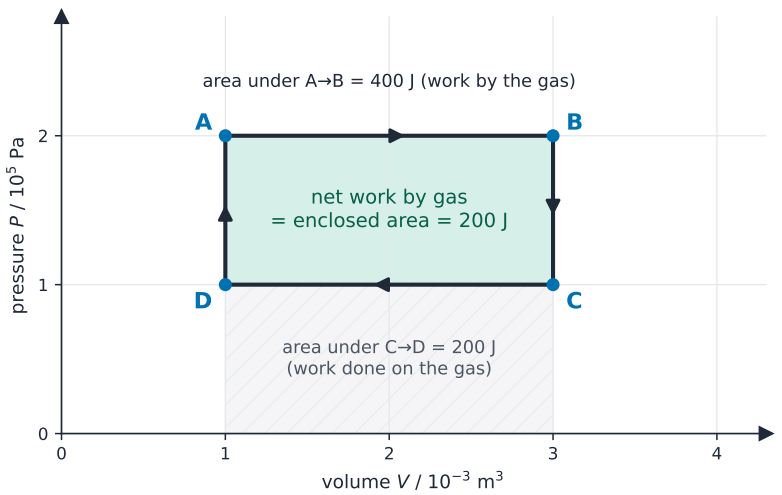
Process	What is fixed	Shape on a PV diagram	First law becomes
isovolumetric 等容	volume	vertical line	$W = 0$, so $\Delta U = Q$
isobaric 等压	pressure	horizontal line	$W = -P\Delta V$
isothermal 等温	temperature	curve $PV =$ constant (an isotherm)	$\Delta U = 0$, so $Q = -W$
adiabatic 绝热	no heat flow, $Q = 0$	steeper curve than an isotherm	$\Delta U = W$

Lines of constant temperature on a PV diagram are called **isotherms** 等温线; isotherms farther from the origin are hotter.

> **Worked example.** A gas absorbs 500 J of heat and, as it expands, does 200 J of work **on its surroundings**. Find the change in its internal energy. Work done *on* the gas is $W = -200$ J (it does work, so it loses that energy):
>
> $$\Delta U = Q + W = 500 + (-200) = 300 \text{ J}.$$
>
> The internal energy rises by 300 J, so the gas ends up hotter. Getting the sign of W right is the whole game in first-law problems.

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Worked example (a cycle on a PV diagram). A sample of monatomic ideal gas goes round the cycle A → B → C → D → A shown below.



Going clockwise round a cycle, the gas does more work expanding than is done on it during compression; the difference is the enclosed area

- **A → B** (constant pressure, expanding): work done *by* the gas $= P\Delta V = (2.0 \times 10^5)(2.0 \times 10^{-3}) = 400$ J, so $W = -400$ J.
- **B → C** and **D → A** (constant volume): no area under the line, so $W = 0$.
- **C → D** (constant pressure, compressed): $W = +(1.0 \times 10^5)(2.0 \times 10^{-3}) = +200$ J done on the gas.
- **Whole cycle:** $W_{\text{net}} = -400 + 200 = -200$ J, so the gas does a net 200 J of work, the enclosed area. It returns to A, so $\Delta U = 0$ and $Q_{\text{net}} = \Delta U - W = +200$ J: a clockwise cycle takes in heat and gives out work, like an engine.
- **Temperatures without numbers for n:** $T \propto PV$. In joules, PV is 200 at A, 600 at B, 300 at C and 100 at D, so B is the hottest state and D the coldest.
- **Heat in A → B:** for a monatomic gas $U = \frac{3}{2}nRT = \frac{3}{2}PV$, so $\Delta U = \frac{3}{2}(600-200) = 600$ J, and $Q = \Delta U - W = 600 - (-400) = 1000$ J.

Worked example (a cycle with a slanted side). Two moles of a monatomic ideal gas are held by a piston of area 5.0×10^{-3} m^2. State A is 0.040 m^3 at 1.0×10^5 Pa; A → B expands at constant pressure to 0.10 m^3; B → C cools at constant volume to 0.50×10^5 Pa; C → A returns along a straight line.

- *Force on the piston at A:* $F = PA = (1.0 \times 10^5)(5.0 \times 10^{-3}) = 500$ N. It comes from the momentum the molecules give the piston each time they bounce off it.

- *Temperature at B:* $T = \dfrac{PV}{nR} = \dfrac{(1.0 \times 10^5)(0.10)}{2(8.31)} = 600$ K. Temperature measures the average kinetic energy of the atoms.
- *A → B:* PV grows from 4000 to 10000 J, so the temperature and the internal energy **rise**.
- *Heat along A → B → C:* $\Delta U = \frac{3}{2}(P_C V_C - P_A V_A) = \frac{3}{2}(5000-4000) = 1500$ J. Work on the gas: $-P\Delta V = -(1.0 \times 10^5)(0.060) = -6000$ J for A → B, and zero for B → C. So $Q = \Delta U - W = 1500 + 6000 = 7500$ J.
- *C → A directly:* the total kinetic energy of the atoms is the internal energy, so it changes by -1500 J.

Worked example (a piston with a weight on it). A monatomic gas is sealed under a piston of mass M and area A, in a water bath at T_0; the gas fills V_0. Three forces act on the piston: its weight Mg and the air's push $P_{\text{atm}}A$ down, and the gas's push PA up. In equilibrium, $P = P_{\text{atm}} + \dfrac{Mg}{A}$, and $U = \frac{3}{2}PV_0 = \frac{3}{2}\left(P_{\text{atm}} + \dfrac{Mg}{A}\right)V_0$.

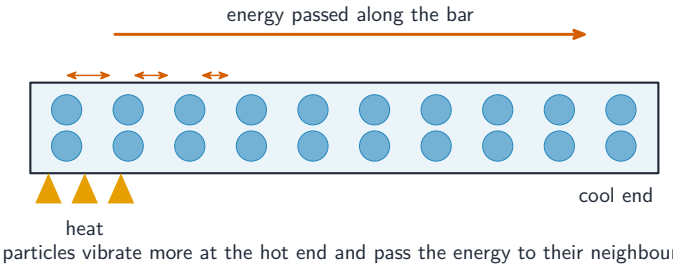
- A block is lowered slowly onto the piston. The bath keeps the temperature at T_0, so this is an **isothermal compression**: on a PV diagram the state moves up and to the left along an isotherm.
- To bring the gas back to V_0 at this **higher** pressure, PV must be larger, so the bath's new temperature must be **higher** than T_0.

Worked example (heating a rigid chamber). A gas in a sealed, rigid chamber is heated. The volume cannot change, so on a PV diagram the process is a **vertical** line going up. On a graph of U against V it is also a vertical line going up: $U = \frac{3}{2}PV$ rises because P rises at fixed V. If the pressure goes from P to $3P$, then $\Delta T = \dfrac{\Delta(PV)}{nR} = \dfrac{2PV}{nR}$.

Specific Heat and Thermal Conductivity

- **Specific heat** 比热容 c is the heat needed to raise one kilogram by one degree: $Q = mc\,\Delta T$. A high specific heat (like water's) means a substance resists temperature change.
- **Thermal conductivity** 热导率 measures how fast heat conducts through a material; the rate of conduction rises with area and temperature difference and falls with thickness.

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Conduction: vibrating particles pass energy along a metal bar

Worked example. How much heat raises the temperature of 2.0 kg of water from 20 °C to 80 °C? Water's specific heat is $c = 4200$ J/(kg °C):

$$Q = mc\,\Delta T = 2.0 \times 4200 \times (80 - 20) = 5.0 \times 10^5 \text{ J}.$$

Water's large specific heat is why it is used as a coolant and why coastal climates are mild.

Specific heat and thermal conductivity are both **intrinsic** properties: they belong to the material and depend on how its atoms are arranged and how they interact, not on the size of the sample.

Worked example (same heat, different temperature changes). A hot metal sphere is dropped into a larger mass of cool liquid in an insulated cup; the sphere has the smaller specific heat. The energy the sphere loses equals the energy the liquid gains, $m_S c_S|\Delta T_S| = m_L c_L|\Delta T_L|$. The sphere has both the smaller mass and the smaller specific heat, so a given amount of energy changes its temperature more: $|\Delta T_S| > |\Delta T_L|$.

Rate of conduction. Energy flows through a slab of material at a rate

$$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L},$$

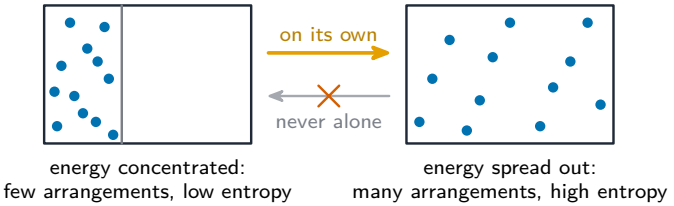
where k is the thermal conductivity, A the area, ΔT the temperature difference across the slab and L its thickness. A thicker slab lets less energy through.

Measuring k (a lab). Boiling water keeps the bottom of a plastic slab at 100 °C, and melting ice keeps its top at 0 °C. The rate of energy flow is found from how fast the ice melts. Repeat with stacks of 1 to 5 slabs. Since $\frac{Q}{\Delta t} = kA\Delta T \cdot \frac{1}{L}$, plot $\frac{Q}{\Delta t}$ against $\frac{1}{L}$: a straight line with slope $kA\Delta T$. With $A = 0.025$ m^2 and a slope of 1.0 J m/s, $k = \dfrac{1.0}{(0.025)(100)} = 0.40$ W/(m K). One source of error: some energy also flows from the air into the ice, so more ice melts than the slab alone explains, and k comes out **too large**.

At the microscopic level, particles at the hot face vibrate faster. They collide with their slower neighbours and pass energy along, layer by layer, toward the cold face. This also explains why plastic at room temperature can feel **cooler** to the touch than wood at the same temperature: if the plastic conducts better, it carries energy away from your hand faster, and your skin senses that rate, not the material's temperature.

Entropy and the Second Law of Thermodynamics

Entropy 熵 measures the disorder, or the number of ways to arrange, a system. The **second law** 热力学第二定律: the total entropy of an isolated system never decreases – energy naturally spreads out. This sets the **direction** of processes: heat flows hot→cold on its own, never the reverse, and no cyclic heat engine can convert heat taken from a single reservoir entirely into work without another effect.



Schematic: concentrated energy spreads out on its own because the spread-out state has far more arrangements; it never gathers back by itself

Entropy can be pictured as the **spreading out** of energy: energy that is concentrated in one place tends to spread, and spread-out energy is less available to do work. Four points make the second law precise.

- Entropy is a **state function**: its value depends only on the current state of the system, not on how the system got there.
- An isolated system moves on its own toward **thermodynamic equilibrium**, where its entropy is the **largest** it can be.
- The total entropy of an isolated system stays constant only if every process is **reversible**; real processes always increase it.
- The entropy of a **closed** (not isolated) system **can** decrease, as when a fridge cools its contents. Energy leaves the system, and the entropy of the surroundings rises by more.

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Practice

Answer each before reading the answers below. Take $R = 8.31$ J/(mol K).

1. A gas at 2.0×10^5 Pa fills 3.0×10^{-3} m^3 at 300 K. How many moles of gas are there?
2. The gas in question 1 is heated at constant pressure to 450 K. Find its new volume and the work done **by** the gas.
3. A gas is compressed: 150 J of work is done on it while it gives out 60 J of heat. Find the change in its internal energy. Does its temperature rise or fall?
4. 0.50 kg of water at 80 °C is mixed with 1.0 kg of water at 20 °C in an insulated flask. Find the final temperature.
5. A student says: "Hydrogen and oxygen in the same room are at the same temperature, so their molecules move at the same average speed." Correct the student.

Answers

1. $n = \dfrac{PV}{RT} = \dfrac{(2.0\times10^5)(3.0\times10^{-3})}{(8.31)(300)} = \dfrac{600}{2493} = 0.24$ mol.
2. At constant P, $V \propto T$: $V_2 = 3.0\times10^{-3} \times \dfrac{450}{300} = 4.5\times10^{-3}$ m^3. Work by the gas $= P\Delta V = (2.0\times10^5)(1.5\times10^{-3}) = 300$ J, so $W = -300$ J in $\Delta U = Q + W$.
3. $Q = -60$ J and $W = +150$ J, so $\Delta U = -60 + 150 = +90$ J. The internal energy rises, so the temperature rises.
4. Heat lost by the hot water = heat gained by the cold water: $0.50\,c\,(80 - T) = 1.0\,c\,(T - 20)$. Then $40 - 0.50T = T - 20$, so $1.5T = 60$ and $T = 40$ °C. The c cancels; the final temperature sits nearer the larger mass's start.
5. The same temperature means the same **average kinetic energy**, $\frac{3}{2}k_BT$, not the same speed. A hydrogen molecule has $\frac{1}{16}$ the mass of an oxygen molecule, so v^2 must be 16 times larger: hydrogen molecules move about 4 times faster on average.

Exam tips

- Always convert temperatures to **kelvin** ($T_\text{K} = T_\text{C} + 273$) before using any gas law.
- In the first law $\Delta U = Q + W$, get the **sign of** W right: work done **on** the gas is positive; when the gas expands and does work on its surroundings, W is negative.
- **Temperature** measures the average kinetic energy of particles, so at the same T lighter molecules move faster.
- On a **PV diagram** the work done by the gas is the **area** under the process curve.
- Use $Q = mc\,\Delta T$ for temperature change; the **second law** fixes the direction – heat flows hot→cold and total entropy never decreases.

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Common mistakes

- Using degrees Celsius in $PV = nRT$ or $K_\text{avg} = \frac{3}{2}k_BT$. Temperatures must be in kelvin.
- Getting the sign of W wrong in $\Delta U = Q + W$. In this course W is work done *on* the gas: positive when the gas is compressed, negative when it expands.
- Saying a gas expanding at constant temperature has no heat flow. $\Delta U = 0$, so $Q = -W$: the gas absorbs exactly as much heat as the work it does.
- Reading work from a P–V graph as $P \times V$ at one point. The work is the *area under* the process curve; along a closed cycle it is the enclosed area.
- Confusing temperature with heat. Temperature measures the average kinetic energy of the molecules; heat is energy transferred because of a temperature difference.
- Mixing trials at different volumes on a pressure–temperature graph. Hold the volume (and amount of gas) fixed, or the points will not lie on one line.
- Saying an ideal gas has potential energy between its molecules. In the model the molecules do not interact except in collisions, so U is all kinetic.
- Saying that at thermal equilibrium the molecules stop exchanging energy. Collisions go on; the **net** transfer is zero.
- Thinking a material that feels colder is at a lower temperature. It may just conduct energy away from your hand faster.
- Saying entropy can never decrease anywhere. That is true for an isolated system; a closed system's entropy can fall if energy leaves it.

Electric Force, Field, and Potential

AP Physics 2

Electric Charge and Electric Force

Pull off a wool jumper in a dry room and you hear crackles, and your hair stands up. The same invisible push holds every atom together, and it is far stronger than gravity: this unit is about that force, and the fields and energies that come with it.

Electric charge 电荷 is a **fundamental property** 基本属性 of matter, and it comes in two kinds, positive and negative; **like charges** 同种电荷 **repel** 排斥 and **opposite charges** 异种电荷 **attract** 吸引. Charge is **conserved** 守恒 and **quantized** 量子化 (a multiple of the **elementary charge** 元电荷 e). The force between two **point charges** 点电荷 is **Coulomb's law** 库仑定律:

$$F = \frac{k|q_1 q_2|}{r^2},$$

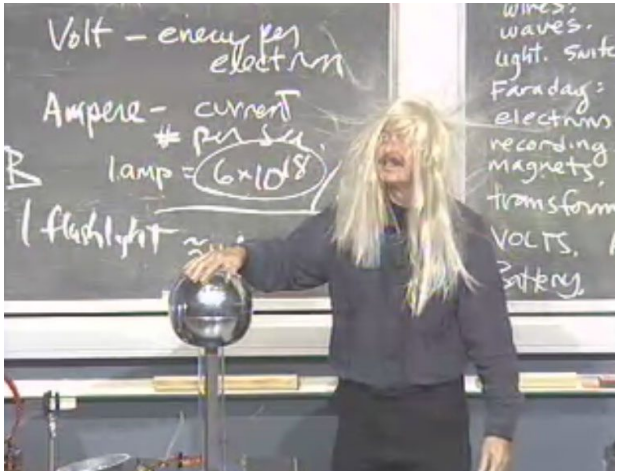
directed along the line joining them – an **inverse-square law** 平方反比定律 like gravity, but it can push or pull.

The constant k hides a property of the medium: the **permittivity** 介电常数. Free space has a fixed **permittivity of free space** 真空介电常数 ε_0 (with $k = 1/4\pi\varepsilon_0$), while the permittivity of **matter** differs from ε_0, depending on the material's composition and arrangement - which is why a material between two charges changes the force. Note also that although the electric force is vastly **stronger** than gravity, **gravity dominates at large scales**, because large objects are usually electrically **neutral** 中性 (equal + and −), leaving only gravity to act.

> **Worked example.** Two point charges, $+3.0\ \mu$C and $-2.0\ \mu$C, sit 0.10 m apart ($k = 9.0 \times 10^9$). The force between them is
>
> $$F = \frac{kq_1q_2}{r^2} = \frac{9.0 \times 10^9 \times (3.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.10)^2} = 5.4\ \text{N},$$
>
> attractive, because the charges have opposite signs. (Use the magnitudes for the size and decide the direction from the signs.)

A **point charge** is a model: we treat a charged object as a point when its size is small compared with the distances in the problem. The electron carries charge $-e$ and the proton $+e$, with $e = 1.6 \times 10^{-19}$ C, and the neutron carries none. Everyday "contact" forces, such as the normal force and friction, are really electric forces between huge numbers of particles, but there are far too many to track one by one, so we treat them as single forces.

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Like charges repel: each hair carries the same charge from the Van de Graaff generator, so they push apart

The Process of Charging

Objects charge by moving electrons. A **conductor** 导体 lets charge move freely; an **insulator** 绝缘体 holds it in place. Three methods:

- **Friction** 摩擦起电: rubbing transfers electrons.
- **Conduction** 接触起电: touching shares charge.
- **Induction** 感应起电: a nearby charge rearranges charge in a neutral object, which can then be **grounded** 接地 to leave it charged.

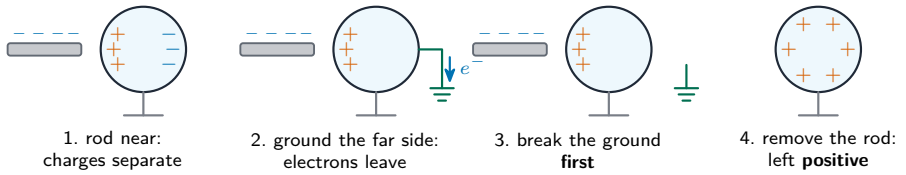
Polarization. A charged object can attract a **neutral** one. Bring a negative balloon near a wall: the balloon pushes the electrons in the wall's molecules slightly away, so the near side of the wall becomes slightly positive and the far side slightly negative. This **induced charge separation** is called **electric polarization** 极化. The positive side is closer to the balloon, so its attraction is stronger than the repulsion of the farther negative side, and the balloon sticks. How easily a material polarizes is measured by its permittivity: in a conductor, electrons move freely; in an insulator, they can only shift a little within each atom.

Charge is conserved. A system's net charge changes only if charge is transferred to or from its surroundings, almost always by moving **electrons**. Rubbing a rod with silk does not create charge: electrons move from the rod to the silk, so the rod becomes positive and the silk equally negative.

Grounding connects a charged object to something huge and nearly neutral, usually the Earth. Charge flows until the object is neutral (or, when a charged rod is held nearby, until the object has the opposite charge to the rod).

Worked example (charging by induction). A negative rod is held near a neutral metal sphere, without touching.

1. Electrons in the sphere are pushed to the far side; the near side becomes positive.
2. With the rod still there, the far side is touched to ground: the pushed electrons flow away into the Earth.
3. The ground connection is removed, **then** the rod is taken away. The sphere is left **positive**, the opposite sign to the rod, and the charge spreads over its surface.



Charging by induction: break the ground connection before removing the rod, and the sphere keeps the opposite sign to the rod

Electric Fields



Lightning: charge builds up until the electric field between cloud and ground is strong enough to tear electrons off air molecules, and a huge current flows

An **electric field** 电场 $\vec{E}$ is the force per unit charge that a small positive **test charge** 试探电荷 would feel:

$$\vec{E} = \frac{\vec{F}}{q}, \qquad E = \frac{kQ}{r^2} \text{ for a point charge.}$$

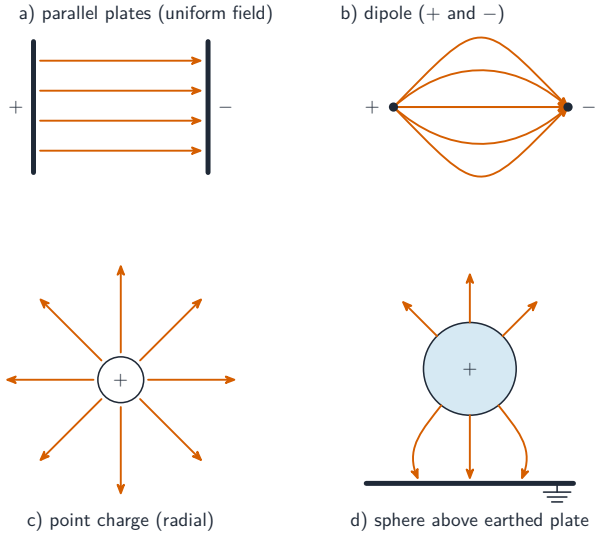
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Electric field lines 电场线 point away from positive charges and toward negative ones; where they are denser, the field is stronger. A charge in a field feels $\vec{F} = q\vec{E}$.

A **test charge** must be small enough that it does not disturb the field it is measuring. Two pictures are used. An **electric field vector map** draws an arrow at many points, its length showing the strength there. A **field line diagram** is a simpler model: one continuous line traced through the arrows. The net field at a point is the **vector sum** of the fields from every nearby charge.

Conductors and insulators in equilibrium. When charges have stopped moving:

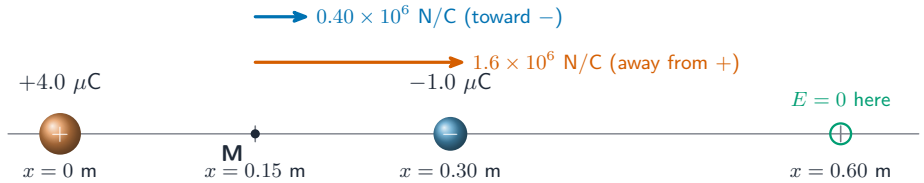
- In a **conductor**, any extra charge sits on the **outer surface**, and the field **inside** the metal is **zero**. (If it were not, free electrons would still be moving.) Just outside, the field is **perpendicular** to the surface.
- In an **insulator**, charges cannot move freely, so extra charge can stay **inside** the material as well as on its surface, and the field inside need not be zero.
- Outside any sphere with its charge spread evenly, the field is the same as for a point charge Q at its center: $E = \dfrac{kQ}{r^2}$.



Electric field-line patterns for parallel plates, a dipole, and a point charge

Worked example. Find the electric field 0.20 m from a +5.0 μC point charge:
$E = \dfrac{kQ}{r^2} = \dfrac{9.0\times10^{9}\times5.0\times10^{-6}}{(0.20)^2} = 1.1\times10^{6}$ N/C, pointing away from the charge.
A +2 nC charge placed there would feel $F = qE = 2\times10^{-9}\times1.1\times10^{6} = 2.2\times10^{-3}$ N.

Worked example (adding fields). A $+4.0\ \mu$C charge sits at $x = 0$ and a $-1.0\ \mu$C charge at $x = 0.30$ m. Find (a) the field at the midpoint M and (b) the point on the line where the field is zero.



Draw each charge's field at the point first; only then add, with signs for direction

(a) Each charge is 0.15 m from M.

$$E_+ = \frac{(9.0\times10^9)(4.0\times10^{-6})}{0.15^2} = 1.6\times10^6\ \text{N/C} \qquad E_- = \frac{(9.0\times10^9)(1.0\times10^{-6})}{0.15^2} = 0.40\times10^6\ \text{N/C}$$

The field of the positive charge points away from it ($+x$); the field of the negative charge points toward it, also $+x$. They **add**: $E = 2.0\times10^6$ N/C in the $+x$ direction.

(b) Between the charges both fields point the same way, so they cannot cancel there. To the left of $x = 0$ the larger charge is always closer, so its field always wins. The zero must be to the right of the smaller charge, at some $x > 0.30$ m, where the two fields point in opposite directions:

$$\frac{k(4.0)}{x^2} = \frac{k(1.0)}{(x-0.30)^2} \Rightarrow \frac{x}{x-0.30} = 2 \Rightarrow x = 0.60\ \text{m}$$

The zero-field point is always nearer the **smaller** charge, outside the pair when the charges have opposite signs.

Worked example (uniform field between plates). Two **parallel plates** 平行板 0.020 m apart are held at a **potential difference** 电势差 of 500 V. The field between them is **uniform** 均匀的:

$$E = \frac{\Delta V}{d} = \frac{500}{0.020} = 2.5\times10^4\ \text{V/m}$$

An electron between the plates feels $F = qE = (1.6\times10^{-19})(2.5\times10^4) = 4.0\times10^{-15}$ N, toward the positive plate, the same anywhere in the gap. (One V/m is the same as one N/C.)

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Electric Potential Energy

Two charges have **electric potential energy** 电势能 stored in their arrangement:

$$U = \frac{kq_1q_2}{r}.$$

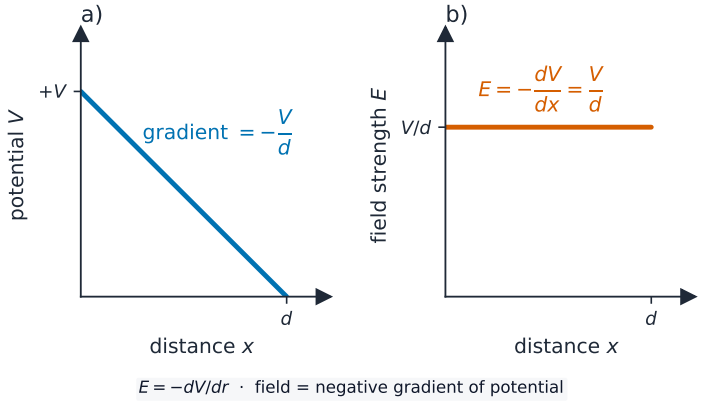
Like charges pushed together store positive energy; opposite charges have negative energy (bound). Moving a charge changes U, and the electric force does work equal to $-\Delta U$.

What U means. The electric potential energy of two point charges is the work an outside force must do to bring them from **infinitely far apart** (where $U = 0$) to their present positions, slowly. For like charges you must push them together, so U is positive; for opposite charges they pull together and you must hold them back, so U is negative. For three or more charges, add U for **every pair**: three charges have three pairs, four charges have six.

Worked example (bringing in a third charge). Particles A and B, each $+Q$, are fixed at two corners of an equilateral triangle of side d. The system's energy is $U_i = \dfrac{kQ^2}{d}$. A third particle is brought from very far away to the third corner.

- If it is $+Q$, two new pairs are added, each $+\dfrac{kQ^2}{d}$, so $U_f = \dfrac{3kQ^2}{d}$: the energy **rises**, because an outside force had to push it in against the repulsion.
- If it is $-Q$, each new pair is $-\dfrac{kQ^2}{d}$, so $U_f = \dfrac{kQ^2}{d} - \dfrac{2kQ^2}{d} = -\dfrac{kQ^2}{d}$: the energy **falls** below zero.

On a bar chart, draw the $+Q$ case as a taller final bar and the $-Q$ case as a bar below the axis.



In a uniform field the potential falls steadily with distance, so E relates to V

Electric Potential

Electric potential 电势 V is the potential energy per unit charge – a scalar field, measured in volts:

$$V = \frac{U}{q} = \frac{kQ}{r}.$$

The potential difference (**voltage** 电压) between two points is the work per unit charge to move between them: $\Delta V = \dfrac{\Delta U}{q}$, and $U = qV$. Positive charges move from high to low potential on their own. Because potential is a **scalar** 标量, adding the potentials from several charges is much easier than adding field **vectors** 矢量.

> **Worked example.** For the two charges in the field example above, the potential at the midpoint M is

$$V = \frac{k(4.0\times10^{-6})}{0.15} + \frac{k(-1.0\times10^{-6})}{0.15} = \frac{(9.0\times10^{9})(3.0\times10^{-6})}{0.15} = 1.8\times10^{5}\ \text{V}.$$

Keep the sign of each charge and add; there are no directions to worry about. The field at M is not zero, but that tells you nothing about V: field and potential are different quantities.

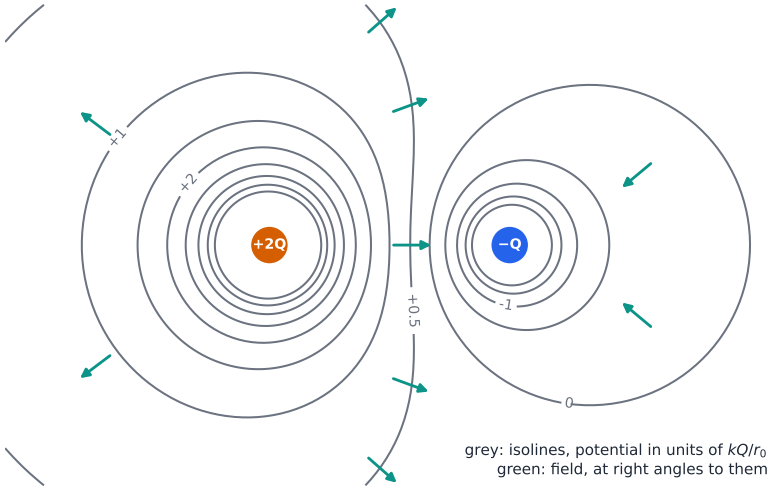
> **Worked example (field zero, or potential zero?).** Return to the triangle: $+Q$ at A and at B, and point P at the center, a distance r from each corner. What third charge at the last corner makes the **field** at P zero, and what makes the **potential** zero?

- *Field.* Three equal charges at the corners give three fields of the same size at P, 120° apart, and they cancel. So a third charge of $+Q$ makes $E = 0$. A student who says $+2Q$ is wrong: that charge's field would be twice as large, and the net field would be $\dfrac{kQ}{r^2}$, pointing away from the third corner.
- *Potential.* Potential is a scalar: $\dfrac{kQ}{r} + \dfrac{kQ}{r} + \dfrac{kq_3}{r} = 0$ needs $q_3 = -2Q$. A student who says $-Q$ is wrong: that leaves $V = \dfrac{kQ}{r}$.

The two answers differ because field adds as vectors (directions matter) and potential adds as numbers.

Equipotential lines. Points at the same potential can be joined into **equipotential lines** 等势线, also called **isolines**. They are like contour lines on a map, with potential in place of height.

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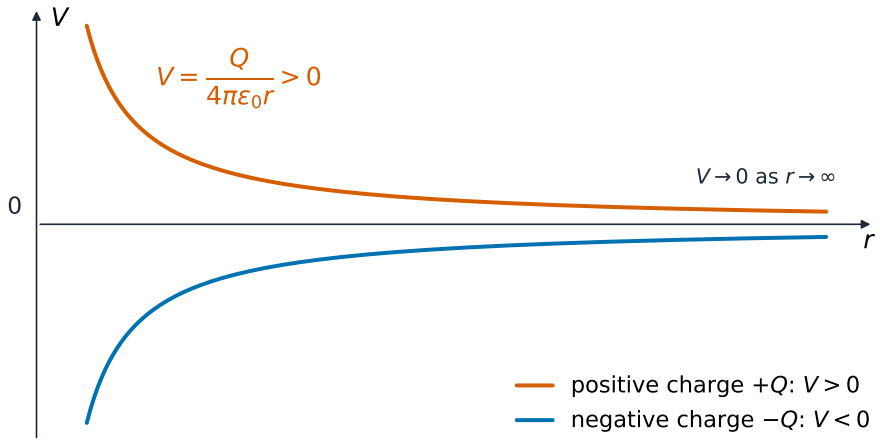
Field arrows cross the isolines at right angles and point downhill in potential; where the lines crowd, the field is strong

- The field is always **perpendicular** to the isolines, and it points from **higher** to **lower** potential. There is no field component **along** an isoline, so moving a charge along one takes no work.
- Where isolines are **close together**, the field is **strong**. Along the field direction, the average field magnitude over a distance d is $E_{\text{avg}} = \dfrac{|\Delta V|}{d}$.
- More isolines around one charge, reaching farther out, mean that charge is larger. Positive values surround a positive charge; negative values surround a negative one.

> **Worked example (a proton released between the isolines).** A proton $(+q)$ is released from rest and moves through a potential difference of 20 V, from higher to lower potential. The system's electric potential energy changes by $\Delta U = q\,\Delta V = q(-20\ \text{V})$, a **decrease**, and the proton gains the same amount of kinetic energy. If the proton moved a distance d through an average field E_{avg}, the work done on it by the field is $W = qE_{\text{avg}}d$, which also equals $20q$.

Gravitational potential around two masses looks the same as electric potential around two charges of the same sign, because both forces fall off as $1/r^2$. The difference: gravity always attracts, so gravitational isolines never show the "repel" pattern of two like charges pushing a test charge away.

A battery produces a potential difference by **chemical** reactions that push positive and negative charges apart. When two conductors touch, electrons flow between them until both surfaces are at the **same potential**.



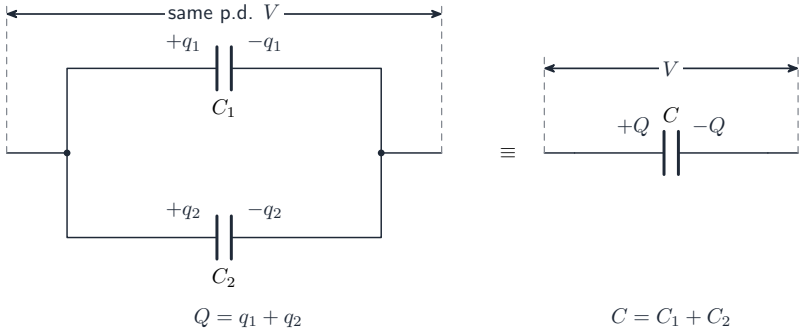
The potential near a point charge varies as 1/r

Capacitors

A **capacitor** 电容器 stores charge and energy on two conductors separated by a gap. Its **capacitance** 电容 relates charge to voltage:

$$C = \frac{Q}{V},$$

and the stored energy is $U = \frac{1}{2}CV^2$. Capacitance depends on the plates' **geometry** 几何形状 and the material between them, not on the charge placed on it.



Capacitors in parallel share the same p.d., and their charges add

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> **Worked example.** A 100 μF capacitor is charged to 12 V. It holds $Q = CV = 100\times10^{-6}\times12 = 1.2\times10^{-3}$ C of charge and stores $U = \frac{1}{2}CV^2 = \frac{1}{2}\times100\times10^{-6}\times12^2 = 7.2\times10^{-3}$ J of energy.

The parallel-plate capacitor. Two flat plates of area A, a distance d apart, carry equal and opposite charges. Then

$$C = \frac{\kappa\varepsilon_0 A}{d}, \qquad E = \frac{Q}{\varepsilon_0 A} \text{ (in air).}$$

Larger plates hold more charge at the same voltage; plates farther apart hold less. κ is the **dielectric constant** (relative permittivity) of the material between the plates ($\kappa = 1$ for a vacuum, about 1 for air). Between the plates, away from the edges, the field is **uniform**: the same size and direction everywhere.

- **Energy.** The energy $\frac{1}{2}QV = \frac{1}{2}CV^2$ is the work needed to separate the charges against the capacitor's increasing voltage. A fixed-voltage battery supplies QV when charging an initially uncharged capacitor through a resistor; half is stored and half becomes heat.
- **Dielectric.** Sliding an insulator between the plates polarizes it. Its induced charges make a field **opposite** to the plates' field, so for the same charge the voltage falls, and $C = Q/V$ **rises** by the factor κ.

> **Worked example (electrons through two plates).** Two plates of area 0.25 m^2, 0.010 m apart, have holes in their centers. Fast electrons enter through the top plate and are **slowed down** as they cross toward the bottom one.

- The force on each electron points **up**, against its motion. An electron's force is opposite to the field, so the field points **down**: from the top plate to the bottom. The field points from + to −, so the **bottom** plate is negative.
- If the potential difference is 200 V: $E = \dfrac{200}{0.010} = 2.0\times10^4$ V/m.
- Charge on each plate: $Q = \varepsilon_0 AE = (8.85\times10^{-12})(0.25)(2.0\times10^4) = 4.4\times10^{-8}$ C.

Capacitors in series. Connected in series to a battery, each capacitor holds the **same charge** Q (the charge that leaves one plate arrives at the next), and the voltages add to the battery's. Two identical capacitors in series have half the capacitance of one. A single replacement with the same C could have half the plate area, or twice the plate spacing.

> **Worked example (finding an unknown capacitance).** A 4.5 V battery is connected in series with an unknown capacitor C_U and a known capacitor C_K, using several different C_K. The voltmeter reads ΔV_K across the known one; the rest, $\Delta V_U = 4.5 - \Delta V_K$, is across the unknown.

- Same charge on both: $Q = C_K\Delta V_K = C_U\Delta V_U$. With $C_K = 200$ μF and $\Delta V_K = 1.5$ V, $Q = 3.0\times10^{-4}$ C.

- Plot Q (calculated as $C_K \Delta V_K$) on the vertical axis against ΔV_U on the horizontal: a straight line through the origin with slope C_U.
- In **parallel**, both capacitors would simply have the battery's voltage, 4.5 V, whatever C_U is, so a voltmeter alone would learn nothing about it.

The same idea works for a single capacitor: plot the charge q against $|\Delta V|$, and the slope is C.

Conservation of Electric Energy

Energy is conserved for charges just as for masses. A charge released in a field converts electric potential energy into **kinetic energy** 动能:

$$\Delta K = -q\,\Delta V \quad \text{(electric potential energy lost becomes kinetic energy).}$$

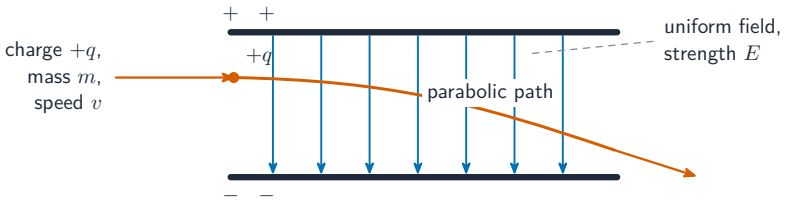
Worked example. An electron (charge magnitude $|q| = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg) is accelerated from rest through a potential difference of 100 V. Its final speed is

$$v = \sqrt{\frac{2|q|\,|\Delta V|}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{9.1 \times 10^{-31}}} = 5.9 \times 10^{6} \text{ m/s}.$$

This is exactly how the **electron gun** 电子枪 in an old television or an electron microscope works.

Kinetic energy depends on charge, not mass. Accelerated from rest through the same $|\Delta V|$, a particle gains $K = |q|\,|\Delta V|$. A particle with twice the charge gains twice the kinetic energy, whatever its mass; the **speed** it reaches does depend on the mass.

A charge in a uniform field moves like a projectile. Between parallel plates the force qE is constant, so the acceleration qE/m is constant. A charge that enters moving parallel to the plates keeps its sideways velocity and accelerates steadily toward one plate: its path is a **parabola**, exactly like a ball thrown horizontally under gravity.



Constant force qE, constant acceleration: a charge crossing a uniform field follows a parabola, like a horizontally thrown ball

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Worked example (an electron orbiting a nucleus). In a simple model of hydrogen, the electron moves in a circle of radius r around the proton. The electric attraction is the centripetal force:

$$\frac{ke^2}{r^2} = \frac{mv^2}{r} \;\Rightarrow\; v = \sqrt{\frac{ke^2}{mr}}.$$

Then $K = \frac{1}{2}mv^2 = \dfrac{ke^2}{2r}$ and $U = -\dfrac{ke^2}{r}$, so the total energy is $E = K + U = -\dfrac{ke^2}{2r}$. It is **negative**, because the electron is bound. When the atom absorbs a photon, the electron moves to a larger r: U rises toward zero, K falls, and the total energy rises by the photon's energy.

Practice

Answer each before reading the answers below. Take $k = 9.0 \times 10^9$ N m^2/C^2 and $e = 1.6 \times 10^{-19}$ C.

1. Two $+2.0$ μC charges are 0.30 m apart. Find the force between them, and say whether it attracts or repels.
2. The distance in question 1 is tripled. Find the new force without starting again.
3. Find the electric field 0.50 m from a -8.0 nC point charge, with its direction.
4. Two parallel plates 0.010 m apart have a potential difference of 200 V. Find the field between them, the force on a proton there, and the kinetic energy a proton gains crossing from the positive plate to the negative plate.
5. A student says: "At a point where the electric potential is zero, the electric field must be zero too." Correct the student.

Answers

1. $F = \dfrac{kq_1q_2}{r^2} = \dfrac{(9.0 \times 10^9)(2.0 \times 10^{-6})^2}{0.30^2} = 0.40$ N, repulsive (like charges).
2. $F \propto 1/r^2$, so tripling r divides F by 9: $0.40/9 = 0.044$ N.
3. $E = \dfrac{kQ}{r^2} = \dfrac{(9.0 \times 10^9)(8.0 \times 10^{-9})}{0.50^2} = 2.9 \times 10^2$ N/C, pointing **toward** the charge, because it is negative.
4. $E = \dfrac{\Delta V}{d} = \dfrac{200}{0.010} = 2.0 \times 10^4$ V/m. $F = eE = (1.6 \times 10^{-19})(2.0 \times 10^4) = 3.2 \times 10^{-15}$ N. Energy gained $= e\,\Delta V = (1.6 \times 10^{-19})(200) = 3.2 \times 10^{-17}$ J.
5. Potential and field are different quantities. Midway between equal charges $+q$ and $-q$, the two potentials cancel, so $V = 0$. But both fields there point from $+q$ toward $-q$ and add, so E is not zero. The field depends on how fast V changes with position, not on the value of V.

Exam tips

- **Coulomb's law** and the point-charge field are **inverse-square** – doubling the separation makes the force (or field) four times smaller.
- Use magnitudes for the size of a force and decide direction from the **signs**; the field points the way a **positive** test charge would move.
- **Potential (V) is a scalar** so potentials from several charges simply add; the **field (E) is a vector** and must be added by direction.
- For a charge accelerated through a voltage use energy conservation $\Delta K = -q\,\Delta V$; from rest, $\frac{1}{2}mv^2 = |q|\,|\Delta V|$ when the field accelerates it.
- Capacitor relations: $C = Q/V$ (capacitance is fixed by geometry) and energy $U = \frac{1}{2}CV^2$.

Common mistakes

- Adding field magnitudes without thinking about direction. Draw each field at the point first; fields from a + and a − charge point the *same* way between them.
- Putting the sign of a charge into $E = kQ/r^2$ and then also adding a direction. Use magnitudes for size, then choose the direction from the sign.
- Using $1/r$ for the field or $1/r^2$ for the potential. Field and force go as $1/r^2$; potential and potential energy as $1/r$.
- Saying that where $E = 0$ the potential must be zero (or the reverse). They are independent: at M above, $E \neq 0$ and $V \neq 0$; at the midpoint of two equal opposite charges $V = 0$ but $E \neq 0$.
- Forgetting the micro- prefix: 1 μC $= 10^{-6}$ C, 1 nC $= 10^{-9}$ C.
- Thinking capacitance changes when the charge or voltage changes. C is set by the plates and the material between them; Q and V change together.
- Saying a charged object cannot attract a neutral one. It polarizes the neutral object, and the nearer, opposite induced charge wins.
- Saying the field inside a charged metal object is strong. In equilibrium it is zero inside; the extra charge sits on the surface.
- Drawing field arrows along an equipotential line. The field always crosses isolines at right angles.
- Thinking capacitors in series share the same voltage. In series they carry the same charge; in parallel they share the same voltage.
- Counting only some pairs in the potential energy of three or more charges. Every pair contributes one term.

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Electric Circuits

AP Physics 2

Electric Current



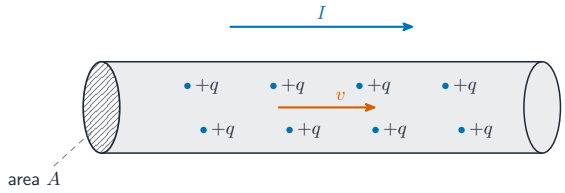
An oscilloscope: circuit behaviour over time is measured as voltage against time

Electric current 电流 is the rate at which charge flows past a point, measured in **amperes** 安培 (A):

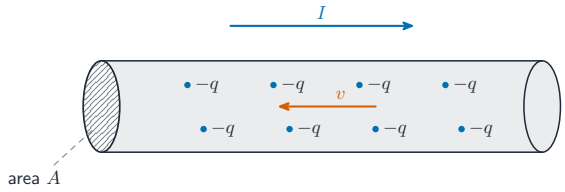
$$I = \frac{\Delta q}{\Delta t}.$$

By convention, current points the way **positive** charge would move (opposite to the electrons in a wire). A steady current needs a complete loop and an energy source (a battery's **electromotive force** 电动势, or emf).

Zero current does not mean the electrons are still. In a wire with no current, the free electrons still race about at very high speeds, but in random directions, so there is no **net** flow of charge. A potential difference adds a slow average **drift** in one direction on top of that random motion; the drift is the current. Current has a direction (the way positive charge would move), but it is **not a vector**: at a junction, currents add as numbers, whatever angles the wires make.



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drift slowly through a conductor to make a current

Simple Circuits

A **circuit** 电路 is a closed loop of **conductors** 导体, a source (battery), and components. In a **series** 串联 path the same current flows through each element; in a **parallel** 并联 path the same voltage is across each **branch** 支路. A **circuit diagram** 电路图 uses standard symbols; reading it correctly is the first step of any circuit problem.

Three words describe whether charge can flow round a loop:

- **closed circuit**: there is a complete path, so charge can flow;
- **open circuit**: the path is broken (an open switch, a gap), so no charge flows;
- **short circuit** 短路: a path with (almost) no resistance, so charge flows with **no** change in potential. A resistor or bulb connected across an ideal short circuit has zero potential difference across it, and so no current through it.

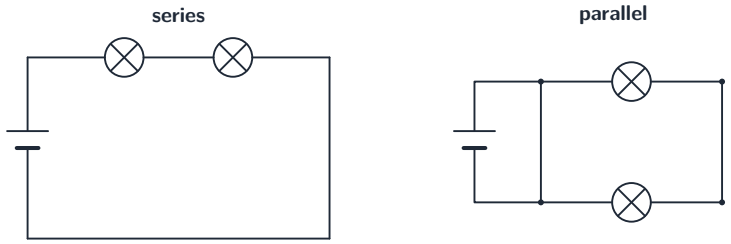
One element can belong to more than one loop, and the arrangement of the elements, not just which elements there are, decides how the circuit behaves.

> **Worked example (shorting out a bulb).** Two identical bulbs are in series with an ideal battery. A switch is connected across bulb 2. Before the switch is closed, the current is $\frac{\varepsilon}{2R}$ and bulb 1 has power $P_1 = \left(\frac{\varepsilon}{2R}\right)^2 R = \frac{\varepsilon^2}{4R}$.

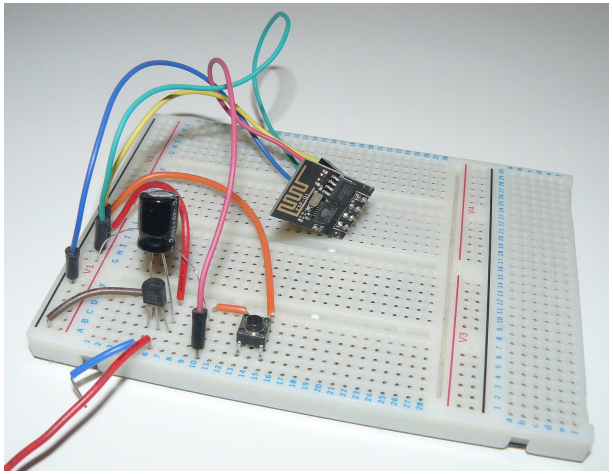
- Closing the switch makes a short circuit across bulb 2: it goes **out**.

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- The total resistance halves, so the current doubles to $\frac{\varepsilon}{R}$.
- Bulb 1's power becomes $\left(\frac{\varepsilon}{R}\right)^2 R = \frac{\varepsilon^2}{R} = 4P_1$: it gets **brighter**, not dimmer, although a bulb has gone out.



Components can be joined in series or in parallel



A real circuit: components pushed into a breadboard and joined by wires so current has a complete path

Resistance, Resistivity, and Ohm's Law

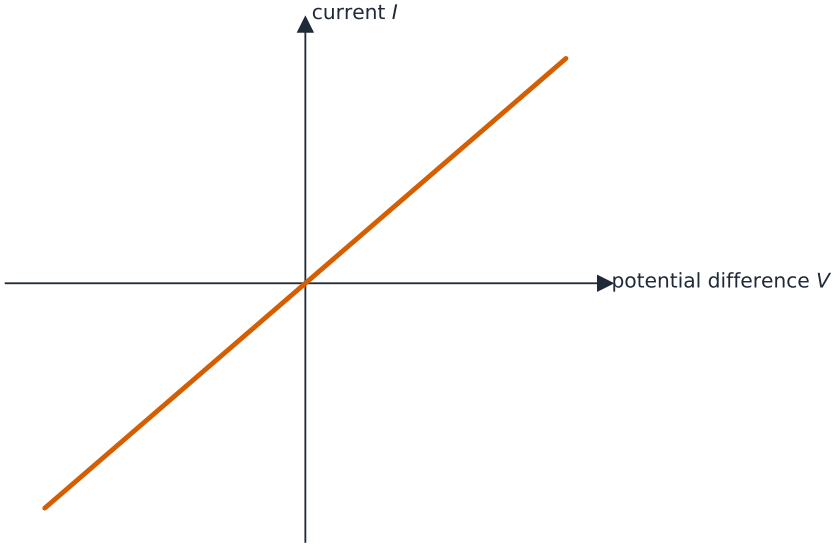
Resistance 电阻 R opposes current, measured in ohms. For an **ohmic** 欧姆性 component under fixed physical conditions, **Ohm's law** 欧姆定律 states that current is proportional to voltage. Its constant resistance relates the quantities:

$$V = IR.$$

A component's resistance depends on the material's **resistivity** 电阻率 ρ, its length, and its **cross-sectional area** 横截面积: $R = \frac{\rho L}{A}$ – longer and thinner means more

resistance.

Resistivity is a property of the **material**, set by its atomic structure. For most metals it **rises with temperature**: hotter atoms vibrate more and get in the way of the drifting electrons more often. An **ohmic** material keeps a constant resistance, so its graph of current against potential difference is a straight line through the origin with slope $\frac{1}{R}$. A filament bulb is not ohmic: as it heats up its resistance rises, and the graph curves.



The I-V line of an ohmic conductor is straight through the origin

> **Worked example.** A 2.0 A current flows through a 6.0 Ω **resistor** 电阻器. The voltage across it is $V = IR = 2.0 \times 6.0 = 12$ V.

Measuring resistivity (a lab). Use rods of the same material with different lengths L and cross-sectional areas A. For each rod, connect a variable power supply and an ammeter in series and a voltmeter **across the rod**; record ΔV and I, and find $R = \Delta V/I$ (or take the slope of a ΔV–I graph from several readings). Then plot R against $\frac{L}{A}$: a straight line through the origin with slope ρ.

- The power supply's internal resistance does **not** spoil the result, because the voltmeter measures the potential difference across the rod itself.
- Large currents heat the rods, which raises their resistance. Keep the currents small and take readings quickly.

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Electric Power

Electric power 电功率 is the rate a component converts electrical energy (to heat, light, motion):

$$P = IV = I^2R = \frac{V^2}{R}.$$

Pick the form that uses the quantities you know. A resistor's power all becomes heat.

> **Worked example.** The 6.0 Ω resistor above, carrying 2.0 A, **dissipates** 耗散 $P = I^2R = 2.0^2 \times 6.0 = 24$ W – equivalently $P = IV = 2.0 \times 12 = 24$ W.

In a simple filament-bulb model, a **light bulb** 灯泡 glows as it dissipates electrical power. For comparable bulbs with similar light-conversion behaviour, their **brightness** 亮度 generally rises with **power**. Compare power using resistances at the actual operating conditions. In a series string every bulb carries the same current, so by $P = I^2R$ the bulb with the **largest** resistance is brightest; wired in parallel every bulb gets the full battery voltage, so by $P = V^2/R$ the bulb with the **smallest** resistance is brightest.

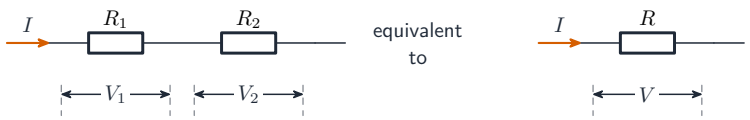


High-voltage power lines: electrical power is transmitted as high voltage so current (and I^2R loss) stays low

Compound DC Circuits

Combine resistors to find an **equivalent resistance** 等效电阻:

- **Series:** $R_{\text{eq}} = R_1 + R_2 + \cdots$ (resistances add).
- **Parallel:** $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ (the total is **less** than the smallest).

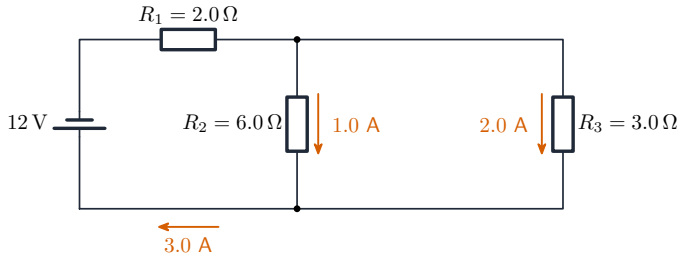


Resistors in series add to a single equivalent resistance

Reduce the network step by step to find the total current from the battery, then work back to each element.

Worked example. A 12 V battery drives a 4.0 Ω and a 12 Ω resistor in **parallel**. First combine them: $\frac{1}{R_{\text{eq}}} = \frac{1}{4} + \frac{1}{12} = \frac{4}{12} \Rightarrow R_{\text{eq}} = 3.0\ \Omega$. The total current from the battery is $I = \frac{V}{R_{\text{eq}}} = \frac{12}{3.0} = 4.0$ A, which splits so that the smaller resistor carries the larger share (3.0 A through the 4 Ω, 1.0 A through the 12 Ω).

Worked example (series and parallel together). A 12 V battery with negligible **internal resistance** 内阻 drives $R_1 = 2.0\,\Omega$ in series with a parallel pair, $R_2 = 6.0\,\Omega$ and $R_3 = 3.0\,\Omega$. Find the current in each resistor and the power each one dissipates.



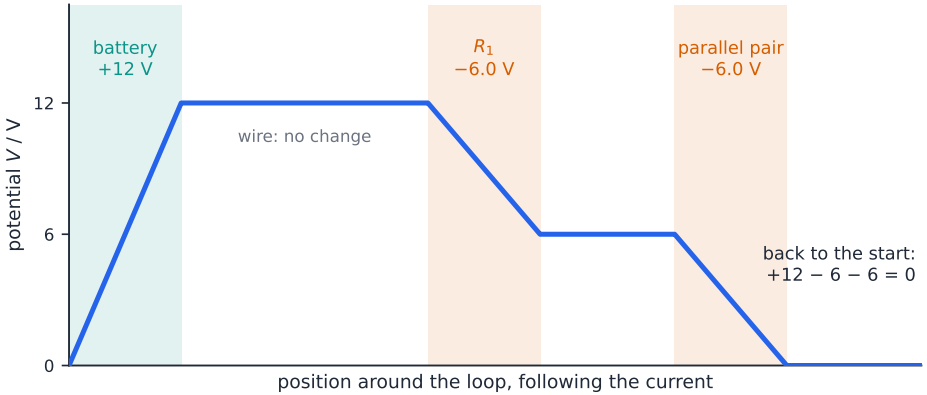
Reduce the parallel pair first, then work back out from the total current

1. Parallel pair: $\frac{1}{R_{23}} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{3}{6.0}$, so $R_{23} = 2.0\ \Omega$.
2. Whole circuit: $R_{\text{eq}} = 2.0 + 2.0 = 4.0\ \Omega$, so $I = \frac{12}{4.0} = 3.0$ A through R_1 (and the battery).
3. Voltage across R_1: $3.0 \times 2.0 = 6.0$ V. The loop rule leaves $12 - 6.0 = 6.0$ V across the pair.
4. Branch currents: $I_2 = \frac{6.0}{6.0} = 1.0$ A and $I_3 = \frac{6.0}{3.0} = 2.0$ A. **Junction** 节点 check: $1.0 + 2.0 = 3.0$ A.
5. Powers: $P_1 = 3.0^2(2.0) = 18$ W, $P_2 = 1.0^2(6.0) = 6.0$ W, $P_3 = 2.0^2(3.0) = 12$ W. Total 36 W = (12 V)(3.0 A), the battery's output.

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If these were identical-type bulbs, R_1 would be brightest: it carries the whole current.

Potential around a loop. Kirchhoff's loop rule can be drawn. Start at the battery's negative terminal, call it 0 V, and follow the current round the circuit above, plotting the potential as you go.



Around any loop, the rises and drops in potential add to zero

The potential **rises** across the battery, stays **flat** along ideal wires, and **drops** across each resistor by IR. Coming back to the start, it must return to 0 V: $+12 - 6.0 - 6.0 = 0$.

Worked example (internal resistance). A battery has emf $\varepsilon = 9.0$ V and internal resistance $r = 0.50\ \Omega$. It is connected to a 4.0 Ω resistor. The internal resistance is in series with the external one:

$$I = \frac{\varepsilon}{R + r} = \frac{9.0}{4.0 + 0.50} = 2.0\text{ A}, \qquad V_{\text{terminal}} = \varepsilon - Ir = 9.0 - (2.0)(0.50) = 8.0\text{ V}.$$

A voltmeter across the battery reads 8.0 V, not 9.0 V, while current flows. The "lost" 1.0 V heats the battery.

Worked example (a qualitative–quantitative question). A battery of emf ε and internal resistance r lights n identical bulbs, each of resistance R, connected in parallel.

(a) *A student says that adding one more bulb in parallel makes the other bulbs dimmer. Without using equations, explain whether the student is correct.* The student is correct. Another parallel branch lowers the total resistance, so the battery delivers more current. More current through the internal resistance means a larger potential drop inside the battery, so the terminal potential difference across the bulbs falls, and each bulb gets less power.

(b) *Derive an expression for the potential difference V across each bulb in terms of ε, r, R and n.* The bulbs in parallel have $R_{\text{eq}} = \dfrac{R}{n}$, in series with r:

$$I = \frac{\varepsilon}{\dfrac{R}{n} + r}, \qquad V = I\,\frac{R}{n} = \frac{\varepsilon R}{R + nr}.$$

(c) *Explain how your expression supports your answer to (a).* n appears only in the denominator, so a larger n gives a smaller V and dimmer bulbs, as argued in (a). It also shows the limit: if $r = 0$ (an ideal battery), $V = \varepsilon$ for any n, and adding bulbs would not dim the others. The dimming comes from the internal resistance.

An ideal battery has no internal resistance, and ideal wires have none either. Wires can be treated as ideal only when the circuit also contains real resistors, whose resistance is much larger. The emf is what a voltmeter reads across the battery when **no** current flows through it. If an ideal battery is replaced by a real one with the same emf, the total resistance rises, the current falls, and every resistor in the circuit dissipates **less** power.

Finding the emf from a linear graph. A battery with internal resistance r drives a variable resistor R_{var}. For each setting, the current is measured. From $\varepsilon = I(R_{\text{var}} + r)$,

$$R_{\text{var}} = \varepsilon \cdot \frac{1}{I} - r.$$

Plot R_{var} on the vertical axis against $\dfrac{1}{I}$ on the horizontal: a straight line whose **slope** is ε and whose vertical intercept is $-r$. A slope of 9.0 Ω A means $\varepsilon = 9.0$ V.

Measuring Current and Voltage

Two meters read a circuit. An **ammeter** 电流表 measures the current at a point, so it must be wired **in series** – the current you want to measure has to flow *through* it. A **voltmeter** 电压表 measures the **potential difference** 电势差 between two points, so it is wired **in parallel**, bridging across the component whose voltage you want.

For a meter to read the true value it must barely disturb the circuit:

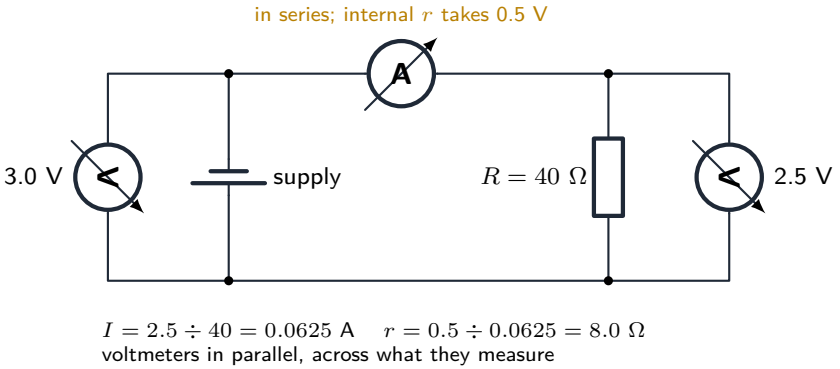
- an **ideal ammeter** has **zero resistance**, so putting it in series does not reduce the current it reads;
- an **ideal voltmeter** has **infinite resistance**, so almost no current is diverted through it.

A real, **nonideal** 非理想 meter is imperfect: a real ammeter has a small resistance (it slightly lowers the current), and a real voltmeter lets a little current leak through (it slightly lowers the voltage it reads). So connecting any meter changes, a little, the very quantity it is measuring.

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Worked example (an ammeter with resistance). A power supply drives a resistor $R = 40\ \Omega$ through an ammeter with internal resistance r. One voltmeter reads 3.0 V across the supply, another 2.5 V across R.

- Current: $I = \dfrac{2.5}{40} = 0.0625$ A.
- The ammeter takes the remaining 0.5 V: $r = \dfrac{0.5}{0.0625} = 8.0\ \Omega$.
- Energy conservation for the loop: $\Delta V_{\text{PS}} = I(R + r)$. On a graph of I against ΔV_{PS}, the slope is $\dfrac{1}{R+r}$, **less** steep than the $\dfrac{1}{R}$ an ideal ammeter would give.



Ammeter in series, voltmeters in parallel. The 0.5 V missing across R is dropped inside the ammeter: r = 0.5 / 0.0625 = 8.0 ohm

Worked example (measuring two resistors you cannot separate). R_1 is in series with a parallel pair made of a switch and R_2, and you can connect only to the two ends. Connect a battery and ammeter in series across the ends, with a voltmeter across the battery.

- Switch **closed**: it short-circuits R_2, so $R_1 = \dfrac{\Delta V}{I_{\text{closed}}}$.
- Switch **open**: all the current goes through both, so $R_1 + R_2 = \dfrac{\Delta V}{I_{\text{open}}}$; subtract to find R_2.

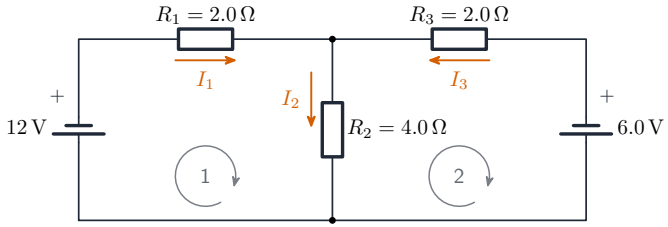
Kirchhoff's Loop Rule

Kirchhoff's loop rule 基尔霍夫电压定律 (energy conservation): around any closed loop, the voltage gains and drops sum to **zero**. Add the battery's emf and subtract each IR drop as you go around. This gives one equation per independent loop.

Kirchhoff's Junction Rule

Kirchhoff's junction rule 基尔霍夫电流定律 (charge conservation): the total current **into** any junction equals the total current **out**. Together with the loop rule, it lets you solve any multi-loop circuit for its unknown currents.

> **Worked example (two loops).** In the circuit below, find the current in each resistor.

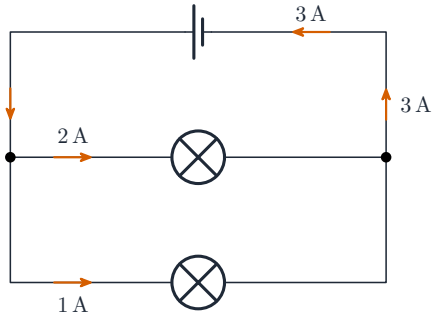


Guess a direction for each current; a negative answer means the real current flows the other way

1. Junction (top middle): $I_2 = I_1 + I_3$.
2. Loop 1, clockwise from the bottom-left corner: up through the 12 V battery (+12), along R_1 with I_1 ($-2.0I_1$), down R_2 with I_2 ($-4.0I_2$): $12 - 2.0I_1 - 4.0I_2 = 0$.
3. Loop 2, clockwise from the bottom of the middle branch: up R_2 against I_2 ($+4.0I_2$), along R_3 against I_3 ($+2.0I_3$), down through the 6.0 V battery from + to − (-6.0): $6.0 - 2.0I_3 - 4.0I_2 = 0$.
4. Substitute I_2: $12 = 6.0I_1 + 4.0I_3$ and $6.0 = 4.0I_1 + 6.0I_3$. Multiply the first by 3 and the second by 2, then subtract: $36 - 12 = 18I_1 - 8.0I_1$, so $I_1 = 2.4$ A. Then $I_3 = \dfrac{6.0 - 4.0(2.4)}{6.0} = -0.60$ A and $I_2 = 2.4 - 0.60 = 1.8$ A.

The minus sign on I_3 means 0.60 A flows to the **right** through R_3 and down into the + terminal of the 6.0 V battery: the bigger battery is charging the smaller one. Check loop 1: $12 - 2.0(2.4) - 4.0(1.8) = 12 - 4.8 - 7.2 = 0$.

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Current divides at a junction: what flows in equals what flows out

RC Circuits

An **RC circuit** RC 电路 contains a resistor and a **capacitor** 电容器. When charging, the capacitor's voltage rises and the current falls, both **exponentially**, over a characteristic time $\tau = RC$. Key limits: at the first instant the uncharged capacitor acts like a plain wire (maximum current); after a long time it is fully charged and blocks current (acts like a break).

Capacitors in combination. Capacitors combine the **opposite** way to resistors:

- **Parallel:** $C_{\text{eq}} = C_1 + C_2 + \cdots$ (more plate area).
- **Series:** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$, less than the smallest. By conservation of charge, every capacitor in series carries the **same** charge.

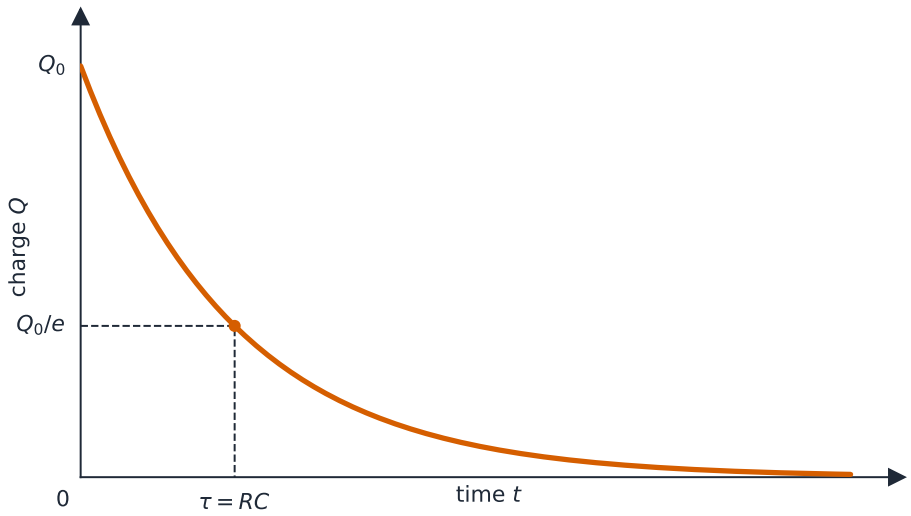
For a circuit reducible to a single resistor and capacitor, use the equivalent values: $\tau = R_{\text{eq}}C_{\text{eq}}$.

What changes, and how, during charging and discharging.

	Just after switching	**During**	**After a long time**
charging: current in the branch	largest, ε/R	falls	zero
charging: ΔV, Q and stored energy	zero	rise	largest
discharging: Q, ΔV, energy and current	largest	fall	zero

Charge, capacitor voltage and current change fastest at the start. During charging, the stored energy initially rises at zero rate because both charge and voltage start at

zero; its rate of increase peaks later. When charging, the charge reaches about 63% of its final value after one time constant; when discharging, about 37% of the charge is left after one time constant. After many time constants, treat the circuit as **steady**: the capacitor is a gap, and only the resistor paths carry current.



The charge on a capacitor decays exponentially as it discharges

Worked example. For $R = 10$ kΩ and $C = 100$ μF, the **time constant** 时间常数 is $\tau = RC = (10 \times 10^3)(100 \times 10^{-6}) = 1.0$ s. After one time constant the capacitor reaches about 63% of the supply voltage; after about 5τ it is essentially fully charged.

Connected to a 9.0 V battery, this circuit starts with the capacitor uncharged, acting as a wire: $I_0 = \dfrac{9.0}{10 \times 10^3} = 0.90$ mA. A long time later no current flows, the capacitor has the full 9.0 V across it, and it stores $Q = CV = (100 \times 10^{-6})(9.0) = 9.0 \times 10^{-4}$ C.

Worked example (two ways to wire a bulb and a capacitor). You have a battery with a small internal resistance, a bulb and an uncharged capacitor.

- *A bulb that lights brightly at once, then fades out.* Put the bulb **in series** with the capacitor. At first the capacitor acts like a wire, so the full current flows and the bulb is bright. As the capacitor charges, its voltage rises, the bulb's share falls, and the current drops to zero.
- *A bulb that starts dark and slowly lights up.* Put the bulb **in parallel** with the capacitor. At first the uncharged capacitor acts like a wire across the bulb, so the current goes round the bulb and it is dark. As the capacitor charges, less current goes into it and more through the bulb, which grows brighter until the capacitor is full and all the current flows through the bulb. (The battery's internal resistance is what stops this being a dead short at the start.)

Is it a resistor or a capacitor? Connect the unknown component in series with a known 500 Ω resistor, a battery and a switch, with a voltmeter across the unknown part. Close the switch and watch the reading. A **resistor** shows a steady voltage straight away. An uncharged **capacitor** starts at 0 V (it has no charge, so no potential difference) and its voltage rises toward the battery's as charge builds up, while the current falls.

Practice

Answer each before reading the answers below.

1. A lamp is rated 60 W at 120 V. Find the current through it and its resistance when it works normally.
2. Resistors of 2.0 Ω, 3.0 Ω and 6.0 Ω are connected in parallel. Find the equivalent resistance. What is it if they are in series instead?
3. A 9.0 V battery (no internal resistance) drives a 3.0 Ω resistor in series with a parallel pair of 4.0 Ω and 12 Ω. Find the current from the battery and the current in each branch of the pair.
4. A battery of emf 6.0 V has a terminal voltage of 5.0 V while it delivers 2.0 A. Find its internal resistance and the external resistance.
5. A student says: "Adding another bulb in parallel makes every bulb dimmer, because the current now has to be shared." Correct the student.

Answers

1. $I = \dfrac{P}{V} = \dfrac{60}{120} = 0.50$ A, and $R = \dfrac{V}{I} = \dfrac{120}{0.50} = 240$ Ω.
2. Parallel: $\dfrac{1}{R} = \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{6} = 1$, so $R = 1.0$ Ω, less than the smallest resistor. Series: $2.0 + 3.0 + 6.0 = 11$ Ω.
3. Pair: $\dfrac{1}{R} = \dfrac{1}{4} + \dfrac{1}{12} = \dfrac{4}{12}$, so 3.0 Ω. Total 6.0 Ω, so $I = 9.0/6.0 = 1.5$ A. The pair has $1.5 \times 3.0 = 4.5$ V across it: $4.5/4.0 = 1.1$ A in the 4.0 Ω branch and $4.5/12 = 0.38$ A in the 12 Ω branch. Check: $1.125 + 0.375 = 1.5$ A.
4. The lost voltage is $6.0 - 5.0 = 1.0$ V $= Ir$, so $r = 1.0/2.0 = 0.50$ Ω. External: $R = 5.0/2.0 = 2.5$ Ω.
5. With an ideal battery, every parallel branch has the full battery voltage across it, so each bulb carries the same current as before and is just as bright. The battery simply supplies more total current. (A real battery's internal resistance makes the bulbs a little dimmer, because a larger total current loses more voltage inside it.)

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Exam tips

- In **series** the current is the same throughout; in **parallel** the voltage is the same across each branch – never mix these up.
- Combine resistors: series **add**; parallel $1/R_{\text{eq}} = \sum 1/R_i$ (the total is **less** than the smallest).
- Apply **Kirchhoff's rules**: junction (current in = current out, charge conserved) and loop (voltages sum to zero, energy conserved).
- Pick the power form that fits your knowns: $P = IV = I^2R = V^2/R$.
- An **ammeter** goes in **series** (ideal: zero resistance); a **voltmeter** goes in **parallel** (ideal: infinite resistance). A real meter slightly disturbs the circuit it measures.
- A bulb is brighter when it dissipates more **power** – in series the biggest resistance (I^2R) glows brightest; in parallel the smallest (V^2/R) does.
- In an **RC circuit** the capacitor acts like a plain wire the instant it starts charging and like a break once fully charged.

Common mistakes

- Adding parallel resistances directly, or forgetting to invert at the end. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{6.0} + \dfrac{1}{3.0} = 0.50\ \Omega^{-1}$ gives $R_{\text{eq}} = 2.0\ \Omega$, not $0.50\ \Omega$.
- Using the battery's emf as the voltage across one resistor in a combination circuit. Find the voltage across that part from IR or the loop rule.
- Saying the current is "used up" along a series circuit. The current is the same everywhere in a series path; energy is transferred.
- Changing a sign halfway round a Kirchhoff loop. Fix the direction of travel first; crossing a resistor *with* the assumed current is a drop ($-IR$), against it is a rise ($+IR$).
- Treating a negative answer from Kirchhoff's rules as an error. It only means the current flows opposite to the direction you guessed.
- Assuming a battery's terminal voltage always equals its emf. With current flowing, $V = \varepsilon - Ir$.
- Putting a voltmeter in series or an ammeter in parallel. An ammeter in parallel (near-zero resistance) short-circuits the component.
- Thinking that closing a switch across one bulb dims the others. Shorting one bulb lowers the total resistance, so the others carry more current and get brighter.
- Combining capacitors like resistors. Capacitances add in **parallel**; in series the reciprocals add.
- Saying the electrons in a wire with no current are at rest. They move fast, but randomly; there is no net drift.
- Saying an uncharged capacitor blocks current at the moment a switch closes. At that instant it acts like a wire; it blocks current only once fully charged.

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Magnetism and Electromagnetism

AP Physics 2

Magnetic Fields

Drop a strong magnet down a copper pipe and it falls slowly, as if through honey, though copper is not magnetic. Pull a magnet away from a coil and a current flows that tries to pull it back. This unit explains both, and the rule behind them: magnetism never gives energy away for free.



An aurora: charged particles from the Sun are steered by Earth's magnetic field toward the poles, where they hit the air and make it glow

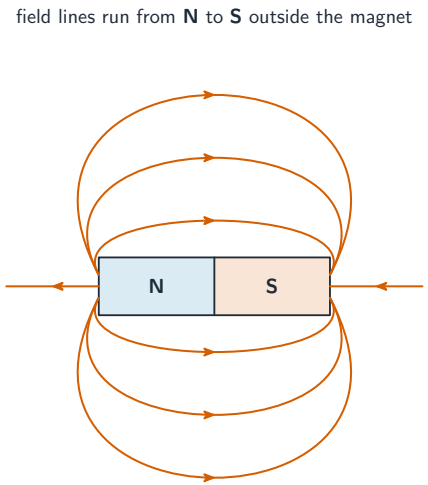
Different materials respond very differently to a magnetic field, and the course names three. **Ferromagnetic** 铁磁性 materials (iron, nickel, cobalt) can develop strongly aligned **magnetic domains** 磁畴 and retain some alignment, so suitable materials can be **permanent magnets** 永磁体. **Paramagnetic** 顺磁性 materials (aluminium, titanium) align only weakly and do not stay aligned. **Diamagnetic** 抗磁性 materials - in fact **all** materials have some diamagnetism - align weakly *opposite* the field. This behaviour comes from a material property, the **permeability** 磁导率: free space has a fixed **permeability of free space** 真空磁导率 μ_0, while the permeability of matter differs from it and is not even constant for a given material.

A **magnetic field** 磁场 $\vec{B}$ surrounds magnets and moving charges. **Magnetic field lines** 磁感线 run from a magnet's **north pole** 北极 to its **south pole** 南极 outside the magnet, and denser lines mean a stronger field. Magnetic poles always come in **pairs** – cutting a magnet in half makes two smaller magnets, never an isolated **pole** 磁极.

Where magnetism comes from. Electric currents create magnetic fields. In a

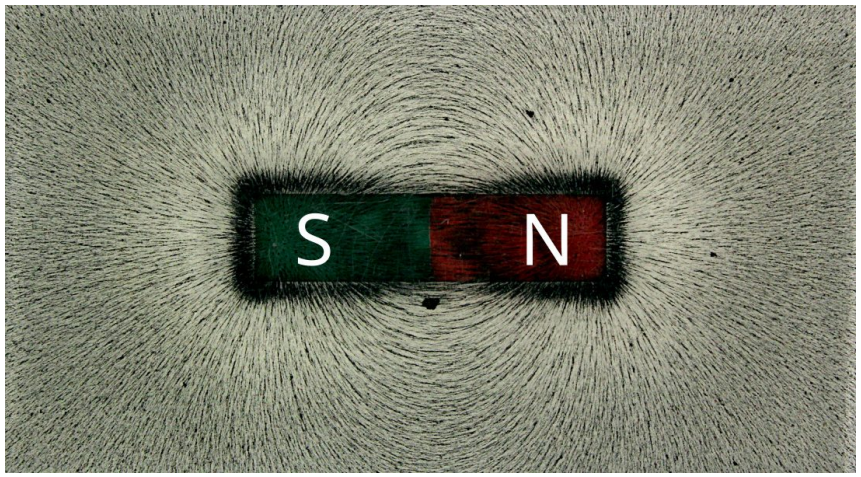
magnet, electrons have magnetic moments from their orbital motion and intrinsic spin: they act as tiny **magnetic dipoles** 磁偶极子, each with a north and a south pole. Spin is a quantum property, not a literal orbit or rotation of a charged ball. A magnet, permanent or induced, is a material in which many of these dipoles line up. That is why a single pole never appears on its own: there are no magnetic "monopoles".

- Field lines form **closed loops**: outside a bar magnet they go from N to S, and **inside** it they continue from S back to N.
- Like poles repel; unlike poles attract. The field of a dipole gets weaker with distance.
- A small dipole, such as a **compass** needle, turns to line up with the field it is in.
- The Earth's field is roughly that of a giant bar magnet. A compass's north pole points to geographic north, so the magnetic pole near geographic north is really a magnetic **south** pole.



Field lines run from N to S outside a bar magnet, closer where the field is stronger

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Iron filings line up along the magnetic field, making the invisible field lines of a bar magnet visible

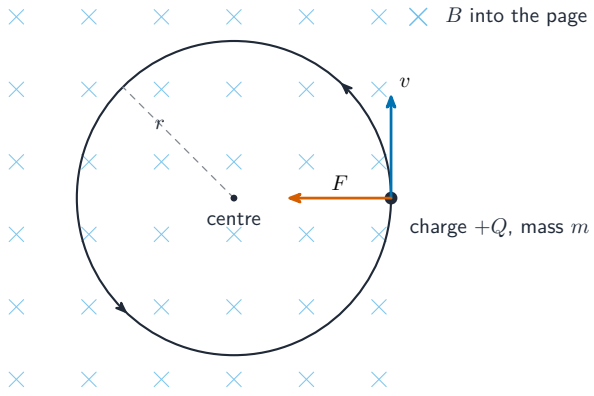
Magnetism and Moving Charges

A charge moving through a magnetic field feels a **magnetic force** 磁力:

$$F = |q|vB\sin\theta,$$

where θ is the angle between the velocity and the field. The force is **perpendicular** 垂直 to both $\vec{v}$ and $\vec{B}$ (use the **right-hand rule** 右手定则 for a positive charge and reverse it for a negative charge), so it changes direction but not speed – a charge moving perpendicular to a **uniform magnetic field** 匀强磁场 travels in a **circle**. A **stationary** 静止的 charge, or one moving **parallel** 平行 to the field, feels no magnetic force.

A moving charge also **makes** a magnetic field. At any point, that field is perpendicular to both the charge's velocity and the line from the charge to the point. It is strongest where those two are at right angles, and it is weaker farther away.



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

A charged particle moving across a magnetic field follows a circular path

Worked example. A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) enters a 0.50 T field at 2.0×10^5 m/s, at right angles to the field. The magnetic force is

$$F = qvB = 1.6 \times 10^{-19} \times 2.0 \times 10^5 \times 0.50 = 1.6 \times 10^{-14} \text{ N}.$$

This force is the **centripetal force** 向心力, so it bends the proton into a circle of radius

$$r = \frac{mv}{qB} = \frac{1.67 \times 10^{-27} \times 2.0 \times 10^5}{1.6 \times 10^{-19} \times 0.50} = 4.2 \times 10^{-3} \text{ m}.$$

Setting $qvB = \dfrac{mv^2}{r}$ and cancelling gives that neat $r = mv/(qB)$ – the principle behind **mass spectrometers** 质谱仪.

Worked example (two particles, the same voltage). Two particles are accelerated from rest through the same potential difference $|\Delta V|$ and then enter the same field B at right angles. Each gains $K = |q||\Delta V|$, so $v = \sqrt{2|q||\Delta V|/m}$, and

$$r = \frac{mv}{|q|B} = \frac{1}{B}\sqrt{\frac{2m|\Delta V|}{|q|}}.$$

A heavier particle with the same charge moves in a **larger** circle. A positive and a negative particle curve in **opposite** directions.

Electric and magnetic forces together. In a region with both fields, a charge feels both forces, independently. For a proton moving right through a magnetic field into the page, the magnetic force is up. An electric field pointing **down** with $E = vB$ makes the forces cancel, so the proton goes straight through. For $v = 3.0 \times 10^5$ m/s and $B = 0.030$ T: $F_B = qvB = 1.4 \times 10^{-15}$ N and $E = vB = 9.0 \times 10^3$ N/C.

This is a **velocity selector**: only particles with speed $v = E/B$ pass straight through. Faster ones feel a larger magnetic force and curve one way; slower ones are pushed the other way by the electric force. The same balance lets you **measure** a magnetic field. Between plates a distance d apart with emf $\mathcal{E}$, $E = \mathcal{E}/d$; adjust $\mathcal{E}$ until the particles go straight, then $\mathcal{E} = B_0 d\, v$. Plot $\mathcal{E}$ against v for several speeds: the slope is $B_0 d$.

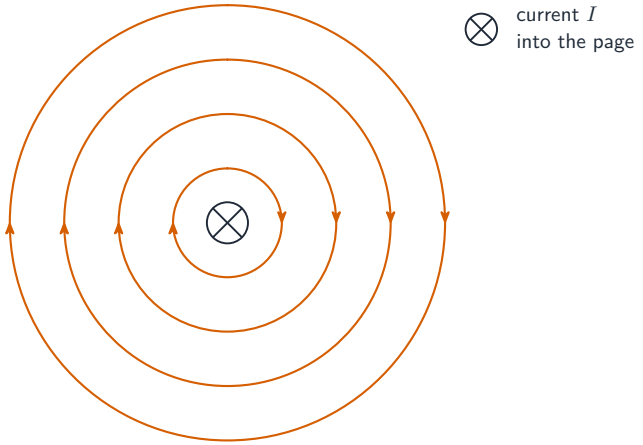
The Hall effect. In a current-carrying strip placed across a magnetic field, the magnetic force pushes the moving charges toward one edge. Charge builds up there until the electric field it makes balances the magnetic force. The result is a small potential difference **across** the strip, the **Hall voltage**, which is how many magnetic field sensors work.

Magnetism and Current-Carrying Wires

Because a current is moving charge, a magnetic field pushes on a **current-carrying wire** 载流导线:

$$F = BIL\sin\theta.$$

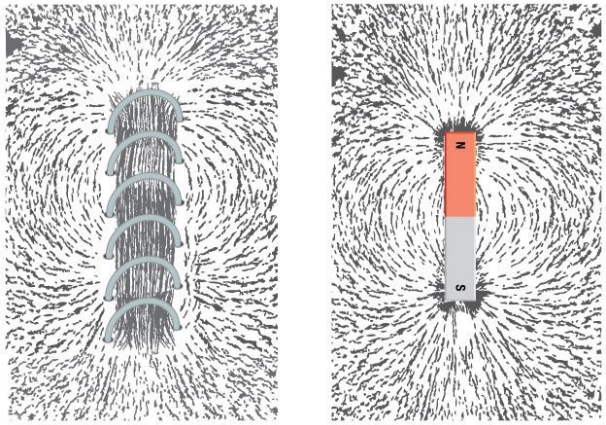
A current also **creates** its own magnetic field: circular field lines wrap around a straight wire (right-hand rule), and a coil (**solenoid** 螺线管) makes a field like a **bar magnet** 条形磁铁. This is how **electromagnets** 电磁铁 and **motors** 电动机 work.



Concentric circular field lines surround a straight current-carrying wire

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Worked example. A 0.30 m length of wire carries 4.0 A at right angles to a 0.20 T field. The force on it is $F = BIL = 0.20 \times 4.0 \times 0.30 = 0.24$ N – the push that turns a motor's coil.



A current-carrying coil (solenoid) makes a magnetic field shaped just like a bar magnet's

Worked example (the field around a wire). A long straight wire carries 10 A. The field 2.0 cm away is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(10)}{2\pi(0.020)} = 1.0 \times 10^{-4} \text{ T},$$

about twice the Earth's field, which is why a **compass** 指南针 near a current-carrying wire swings. The field falls off as $1/r$: twice as far away, half as strong.

Around a long straight wire the field lines are **circles** centered on the wire: the field has no part pointing toward or away from the wire, or along it. Grip the wire with your right thumb along the current; your fingers curl the way the field goes. At the center of a single loop of wire, the field points along the loop's axis. Near several wires, add the fields as **vectors**.

Parallel wires. Each wire sits in the other's field, so each feels a force. Currents in the **same** direction **attract**; opposite currents repel. Since $B \propto I/r$, the force on a length of wire is proportional to the other current and falls off as $1/r$.

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Worked example (cancelling the force on a wire). Wire 1 lies along $y = 0$ and wire 2 along $y = +d$, each carrying I in the $+x$ direction, so wire 2 pulls wire 1 **up**. A third wire carrying $2I$, also in $+x$, must pull wire 1 **down** by the same amount, so it goes **below** wire 1. Equal forces need $\frac{I}{d} = \frac{2I}{r_3}$, so $r_3 = 2d$: wire 3 lies along $y = -2d$.

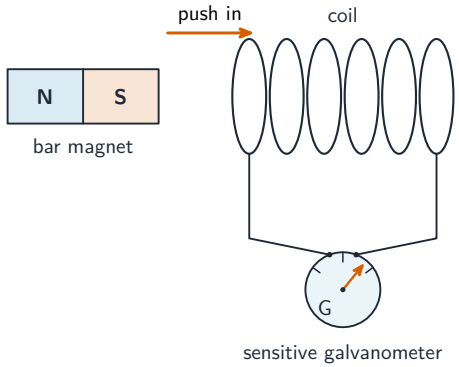
Worked example (a charge moving steadily near a wire). A negative charge moves at constant velocity parallel to a long wire carrying current I, a distance d away, in a region with an electric field E. Constant velocity means the forces balance: $|q|E = |q|vB$, with $B = \frac{\mu_0 I}{2\pi d}$. So $v = \frac{E}{B} = \frac{2\pi d E}{\mu_0 I}$.

Electromagnetic Induction and Faraday's Law

Changing magnetic **flux linkage** 磁链 produces an **emf** 电动势 – **electromagnetic induction** 电磁感应. A current flows if the conducting circuit is closed. In a uniform field, **magnetic flux** 磁通量 is $\Phi = BA\cos\theta$, where θ is measured between the field and the loop's **normal** 法线. For N turns sharing the same flux, **Faraday's law** 法拉第定律 gives the average **induced emf** 感应电动势:

$$\varepsilon_{\text{avg}} = -N\frac{\Delta\Phi}{\Delta t}.$$

The flux changes if the field, the area, or the loop's orientation changes. **Lenz's law** 楞次定律 (the minus sign) says the **induced current** 感应电流 produces a magnetic field that **opposes** the flux change that caused it – the basis of **generators** 发电机.



Moving a magnet into a coil induces an e.m.f. that drives a current

Worked example. The magnetic flux through a single loop drops from 0.020 Wb to 0.008 Wb in 0.030 s. The average induced emf is

$$\varepsilon = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{0.020 - 0.008}{0.030} = 0.40 \text{ V}.$$

A coil of N turns would give N times this – which is why generators and **transformers** 变压器 use many-turn coils.

Flux has a sign. The flux uses an **area vector**, drawn perpendicular to the loop. The flux is positive when the field points the same way as the area vector and negative when it points the opposite way, so a field that reverses makes the flux pass through zero.

Worked example (a coil near a wire). A square coil of side $2L$ lies beside a long straight wire, its near edge L from the wire and its far edge $3L$ away. The wire's field is $3B_0$ at the near edge and B_0 at the far edge. Because $B \propto 1/r$, the field is $2B_0$ at $r = \frac{3}{2}L$, a quarter of the way across the coil from its near edge. Now the current in the wire is reduced so that the flux through the coil falls from 5.0×10^{-5} to 1.0×10^{-5} T m^2 in 2.0 s. The induced emf is $\dfrac{4.0 \times 10^{-5}}{2.0} = 2.0 \times 10^{-5}$ V, and with a resistance of 10 Ω, $I = 2.0 \times 10^{-6}$ A.

A bulb connected to a coil next to another coil stays lit **only while** the current in the other coil keeps changing, because only a changing current makes a changing flux.



A substation transformer: changing magnetic flux in coils induces the voltages that power the grid

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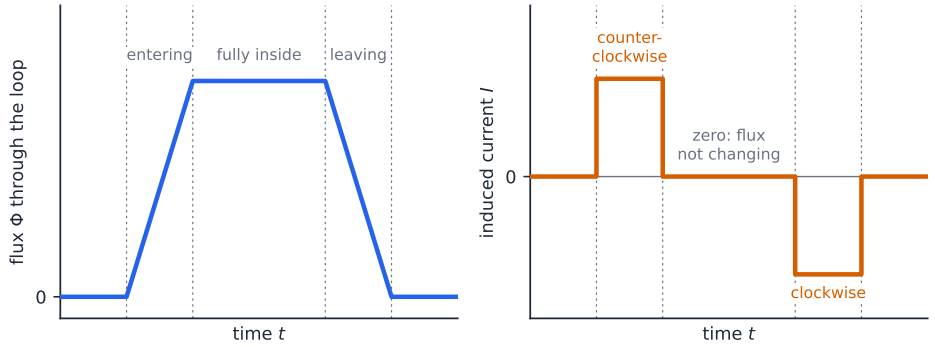
Lenz's law, step by step

Finding the direction of an induced current is a chain of four questions. Push the **north** pole of a magnet toward a coil:

1. **Which way is the flux through the coil?** Field lines leave a north pole, so the flux through the coil points away from the magnet.
2. **Is it increasing or decreasing?** Increasing, because the magnet is getting closer.
3. **What does the induced current do?** It makes its own field **opposing the change**, pointing back toward the magnet.
4. **So the coil face nearest the magnet becomes a north pole**, and it **repels** the approaching magnet.

You must push to move the magnet in; that work becomes the electrical energy in the coil. If the induced current helped the motion instead, you would get energy for nothing. That is why the falling magnet in the copper pipe is slowed.

A loop pulled through a field. A rectangular loop is pulled at constant speed into, through and out of a region of uniform field pointing into the page.



A current flows only while the flux is changing: entering and leaving, in opposite directions

- *Entering:* the inward flux grows at a steady rate, so a constant current flows. It is **counterclockwise**, making an outward field that opposes the growth.
- *Fully inside:* the flux is large but **not changing**, so there is no current.
- *Leaving:* the inward flux falls at the same rate, so the current has the same size but is **clockwise**, trying to keep the inward flux.

Motional emf

A **conducting rod** 导体棒 of length L sliding at speed v, with the rod, velocity and field B mutually perpendicular, sweeps out area, so the flux through the circuit changes. This **motional emf** 动生电动势 is $\varepsilon = BLv$.

> **Worked example (a rod on rails).** A 0.50 m rod slides at 4.0 m/s along rails in a 0.20 T field. The circuit has a resistance of 2.0 Ω.

- $\varepsilon = BLv = (0.20)(0.50)(4.0) = 0.40$ V, so $I = \varepsilon/R = 0.20$ A.
- The current in the rod feels a force $F = BIL = (0.20)(0.20)(0.50) = 0.020$ N, opposing the motion (Lenz again).
- To keep the rod moving steadily you must push with 0.020 N, a power of $Fv = 0.080$ W. The **resistor** 电阻器 **dissipates** 耗散 $I^2R = (0.20)^2(2.0) = 0.080$ W. The two match: every joule of electrical energy came from your push.

Practice

Answer each before reading the answers below. Electron: $q = 1.6 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg.

1. An electron moves at 3.0×10^6 m/s at right angles to a 0.010 T field. Find the force on it and the radius of its circle.
2. A 0.40 m wire carries 3.0 A at 30° to a 0.50 T field. Find the force on it.
3. The flux through each turn of a 50-turn coil falls by 0.0040 Wb in 0.020 s. Find the average induced emf.
4. The north pole of a magnet is pulled **away** from a coil. What pole forms on the coil face nearest the magnet, and why?
5. A student says: "A strong magnet held still inside a coil induces a large current." Correct the student.

Answers

1. $F = qvB = (1.6 \times 10^{-19})(3.0 \times 10^6)(0.010) = 4.8 \times 10^{-15}$ N. $r = \dfrac{mv}{qB} = \dfrac{(9.11 \times 10^{-31})(3.0 \times 10^6)}{(1.6 \times 10^{-19})(0.010)} = 1.7 \times 10^{-3}$ m.
2. $F = BIL\sin\theta = (0.50)(3.0)(0.40)\sin 30° = 0.30$ N.
3. $\varepsilon = N\dfrac{\Delta\Phi}{\Delta t} = 50 \times \dfrac{0.0040}{0.020} = 10$ V.
4. A south pole. The flux from the magnet is decreasing, so the induced current tries to keep it: the coil attracts the departing north pole, opposing the change.
5. Induction needs a **changing** flux. A magnet held still gives a constant flux, so the induced emf and current are zero, however strong the magnet.

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Exam tips

- The magnetic force $F = qvB\sin\theta$ is **perpendicular** to the velocity, so it changes direction but not speed (a circle for motion perpendicular to a uniform field); a stationary charge or one moving **along** the field feels no force.
- Use $F = BIL\sin\theta$ for the force on a current-carrying wire; $F = BIL$ when the wire is perpendicular to the field (the motor effect).
- A current **creates** a magnetic field (circles around a wire; a solenoid acts like a bar magnet).
- **Induction** needs a **changing** flux $\Phi = BA\cos\theta$ – a stationary magnet in a coil induces nothing.
- **Faraday**: $\varepsilon = \Delta\Phi/\Delta t$ (times N turns); **Lenz**: the induced current opposes the change (energy conservation).

Common mistakes

- Using the right-hand rule for a negative charge without reversing the answer. For an electron, the force is opposite to the direction the rule gives.
- Saying a magnetic force speeds up or slows down a charge. It is always perpendicular to the velocity, so it changes only the direction; it does no work.
- Expecting an emf from a large but steady flux. The induced emf depends on the *rate of change* of flux, $\varepsilon = -\Delta\Phi_B/\Delta t$.
- Using B alone instead of the flux. $\Phi_B = BA\cos\theta$: the emf also changes if the loop's area or angle changes.
- Reading Lenz's law as "the induced field opposes the field". It opposes the *change* in flux: if the flux is falling, the induced field points the same way as the original field.
- Forgetting $\sin\theta$ in $F = qvB\sin\theta$ or $F = BIL\sin\theta$. Motion or current parallel to the field feels no force.
- Drawing magnetic field lines that start or stop. They always form closed loops, running from S to N inside a magnet.
- Saying a loop fully inside a strong field has a large current. With constant flux, there is none.
- Saying parallel wires with currents in the same direction repel. They attract.
- Calling the magnetic pole near the geographic North Pole a north pole. It attracts the compass's north end, so it is a magnetic south pole.

Geometric Optics

AP Physics 2

Reflection

A straw in a glass of water looks bent. A shop's round security mirror shows the whole aisle, but small. A **magnifying glass** 放大镜 makes print larger, yet moved a little further away, the same lens turns the room upside down. One rule for bending light and one equation explain all of these.

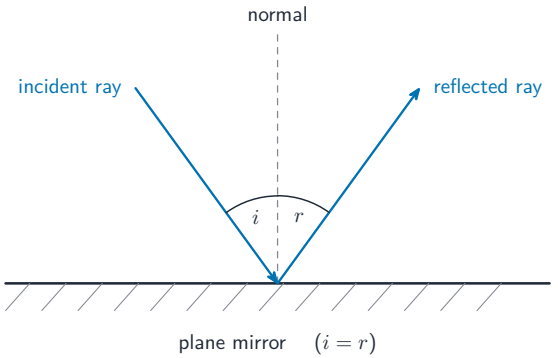
Geometric optics 几何光学 treats light as straight-line rays. The **law of reflection** 反射定律: when light bounces off a surface, the **angle of incidence** 入射角 equals the **angle of reflection** 反射角, both measured from the **normal** 法线 (the line perpendicular to the surface). Smooth surfaces reflect a clear image; rough surfaces scatter the rays.

What a ray is. A ray is a line drawn **perpendicular** to the **wavefronts** 波前 (the crests of the light wave), pointing the way the wave travels. A thin **laser** beam is a good real-life ray. The ray model works when mirrors, lenses and gaps are much larger than the wavelength. It cannot explain interference or diffraction; those need the wave model of unit 14.

Two kinds of reflection. Every single ray obeys the law of reflection. On a smooth surface such as a mirror, the normals are all parallel, so parallel rays leave parallel: **specular reflection** 镜面反射, which forms images. On a rough surface such as paper, the normals point in many directions, so parallel rays leave in many directions: **diffuse reflection** 漫反射. That is why you can read a page from any angle but see no image in it.

The plane mirror. A flat **plane mirror** 平面镜 forms a virtual, upright image the same size as the object. The image is as far behind the mirror as the object is in front. Parallel rays stay parallel after reflection, so its focal length is infinite: put $f \to \infty$ in the mirror equation and you get $d_i = -d_o$ and $m = +1$.

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The law of reflection: the angle of incidence equals the angle of reflection

Images Formed by Mirrors

For rays near the **principal axis** 主光轴, a **spherical mirror** 球面镜 makes parallel rays converge at a real **focal point** 焦点, or appear to diverge from a virtual focus, a distance f from the mirror. The **mirror equation** 面镜公式 relates object and image distances:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \qquad m = -\frac{d_i}{d_o}.$$

A **concave mirror** 凹面镜 (converging) can form a **real image** 实像 (rays actually meet, projectable, **inverted** 倒立); a **convex mirror** 凸面镜 always forms a **virtual image** 虚像 (rays only appear to meet, **upright** 正立, reduced). The **magnification** 放大率 m gives the image's size and orientation. For a spherical mirror of radius R, the focal point lies halfway between the mirror and its **centre of curvature** 曲率中心 C:

$$f = \frac{R}{2}.$$

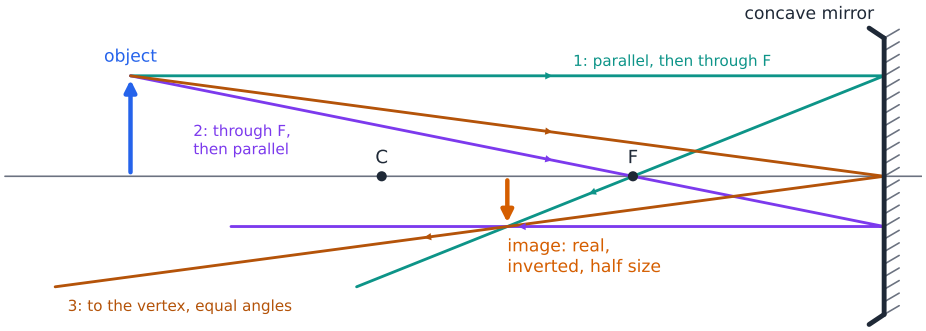
Sign convention 符号规则 matters: with the real-is-positive convention, a **negative** image distance means a virtual image. It lies behind a mirror, or on the object side of a lens.

Three principal rays. To draw a concave mirror's image, send rays from the top of the object. Any two of these three find the image; the third checks it.

1. A ray parallel to the axis reflects through F.
2. A ray through F reflects parallel to the axis.
3. A ray to the vertex (the centre of the mirror) reflects at an equal angle on the other side of the axis.

For a convex mirror the same rules hold, but F is behind the mirror: ray 1 reflects as if it came *from* F, and the rays only appear to meet behind the mirror (a virtual

image).



Three principal rays from the top of the object meet at a real, inverted, half-size image

Worked example. In the diagram, $f = 10$ cm and $d_o = 30$ cm: $\frac{1}{d_i} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}$, so $d_i = +15$ cm and $m = -\frac{15}{30} = -0.5$. The rays in the figure cross at exactly that point. An object beyond C always gives an image between F and C: real, inverted and smaller.

Refraction

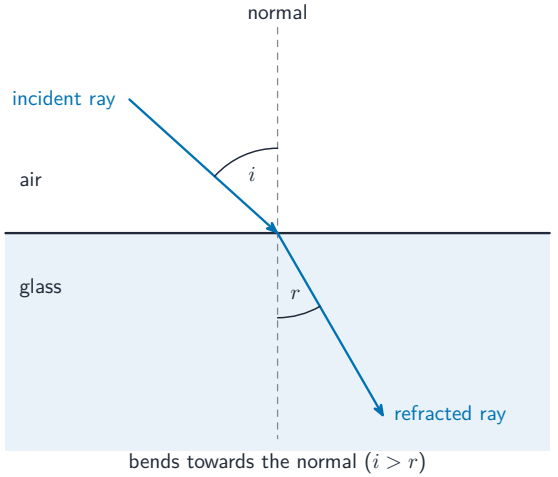
Light bends when it passes between materials because its speed changes – **refraction** 折射. Each material has an **index of refraction** 折射率 $n = \dfrac{c}{v}$ (how much it slows light). **Snell's law** 斯涅尔定律:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

At oblique incidence, light entering a higher-index **medium** 介质 bends **toward** the normal. Beyond the **critical angle** 临界角, light going from a higher to a lower refractive index reflects entirely – **total internal reflection** 全反射, used in **optical fibres** 光纤.

Since $n = c/v$, a larger index means a slower speed. A ray that hits a boundary **along the normal** ($\theta_1 = 0$) slows down but does not bend. At exactly the critical angle the refracted ray travels along the surface ($\theta_2 = 90°$).

Partial reflection. At an air–glass boundary below the critical angle, light generally partly reflects and partly refracts. So a laser pointed at a glass block makes a bright refracted spot and a weaker reflected spot. If the angle inside the glass passes the critical angle, the refracted spot disappears and the reflected spot gets brighter, because all the light now reflects.



Light refracts, bending towards the normal as it enters glass

Worked example. A ray in air ($n_1 = 1.00$) strikes water ($n_2 = 1.33$) at 40° to the normal. By Snell's law,

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.00}{1.33}\sin 40° = 0.483 \;\Rightarrow\; \theta_2 = 29°.$$

The ray bends toward the normal, as expected going into the denser medium.

Worked example (critical angle). For light trying to leave glass ($n = 1.50$) for air, total internal reflection begins at the critical angle θ_c where the refracted ray grazes the surface:

$$\sin\theta_c = \frac{1}{n} = \frac{1}{1.50} = 0.667 \;\Rightarrow\; \theta_c = 42°.$$

Any ray hitting the inside surface at more than 42° to the normal is totally internally reflected – the reason optical fibres carry light for kilometres.

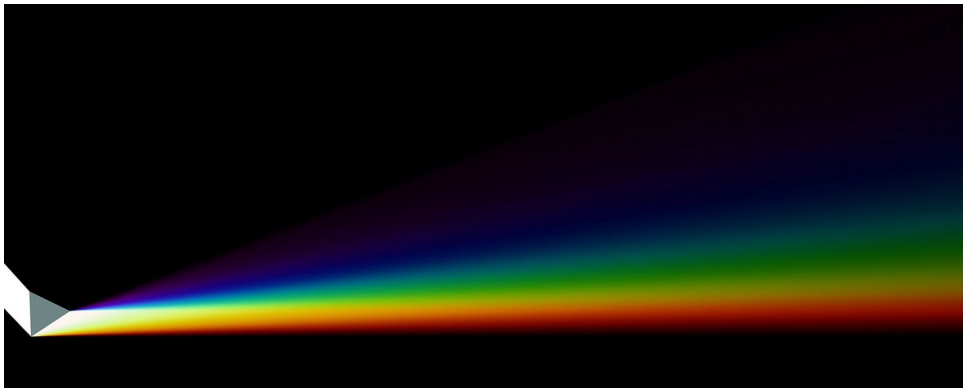
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Worked example (parallel layers). Light from air enters a tank where sugar water forms flat layers, with n rising from 1.35 at the top to 1.45 at the bottom. It enters at 50°. At every flat boundary $n \sin\theta$ is passed on unchanged, so it stays the same all the way down:

$$n_{\text{air}} \sin 50^\circ = n_{\text{bottom}} \sin\theta_{\text{bottom}} \;\Rightarrow\; \sin\theta_{\text{bottom}} = \frac{1.00 \times 0.766}{1.45} = 0.528, \quad \theta_{\text{bottom}} = 32^\circ.$$

The middle layers do not change the final angle, only the path on the way. The ray bends more toward the normal at each step down, so its path curves.

What changes, and what does not. Entering glass, light slows to $v = c/n$ and its **wavelength** 波长 shrinks to $\lambda_n = \lambda/n$. The **frequency** 频率 stays the same: it is set by the source, and the waves cannot pile up at the boundary. So the colour of light, which depends on frequency, does not change inside the glass.



Refraction bends each colour by a different amount, so a prism spreads white light into a rainbow

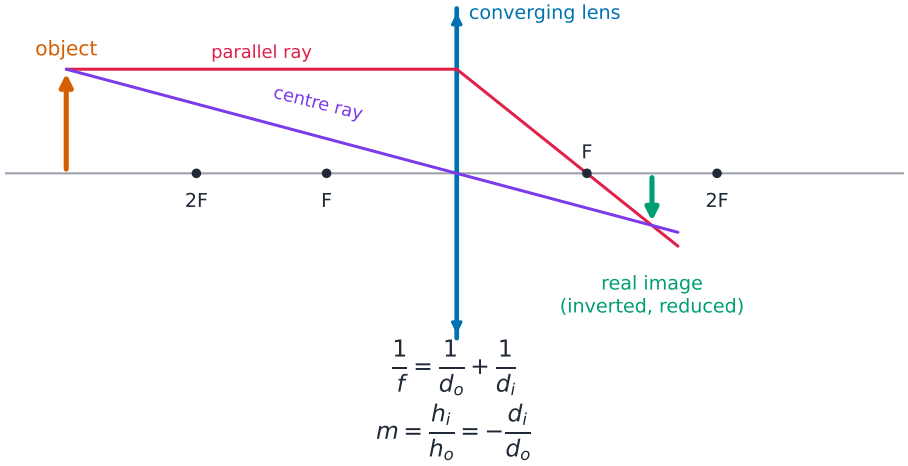
Images Formed by Lenses

A **lens** 透镜 bends light through refraction. The **thin-lens equation** 薄透镜方程 has the same form as the mirror equation:

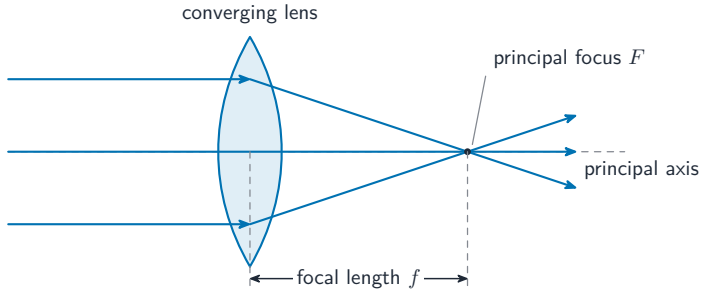
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \qquad m = -\frac{d_i}{d_o}.$$

A **converging** 会聚 (convex) lens can form a real, inverted image or, for a close object, a virtual, upright, enlarged image (a magnifying glass). A **diverging** 发散 (concave) lens always forms a virtual, upright, reduced image. **Ray diagrams** 光路图 locate the image quickly using two easy rays: one parallel to the axis that bends to pass through the far focus, and one straight through the centre of the lens (**undeviated** 不偏折). A lens has a focal point on **each** side, at the same distance f, because light can pass through it either way.

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Two rays locate the image: parallel-then-focus, and straight through the centre



A converging lens brings parallel rays to its principal focus

Worked example. An object sits 30 cm from a converging lens of **focal length** 焦距 $f = 10$ cm. Find the image. From the thin-lens equation,

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30} \;\Rightarrow\; d_i = 15 \text{ cm},$$

and the magnification is $m = -d_i/d_o = -15/30 = -0.5$. The image is real (positive d_i), inverted (negative m), and half the object's size – just what a camera lens does.

Worked example (a lens lab). A student puts a candle 20 cm from a lens and finds a sharp image on a screen 30 cm away on the other side.

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{30} = \frac{5}{60} \;\Rightarrow\; f = 12 \text{ cm}, \qquad m = -\frac{30}{20} = -1.5.$$

The image is real (it is on a screen), inverted and 1.5 times larger. With several measurements, plot $\frac{1}{d_i}$ against $\frac{1}{d_o}$: the points lie on a straight line of gradient -1, and both intercepts equal $\frac{1}{f}$.

A lens's surroundings matter. A lens bends light because its index differs from that of the medium around it. Put a glass lens in water and the difference is smaller, so it bends light less: its focal length gets longer, and the image of a distant object forms farther away. Bending depends on the index *difference*, not just the glass. An air bubble in water is a convex shape with a *lower* index than its surroundings, so it acts as a **diverging** lens. A prism bends light more in air than in a liquid, for the same reason.



A convex lens forms a real, inverted image of a distant scene, exactly as the ray diagram predicts

Signs decide the answer

Every mirror and lens problem uses the same equation, so the **signs** carry the physics.

Quantity	Positive	Negative
f	converging lens, concave mirror	diverging lens, convex mirror
d_i	real image (can be projected)	virtual image (cannot be projected)
m	upright image	inverted image
$\lvert m\rvert$	> 1: enlarged	< 1: reduced

Worked example (a magnifying glass). An object sits 6.0 cm from a converging lens with $f = 10$ cm, inside the focal length.

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{6.0} = -\frac{1}{15} \;\Rightarrow\; d_i = -15 \text{ cm}, \qquad m = -\frac{-15}{6.0} = +2.5.$$

The image is virtual (negative d_i, on the same side as the object), upright and enlarged 2.5 times: a magnifier. Move the object outside f and d_i turns positive: the image becomes real and inverted, as in the camera example above.

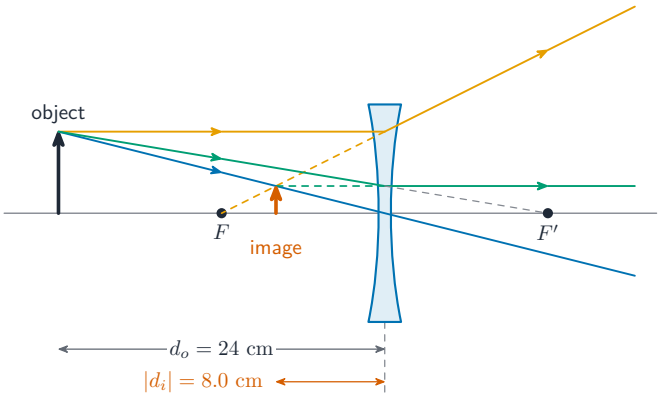
Worked example (two mirrors). A concave mirror has $f = +20$ cm and an object 60 cm away: $\frac{1}{d_i} = \frac{1}{20} - \frac{1}{60} = \frac{1}{30}$, so $d_i = +30$ cm and $m = -0.5$, a real, inverted, half-size image. A convex security mirror has $f = -15$ cm and a shopper 30 cm away: $\frac{1}{d_i} = -\frac{1}{15} - \frac{1}{30} = -\frac{1}{10}$, so $d_i = -10$ cm and $m = +0.33$. The image is virtual, upright and small, which is why the whole aisle fits in the mirror.

Worked example (a diverging lens). An object stands 24 cm from a diverging lens with $f = -12$ cm.

$$\frac{1}{d_i} = \frac{1}{-12} - \frac{1}{24} = -\frac{3}{24} \;\Rightarrow\; d_i = -8.0 \text{ cm}, \qquad m = -\frac{-8.0}{24} = +0.33.$$

The image is virtual, upright and one-third the size, 8.0 cm from the lens on the object's side. Whatever the object distance, d_i stays negative and $|d_i| < |f|$: a diverging lens always makes a small, upright, virtual image between the lens and its focal point.

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Behind a diverging lens the rays spread out; trace them back (dashed) and they meet at a small, upright, virtual image

The ray diagram checks the algebra: the image sits a third of the way from the lens to the object, and it is a third of the object's height.

Practice

Answer each before reading the answers below. Take $c = 3.0 \times 10^8$ m/s.

1. Light passes from air into glass ($n = 1.5$) at 30° to the normal. Find the angle of refraction.
2. Find the speed of light in that glass.
3. Diamond has $n = 2.42$. Find its critical angle with air, and explain why diamonds sparkle.
4. An object is 20 cm from a converging lens with $f = 15$ cm. Find the image distance and magnification, and describe the image.
5. A student says: "A diverging lens can project an image onto a screen if the object is placed carefully." Correct the student.
6. A plane mirror is fixed to a wall. You stand 1.5 m in front of it. How far are you from your image, and is the image real or virtual?
7. A concave mirror has radius of curvature $R = 40$ cm. An object stands 60 cm away. Find f, the image distance and the magnification.
8. A glass lens has $f = 20$ cm in air. It is put under water. Will its focal length increase, decrease or stay the same? Explain.

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Answers

1. $\sin\theta_2 = \dfrac{1.0}{1.5}\sin 30° = 0.333$, so $\theta_2 = 19.5°$, toward the normal.
2. $v = \dfrac{c}{n} = \dfrac{3.0\times10^8}{1.5} = 2.0\times10^8$ m/s.
3. $\sin\theta_c = \dfrac{1}{2.42} = 0.413$, so $\theta_c = 24°$. Such a small critical angle means most light that enters is totally internally reflected several times before it escapes, so diamonds flash brightly.
4. $\dfrac{1}{d_i} = \dfrac{1}{15} - \dfrac{1}{20} = \dfrac{1}{60}$, so $d_i = +60$ cm and $m = -3$: real, inverted and three times larger, as in a projector.
5. A diverging lens has negative f, so d_i is negative for every real object: the image is always virtual and cannot be formed on a screen.
6. The image is 1.5 m behind the mirror, so you are 3.0 m from it. It is virtual: no light actually reaches the point behind the mirror.
7. $f = R/2 = 20$ cm. $\frac{1}{d_i} = \frac{1}{20} - \frac{1}{60} = \frac{2}{60}$, so $d_i = +30$ cm and $m = -0.5$: real, inverted, half size.
8. It increases. Water's index is closer to glass's, so light bends less at each surface and the rays meet farther from the lens.

Exam tips

- Measure all angles from the **normal**, not the surface.
- Light entering a **denser** medium (larger n) bends **toward** the normal; use Snell's law $n_1\sin\theta_1 = n_2\sin\theta_2$.
- Total internal reflection happens only going into a **less** dense medium, beyond the critical angle ($\sin\theta_c = n_2/n_1$, with $n_1 > n_2$).
- For mirrors and lenses use $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ and $m = -d_i/d_o$: a **positive** d_i is a real image, negative is virtual.
- $n = c/v$, so a bigger index means a slower speed of light in the material.

Common mistakes

- Measuring angles from the surface in Snell's law. Use the angle to the normal; if the question gives the angle to the surface, subtract it from 90°.
- Dropping the minus sign of a diverging lens or convex mirror focal length. Put $f < 0$ into the equation, or the image comes out real.
- Forgetting to invert at the end: $\dfrac{1}{d_i} = \dfrac{2}{30}$ means $d_i = 15$ cm, not 0.067 cm.
- Saying the frequency (colour) of light changes in glass. Speed and wavelength drop by the factor n; frequency does not change.

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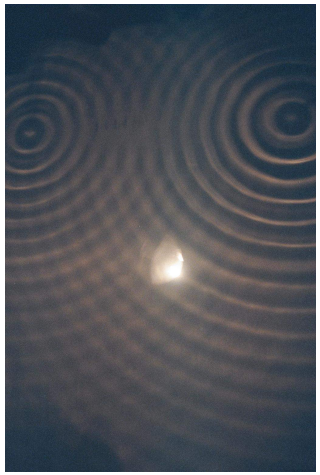
- Expecting total internal reflection when light goes from air into glass. It needs light travelling toward a *lower* index, at more than the critical angle.
- Drawing the ray through the centre of a lens with a bend. It goes straight on; only the parallel ray and the focal ray bend.
- Using a single "n" in Snell's law when the lens or prism is in water. The bending depends on both indices; a lens in water has a longer focal length than in air.
- Calling a plane-mirror image real. It is virtual: it sits behind the mirror where no light goes, at the same distance as the object is in front.

Waves, Sound, and Physical Optics

AP Physics 2

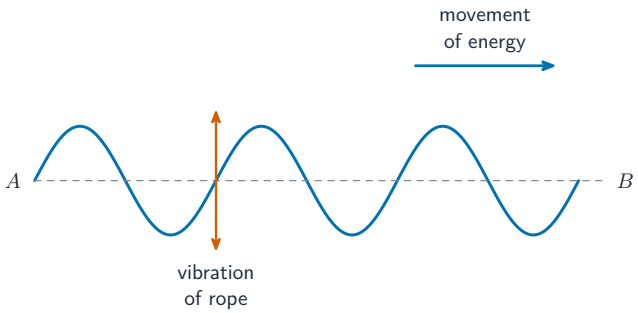
Properties of Wave Pulses and Waves

A guitar string, a microwave oven, polarized sunglasses and the colours on a soap bubble all work by the same few wave ideas. A wave carries energy from place to place. When two waves meet they simply add. This unit builds from a single pulse on a rope to the bright and dark bands that proved light is a wave.



Ripple-tank interference: wave paths add constructively and destructively

A **wave** 波 carries **energy** through a medium (or space) without carrying the matter along. A single travelling disturbance is a **pulse**; a repeating disturbance forms a periodic wave. Two types:



A transverse wave: the medium moves at right angles to the wave's travel

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- **Transverse** 横波: oscillations are perpendicular to the wave's travel. Rope particles move; for light it is the electric and magnetic fields that oscillate.
- **Longitudinal** 纵波: the medium moves along the direction of travel (sound).

A **wave pulse** 波脉冲 is one single disturbance, such as one flick of a rope. Both pulses and waves move energy, not matter: a cork on a pond bobs up and down as ripples pass, but it does not travel with them. **Mechanical waves** 机械波 (on strings, in water, sound) need a medium. Electromagnetic waves do not: they cross empty space at $c = 3.00 \times 10^8$ m/s.

What sets the speed. The medium does. On a string the speed rises with the tension F_T and falls with the **linear mass density** 线密度 μ (mass per length):

$$v = \sqrt{\frac{F_T}{\mu}}.$$

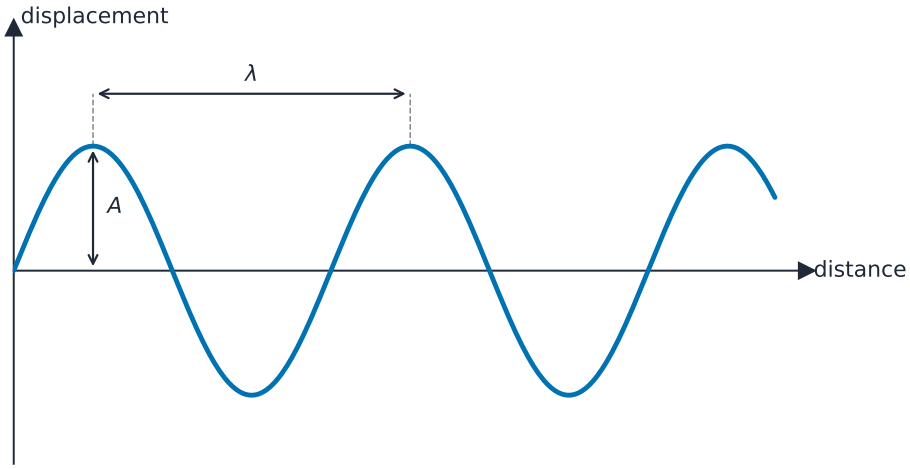
A tight, light string carries waves fast. Sound travels faster in warmer air, because the molecules move faster.

> **Worked example.** A string has $\mu = 5.0 \times 10^{-3}$ kg/m and tension 80 N: $v = \sqrt{80/0.0050} = \sqrt{16\,000} = 126$ m/s. Making the tension four times larger doubles the speed.

Sound as a pressure wave. Sound is a longitudinal mechanical wave. Air molecules crowd together in **compressions** 密部 (high pressure) and spread apart in **rarefactions** 疏部 (low pressure). The amplitude of a sound wave is the largest rise (or fall) in pressure from normal air pressure. A larger amplitude sounds **louder** and carries more energy.

Periodic Waves

A repeating wave is described by:



A displacement-distance graph shows the amplitude and wavelength

- **wavelength** 波长 λ (distance between repeats),
- **frequency** 频率 f (cycles per second) and **period** $T = 1/f$,
- **amplitude** 振幅 A (maximum displacement – related to energy),
- **wave speed** 波速 $v = f\lambda$, set by the **medium**, not the source.

> **Worked example.** A musical note has frequency 340 Hz and the speed of sound is 340 m/s. Its wavelength is $\lambda = v/f = 340/340 = 1.0$ m. If the same note travels into water (where sound moves at ≈ 1500 m/s) the **frequency stays** 340 **Hz** but the wavelength stretches to $1500/340 \approx 4.4$ m – the source sets the frequency, the medium sets the speed and hence the wavelength.

Separate properties. The amplitude does not depend on the period or the frequency: you can pluck the same note softly or loudly. The frequency of a sound sets its **pitch** 音调 (higher frequency, higher note), and its amplitude sets how loud it is. For mechanical waves in the same medium with the same **displacement** amplitude, the one with the higher frequency carries more energy. This does not apply to light at fixed electric-field amplitude, or sound at fixed pressure amplitude.

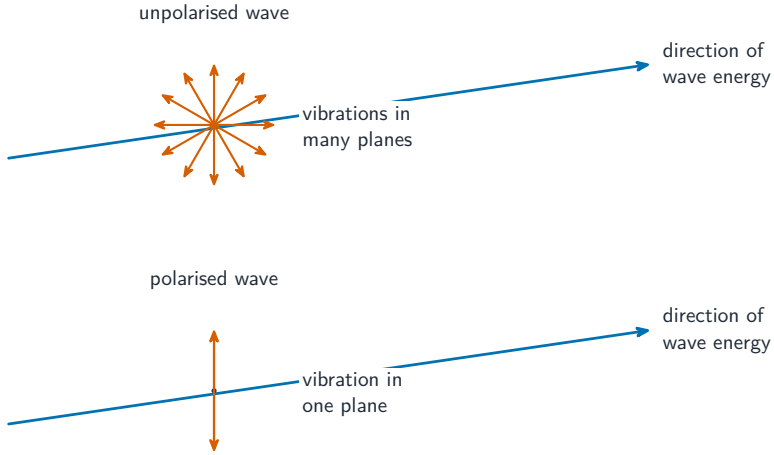
Two graphs, two equations. A sinusoidal wave can be written two ways. At one fixed **place**, the displacement changes with time: $y = A\cos\left(\frac{2\pi t}{T}\right)$. At one fixed **time**, the displacement changes along the wave: $y = A\cos\left(\frac{2\pi x}{\lambda}\right)$. Read the period T from a displacement-time graph and the wavelength λ from a displacement-position graph; check the axis label before reading either.

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Boundary Behavior and Polarization

At a boundary a wave can partly **reflect** and partly **transmit**. For two ropes under the same tension, a displacement pulse going from a light rope into a heavy rope (where it slows down) reflects **inverted**. A pulse going from a heavy rope into a light rope (where it speeds up) reflects **upright**. The transmitted pulse is never inverted, and its frequency never changes at a boundary. **Polarization** 偏振 applies to transverse waves only: a polarizer passes just one direction of oscillation, which is why polarized sunglasses cut glare. Light that reflects off water or a road is partly polarized (mostly horizontal), so sunglasses that pass only vertical oscillations block most of the glare. Longitudinal waves, such as sound, cannot be polarized: the oscillation is already along the direction of travel.

The effect of a polarizer is measured through **intensity** 强度 - the average power a wave delivers **per unit area** over one period, in W m^{-2}. Because a polarizer removes the components of the oscillation that are not aligned with it, it **reduces the wave's intensity**: unpolarized light passing a single ideal polarizer drops to **half** its intensity, and a second polarizer at an angle cuts it further. Intensity also falls with distance from a source, since the same power spreads over a larger area.



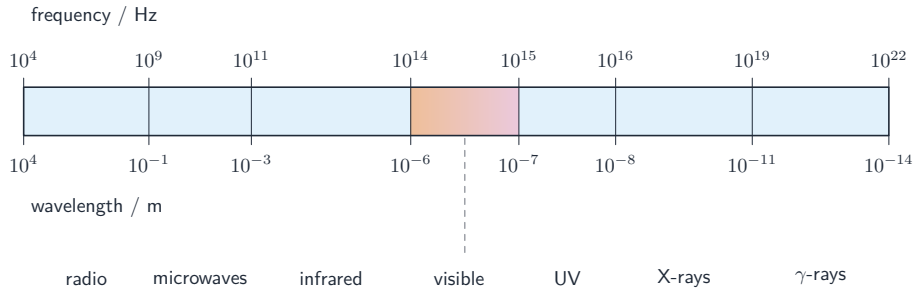
Unpolarised waves vibrate in many planes; a polarised wave vibrates in one

Electromagnetic Waves

Electromagnetic waves 电磁波 are oscillating electric and magnetic fields that travel through vacuum at the speed of light c, needing no medium. They span the spectrum from radio to gamma rays; higher frequency means shorter wavelength and higher photon energy.

The electric field and the magnetic field are perpendicular to each other **and** to the direction of travel, so electromagnetic waves are transverse. Far from the source the wavefronts are flat, so they are treated as **plane waves** 平面波. In order of **decreasing** wavelength the spectrum runs: radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays. Visible light runs from red (about 700 nm) through orange, yellow, green and blue to violet (about 400 nm).

> **Worked example.** Wi-Fi uses $f = 2.4$ GHz: $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}}{2.4 \times 10^{9}} = 0.125$ m, about 12 cm, in the microwave band.

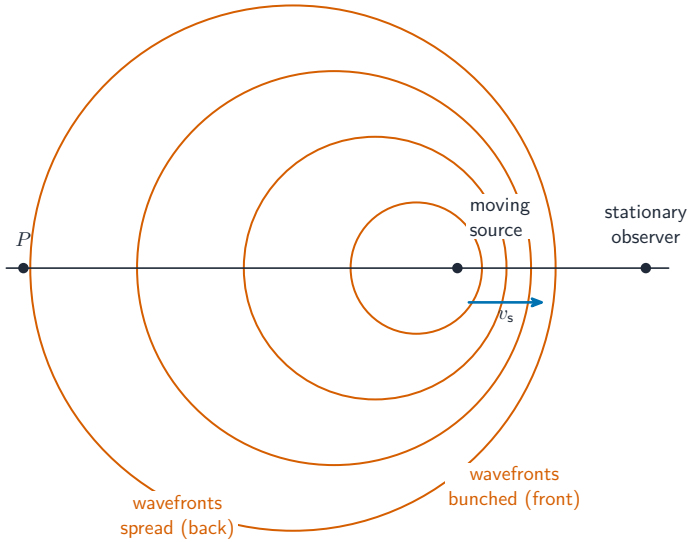


The electromagnetic spectrum, from radio waves to gamma rays

The Doppler Effect

The **Doppler effect** 多普勒效应 is the change in observed frequency when a wave source and observer move relative to each other. Approaching ⇒ higher frequency (shorter wavelength); receding ⇒ lower frequency. It explains a passing siren's drop in pitch and the redshift of receding galaxies.

The frequency the source actually emits is its **rest frequency**. The faster the source and observer approach (or separate), the bigger the difference between the observed frequency and the rest frequency. If source and observer move with the **same** velocity, there is no relative motion and no shift. The wave speed in the medium does not change. Source motion changes the spacing of the wavefronts; observer motion changes how quickly the observer meets them.

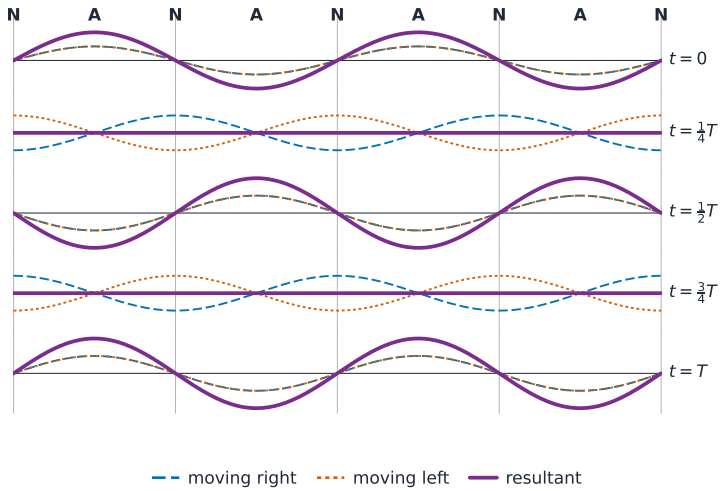


A moving source squashes the wavefronts ahead of it, raising the observed frequency

Wave Interference and Standing Waves

When waves overlap they **superpose** (add). Two pulses meeting on a rope do not bounce off each other: they pass through each other. While they overlap, the displacement at each point is the sum of the two displacements (**superposition** 叠加). Afterwards each pulse carries on unchanged. **Constructive interference** 相长干涉 (crests aligned) gives a bigger wave; **destructive interference** 相消干涉 (crest on trough) cancels. Two waves of the same frequency and speed, with equal amplitudes and opposite directions, can form a **standing wave** 驻波 with fixed **nodes** 波节 (no motion) and **antinodes** 波腹 (maximum motion) – the basis of resonance on strings and in pipes.

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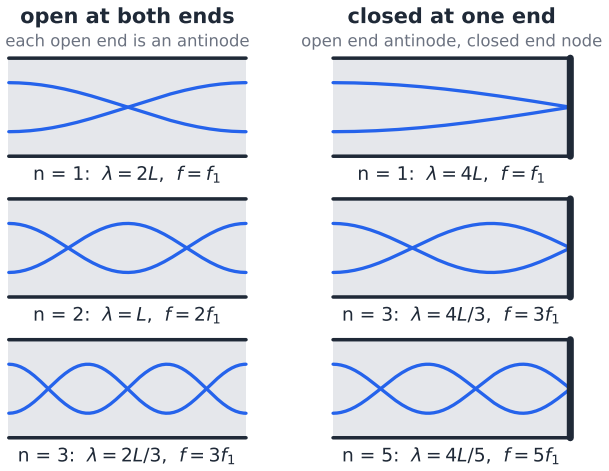
A standing wave forms where two waves travelling in opposite directions overlap

> **Worked example.** A guitar string 0.65 m long is fixed at both ends. Its **fundamental** (first harmonic) fits half a wavelength between the ends, so $\lambda = 2L = 1.30$ m. If waves travel along the string at 260 m/s, the note's frequency is $f = v/\lambda = 260/1.30 = 200$ Hz. Shortening the string (a fret) raises the pitch.

Standing waves in pipes. Air in a pipe can hold standing sound waves too. An open end is always a displacement **antinode**; a closed end is always a **node**. The longest wavelength that fits is the **fundamental**, or first **harmonic** 谐波, and each allowed wavelength gives one harmonic:

- string fixed at both ends, or pipe open at both ends: $\lambda_n = \dfrac{2L}{n}$, $f_n = nf_1$ for $n = 1, 2, 3, \ldots$
- pipe closed at one end: $\lambda_n = \dfrac{4L}{n}$, $f_n = nf_1$ for **odd** $n = 1, 3, 5, \ldots$ only.

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An open end is always an antinode and a closed end a node; a pipe closed at one end has only odd harmonics

> **Worked example.** A pipe 0.85 m long is closed at one end; sound travels at 340 m/s. The fundamental has $\lambda = 4L = 3.4$ m, so $f_1 = 340/3.4 = 100$ Hz. The next possible frequencies are 300 Hz and 500 Hz: 200 Hz cannot form, because a closed end must be a node. Open the closed end and $\lambda = 2L = 1.7$ m, so $f_1 = 200$ Hz.

When two waves of **slightly different** frequency overlap, they drift in and out of step, so the combined sound swells loud and soft in a slow throb called **beats** 拍. The **beat frequency** 拍频 is simply the difference of the two frequencies:

$$f_{\text{beat}} = |f_1 - f_2|.$$

Two tuning forks of 256 Hz and 260 Hz sounded together give $|260 - 256| = 4$ beats each second; a musician tunes an instrument by slowing these beats towards zero against a reference note.

Diffraction

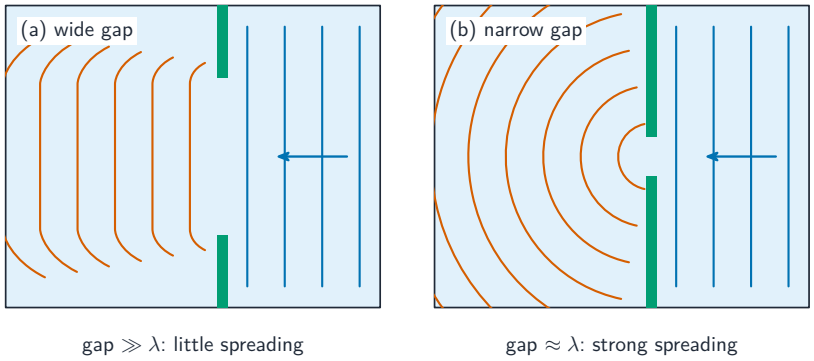
Diffraction 衍射 is the bending and spreading of waves around edges or through openings. The spreading is significant when the opening is comparable to the wavelength – so sound (long wavelength) bends around doorways easily, while light (tiny wavelength) needs a very narrow slit.

The single slit. Light through one narrow slit of width a still makes bright and dark bands, because waves from different parts of the slit interfere. The path difference

between waves from the two edges is $\Delta D = a\sin\theta$. Dark bands appear where

$$a\sin\theta = m\lambda \quad (m = 1, 2, 3, \dots), \qquad y_{\min} \approx \frac{m\lambda L}{a} \text{ (small angles)}.$$

The central bright band is twice as wide as the others. A **narrower** slit gives a **wider** pattern. The pattern also depends on the opening's shape: a round hole gives rings.



Waves spread out (diffract) as they pass through a gap

Double-Slit Interference and Diffraction Gratings

Coherent 相干 light through two closely spaced slits produces a pattern of bright and dark fringes. Bright fringes occur where the path difference is a whole number of wavelengths:

$$d\sin\theta = m\lambda.$$

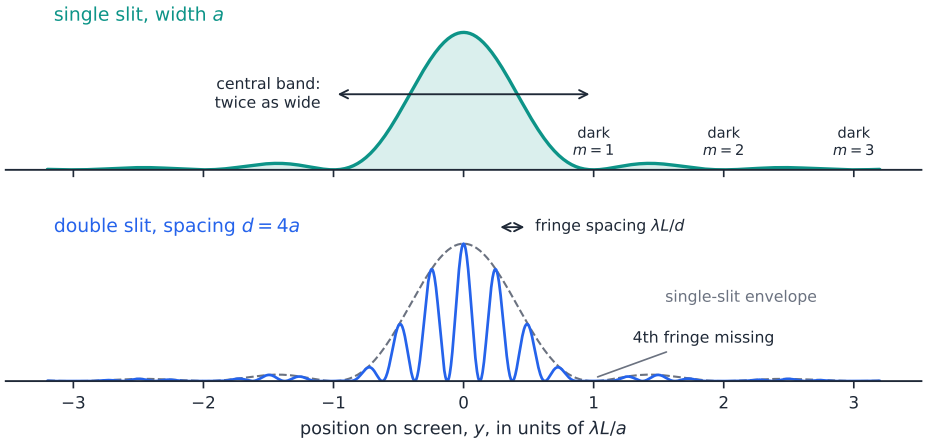
A **diffraction grating** 衍射光栅 has many slits, giving sharp bright lines – useful for separating light into its wavelengths.

For small angles ($\theta < 10°$), $\sin\theta \approx \tan\theta = y/L$, so the bright fringes sit at

$$y_{\max} \approx \frac{m\lambda L}{d}.$$

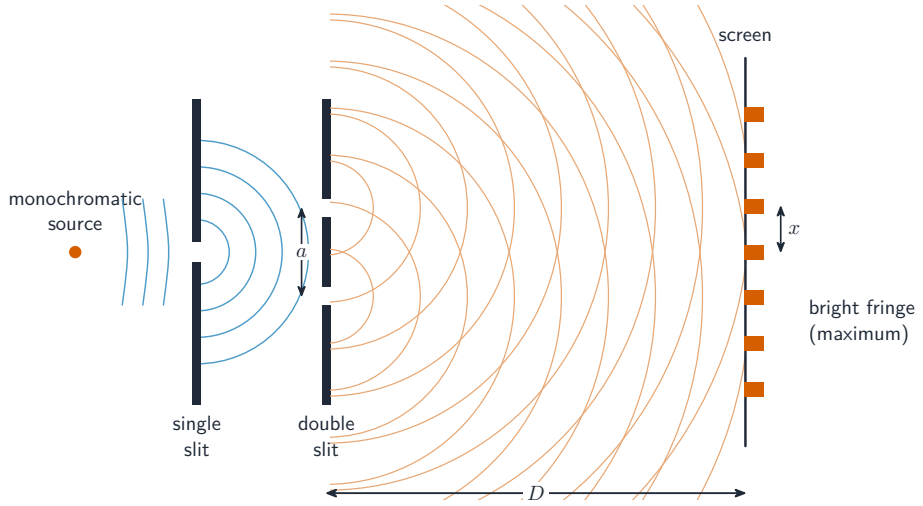
They are evenly spaced, $\lambda L/d$ apart. Each slit also diffracts, so the real double-slit pattern is a set of evenly spaced fringes inside the single-slit envelope. The fringes fade away from the centre, and any fringe that lands on a single-slit dark band is missing. Young's double-slit experiment showed this pattern, and it proved that light is a wave.

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The single-slit pattern sets the envelope; the double slit fills it with evenly spaced fringes

With white light on a grating, the central maximum ($m = 0$) is white, because every colour has zero path difference there. In each higher order the colours spread out into a spectrum, with red (longest wavelength) farthest from the centre.



Young's double slits give an interference pattern of bright and dark fringes

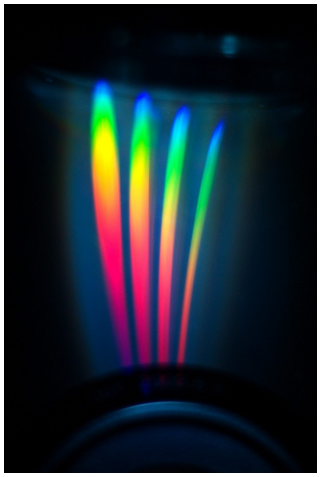
Worked example. Light of wavelength 600 nm passes through two slits 0.20 mm apart. The first bright fringe ($m = 1$) sits at

$$\sin\theta = \frac{m\lambda}{d} = \frac{1 \times 600 \times 10^{-9}}{0.20 \times 10^{-3}} = 3.0 \times 10^{-3} \;\Rightarrow\; \theta = 0.17°.$$

The tiny angle is why the slits must be very close together and the screen far away to see the fringes clearly.

Worked example (red or violet?). A student claims that the first bright band beside the centre is closer to the centre with violet light than with red light. Reason from the path difference, not the formula: at that band the light from one slit travels exactly one wavelength farther than from the other. Violet has the shorter wavelength, so the path difference needed is smaller, and it is reached at a smaller angle, closer to the centre. The claim is correct. The equation agrees: with $\lambda = c/f$, $y_{\max} = \dfrac{mcL}{fd}$, so a higher frequency gives a smaller y.

Worked example (slits under water). Put the whole double-slit set-up under water ($n = 1.33$). The frequency stays the same, but the light slows to c/n, so the wavelength becomes λ/n. Each fringe needs a smaller path difference, so the fringes move closer together, by the factor 1.33. A liquid with a larger index squeezes them even closer.



The fine tracks on a CD act as a diffraction grating, splitting white light into a spectrum

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Thin-Film Interference

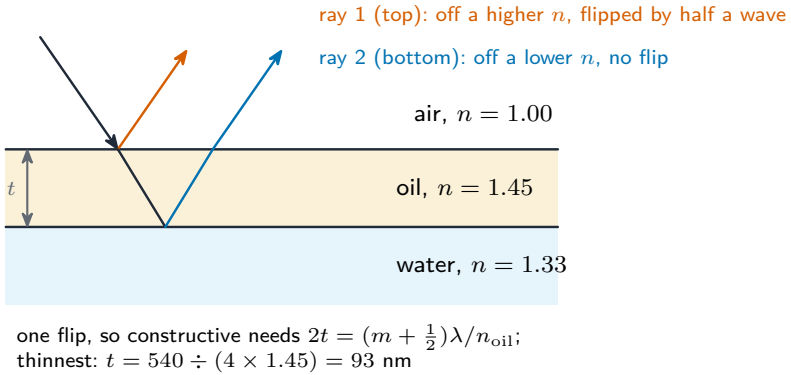
Light reflecting off the top and bottom of a **thin film** 薄膜 (soap bubble, oil slick) interferes with itself. Depending on the film's thickness and a possible half-wavelength phase flip on reflection off a denser medium, particular wavelengths interfere constructively – producing the shifting colours you see.

The phase rules (near-normal incidence, transparent materials). A reflection off a material with a **higher** index of refraction flips the wave by 180° (half a wavelength). A reflection off a **lower** index causes no flip. Transmission across the boundary adds no phase flip; propagation through the film adds phase as the wave travels. The ray that goes down and back through a film of thickness t travels an extra $2t$ (at near-normal incidence), and inside the film the wavelength is λ/n_{film}.

Worked example (oil on water). An oil film ($n = 1.45$) floats on water ($n = 1.33$) and looks green ($\lambda = 540$ nm in air). The top reflection (air to oil) flips; the bottom reflection (oil to water, a lower index) does not. One flip means the two reflected waves are already half a wavelength out of step, so constructive interference needs

$$2t = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_{\text{oil}}}.$$

The thinnest such film ($m = 0$) is $t = \dfrac{\lambda}{4n_{\text{oil}}} = \dfrac{540}{4 \times 1.45} = 93$ nm. Other colours are reinforced where the oil is a different thickness, which is why an oil slick shows bands of colour.



One phase flip (at the top surface only), so green light is reinforced when 2t is half a wavelength in the oil: t = 93 nm

The same physics is put to work in an **antireflection coating** 增透膜 on camera lenses, glasses, and solar cells: a thin transparent layer whose two reflected waves

interfere **destructively**, so almost no light reflects and more is transmitted. For ideal non-absorbing materials at normal incidence, complete cancellation at one chosen vacuum wavelength λ_0 requires a film index $n_f = \sqrt{n_{\text{air}} n_{\text{glass}}}$ and thickness $\lambda_0/(4n_f)$, one **quarter of a wavelength** inside the film. Other wavelengths and incidence angles can still reflect some light.

Practice

Answer each before reading the answers below.

1. A wave on a string has frequency 50 Hz and wavelength 0.80 m. Find its speed.
2. A string 0.90 m long, fixed at both ends, carries waves at 180 m/s. Find the frequency of its fundamental and of its third harmonic.
3. A 440 Hz tuning fork and a guitar string sound together and give 3 beats per second. When the string is tightened slightly, the beats speed up to 5 per second. What was the string's frequency before it was tightened?
4. Light of wavelength 500 nm falls on a grating with 600 lines per millimetre. Find the angles of the first-order and second-order bright lines, and the highest order that can be seen.
5. A student says: "A grating with more lines per millimetre squeezes the bright lines closer together." Correct the student.
6. A pipe 0.50 m long is closed at one end. Sound travels at 340 m/s. Find its fundamental frequency and the next two frequencies it can produce.
7. Light of wavelength 500 nm falls on two slits 0.25 mm apart, and the screen is 2.0 m away. Find the spacing of the bright fringes. What is the spacing if the whole apparatus is put under water ($n = 1.33$)?
8. A soap film ($n = 1.33$) in air is 100 nm thick. Which wavelength (in air) is most strongly reflected?

Answers

1. $v = f\lambda = 50 \times 0.80 = 40$ m/s.
2. $\lambda_1 = 2L = 1.80$ m, so $f_1 = \dfrac{180}{1.80} = 100$ Hz. The third harmonic is $3f_1 = 300$ Hz.
3. The string was at $440 \pm 3 = 437$ or 443 Hz. Tightening raises its frequency, and the beats got faster, so it moved **away** from 440 Hz: it was 443 Hz.
4. $d = \dfrac{1}{600}$ mm $= 1.67 \times 10^{-6}$ m. First order: $\sin\theta = \dfrac{500 \times 10^{-9}}{1.67 \times 10^{-6}} = 0.30$, $\theta = 17°$. Second order: $\sin\theta = 0.60$, $\theta = 37°$. Since $\sin\theta \le 1$, $m \le d/\lambda = 3.3$, so the third order ($\sin\theta = 0.90$, $64°$) is the highest.
5. More lines per millimetre means a **smaller** spacing d. From $d\sin\theta = m\lambda$, a smaller d gives a larger $\sin\theta$, so the bright lines spread **farther** apart (and fewer orders fit).

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6. $\lambda_1 = 4L = 2.0$ m, so $f_1 = 170$ Hz. Only odd harmonics form: $3f_1 = 510$ Hz and $5f_1 = 850$ Hz.
7. $\Delta y = \dfrac{\lambda L}{d} = \dfrac{500 \times 10^{-9} \times 2.0}{0.25 \times 10^{-3}} = 4.0 \times 10^{-3}$ m $= 4.0$ mm. Under water the wavelength is $500/1.33 = 376$ nm, so the spacing is $4.0/1.33 = 3.0$ mm.
8. Air to soap flips the wave; soap to air does not. One flip, so constructive needs $2t = \left(m + \frac{1}{2}\right)\dfrac{\lambda}{n}$. With $m = 0$: $\lambda = 4nt = 4 \times 1.33 \times 100 = 532$ nm, green.

Exam tips

- Use $v = f\lambda$; the **source sets the frequency**, and when a wave enters a new medium the frequency stays fixed while the speed and wavelength change.
- When a question says "without manipulating equations", argue from the **path difference**: a bright band needs a path difference of $m\lambda$, so a shorter wavelength gives bands closer to the centre.
- Distinguish **transverse** (vibration perpendicular, can be polarised) from **longitudinal** (vibration along the travel, e.g. sound).
- For sources in phase, **constructive** interference needs a path difference of a whole number of wavelengths; **destructive** needs a half-odd number.
- On a string fixed at both ends the fundamental fits **half a wavelength** ($\lambda = 2L$).
- **Diffraction** is strong when the gap is comparable to or smaller than the wavelength – sound bends round doorways, light needs a very narrow slit.
- **Beats**: two waves of *slightly different* frequency throb at $f_{\text{beat}} = |f_1 - f_2|$ (e.g. two tuning forks; tuning to zero beats).
- An **antireflection coating** is a quarter-wavelength-thick layer (index between air and glass) whose two reflections cancel, cutting the reflected light.

■ Common mistakes

- Saying the frequency of a wave changes when it enters a new medium. The frequency is set by the source; the speed and the wavelength change.
- Taking the string length as one wavelength for the fundamental. A string fixed at both ends has $L = \lambda/2$ in its fundamental mode.
- Using the slit width in the double-slit equation. $d\sin\theta = m\lambda$ uses the *spacing* between the slits.
- Forgetting the unit prefixes in interference: 1 nm $= 10^{-9}$ m, 1 mm $= 10^{-3}$ m.
- Saying a moving source changes the wave's speed. In the Doppler effect the speed in the medium is unchanged. Source motion changes wavefront spacing; observer motion changes how quickly wavefronts are met.
- Saying sound can be polarized. Only transverse waves can be polarized; sound in air is longitudinal.

- Giving a pipe closed at one end even harmonics. A closed end must be a node, so only odd multiples of f_1 fit.
- In thin films, forgetting the phase flips or using the wavelength in air. Count the flips at the two surfaces first, then use λ/n_{film} inside the film.
- Confusing the single-slit and double-slit formulas. $a\sin\theta = m\lambda$ gives single-slit **dark** bands; $d\sin\theta = m\lambda$ gives double-slit **bright** fringes.

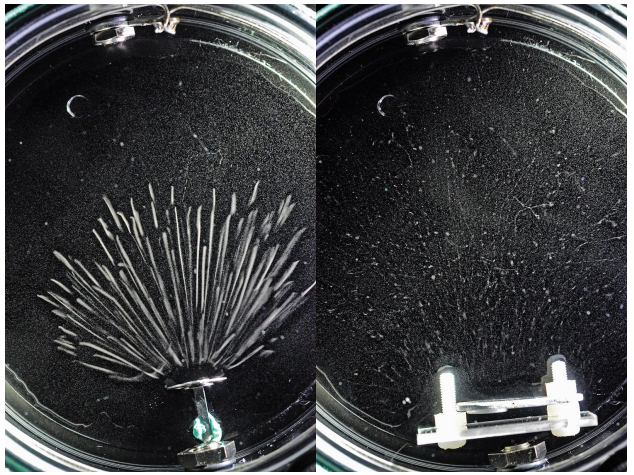
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Modern Physics

AP Physics 2

Quantum Theory and Wave-Particle Duality

Around 1900, three experiments broke classical physics: the colours of glowing hot objects, the sharp lines in the light from gases, and electrons knocked out of metal by light. Each one needed a new idea: energy comes in packets. The same ideas now run solar panels, LEDs, medical scanners and nuclear power stations.



A cloud chamber: modern physics tracks charged particles from radioactive decays

At tiny scales, energy comes in discrete packets called **quanta** 量子. Light is carried by **photons** 光子, each with energy set by its frequency:

$$E = hf,$$

where h is Planck's constant. **Wave–particle duality** 波粒二象性: light and matter each show both wave behavior (interference, diffraction) and particle behavior (photons, electrons as localized hits). A particle also has a **matter wavelength** $\lambda = \dfrac{h}{p}$, and particles genuinely produce interference - a variation of Young's **double-slit experiment** 双缝实验 done with electrons builds up the same fringe pattern as light.

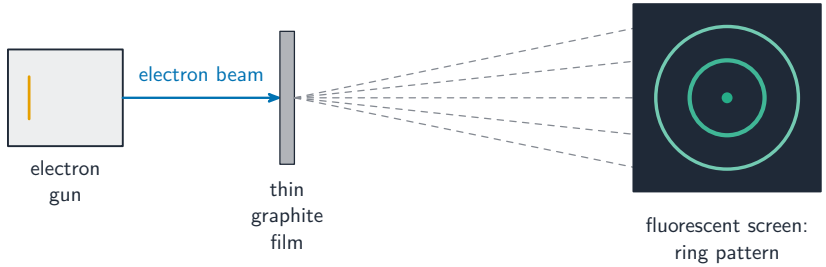
What a photon is. A photon has no mass and no charge, and its energy is proportional to its frequency. It travels in a straight line until it interacts with matter. In empty space every photon travels at $c = 3.00 \times 10^8$ m/s; in a material its speed is c/n, inversely proportional to the index of refraction. A photon also carries momentum, $p = \dfrac{h}{\lambda}$, even though it has no mass.

Matter waves. Every moving particle has a **de Broglie wavelength** 德布罗意波长

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

A smaller momentum means a **longer** wavelength. Quantum theory is needed when this wavelength is about as large as the system: an electron in an atom, yes; a thrown ball, no. For an electron **bound** in an atom, the allowed bound-state energies are **quantized** 量子化. This does not mean that a measurement of its linear momentum can give only discrete values.

> **Worked example.** An electron ($m = 9.11 \times 10^{-31}$ kg) has a wavelength of 5.0 nm. Its speed is
>
> $$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 5.0 \times 10^{-9}} = 1.5 \times 10^{5} \text{ m/s}.$$
>
> A 0.15 kg ball at 40 m/s has $\lambda = \dfrac{6.63 \times 10^{-34}}{0.15 \times 40} = 1.1 \times 10^{-34}$ m, far too small ever to show wave effects.



Electrons form a diffraction pattern, showing that particles have a wave nature

> **Worked example.** Find the energy of a photon of orange light with frequency 5.0×10^{14} Hz ($h = 6.63\times10^{-34}$ J s): $E = hf = 6.63\times10^{-34}\times5.0\times10^{14} = 3.3\times10^{-19}$ J, which is about 2.1 eV (dividing by 1.6×10^{-19}). Visible-light photons carry a few electronvolts – just right to trigger the chemistry of vision and photosynthesis.

The Bohr Model of Atomic Structure

Inside the atom. An atom has a tiny, positive nucleus made of protons and neutrons (together, **nucleons** 核子), with negative electrons around it. Nearly all of its mass is in the nucleus. The number of protons Z decides the element. The total number of nucleons A decides the **isotope** 同位素: carbon-12 and carbon-14 both have $Z = 6$, but 6 and 8 neutrons. Nuclear notation writes this as ${}^{A}_{Z}\mathrm{X}$, for example ${}^{14}_{6}\mathrm{C}$. An atom

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that has gained or lost electrons has a net charge and is called an **ion** 离子.

The **Bohr model** 玻尔模型 pictures electrons orbiting the nucleus only in certain allowed **energy levels** 能级. An electron can jump between levels only by absorbing or emitting a photon whose energy exactly matches the gap:

$$E_{\text{photon}} = |E_{\text{final}} - E_{\text{initial}}|.$$

Because the levels are discrete, only specific photon energies are allowed.

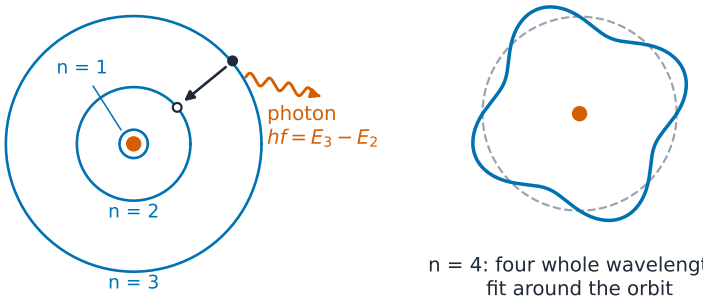
Where the levels come from. Bohr treated the electron as moving in a circle, with the electric force from the nucleus as the centripetal force:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \;\Rightarrow\; v = \sqrt{\frac{ke^2}{mr}}, \qquad K = \tfrac{1}{2}mv^2 = \frac{ke^2}{2r}.$$

Adding the electric potential energy $U = -\dfrac{ke^2}{r}$ gives the total energy of the atom:

$$E = K + U = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}.$$

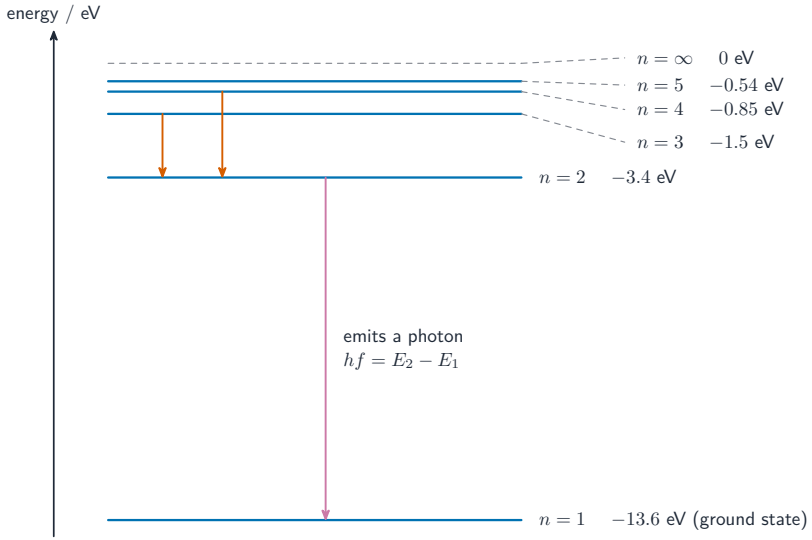
The energy is negative because the electron is bound. A **larger** orbit makes E less negative, so the atom's energy **increases**: when an atom absorbs a photon, the electron moves to a larger orbit. As r grows, K falls and U rises (toward zero) by twice as much. Only certain orbits are allowed because the electron behaves as a standing wave: the circumference must fit a whole number of de Broglie wavelengths, $2\pi r = n\lambda$.



Allowed orbits: a jump between levels emits or absorbs one photon, and an allowed orbit fits a whole number of de Broglie wavelengths

Emission and Absorption Spectra

- An **emission spectrum** 发射光谱 is the set of bright lines given off when electrons drop to lower levels – each line a specific wavelength.
- An **absorption spectrum** 吸收光谱 is the set of dark lines where those same wavelengths are absorbed from a continuous source.



The discrete energy levels of hydrogen produce a line spectrum

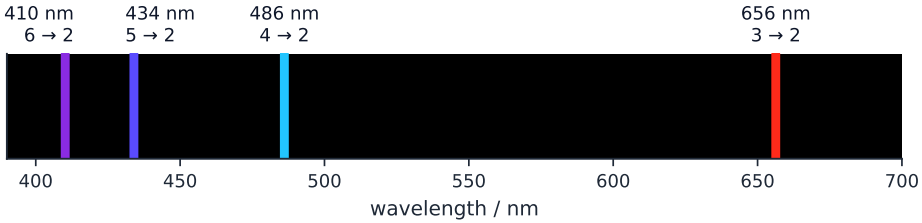
The line pattern is a **fingerprint** of the element, since every element has its own energy levels. An emission spectrum tells you which elements are in a light source, such as a star; an absorption spectrum tells you which elements in a gas absorbed the light.

Ground state and ionization. The lowest level is the **ground state** 基态; higher levels are **excited states** 激发态. The energy needed to remove the electron completely is the **ionization energy** 电离能 (the binding energy of the electron): for hydrogen in its ground state, 13.6 eV. It is largest from the ground state, because that level is farthest below zero.

> **Worked example (red light from hydrogen).** A lamp contains hydrogen atoms in the ground state. To give off red light (656 nm, 1.89 eV), an atom must first gain energy, from a collision or by absorbing a photon, and jump from $n = 1$ to $n = 3$ ($-13.6 \to -1.51$ eV, a gain of 12.1 eV). It then drops from $n = 3$ to $n = 2$ and emits the red photon, $-1.51 - (-3.40) = 1.89$ eV, before falling back to $n = 1$.

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> **Worked example (could a photon add an electron?).** A ground-state hydrogen atom is illuminated by a photon of 3.2×10^{15} Hz: $E = hf = 6.63 \times 10^{-34} \times 3.2 \times 10^{15} = 2.1 \times 10^{-18}$ J (13 eV). Making an electron needs its rest energy $mc^2 = 9.11 \times 10^{-31} \times (3.0 \times 10^8)^2 = 8.2 \times 10^{-14}$ J, about 40 000 times more. Creating an electron alone would also violate charge conservation; electron–positron pair creation needs at least $2mc^2$. This photon cannot create such a pair or ionize ground-state hydrogen (13.6 eV needed). It can excite the atom only if its energy matches an allowed level gap; otherwise it is not absorbed.



Hydrogen's emission spectrum: each bright line is light from an electron dropping between fixed energy levels

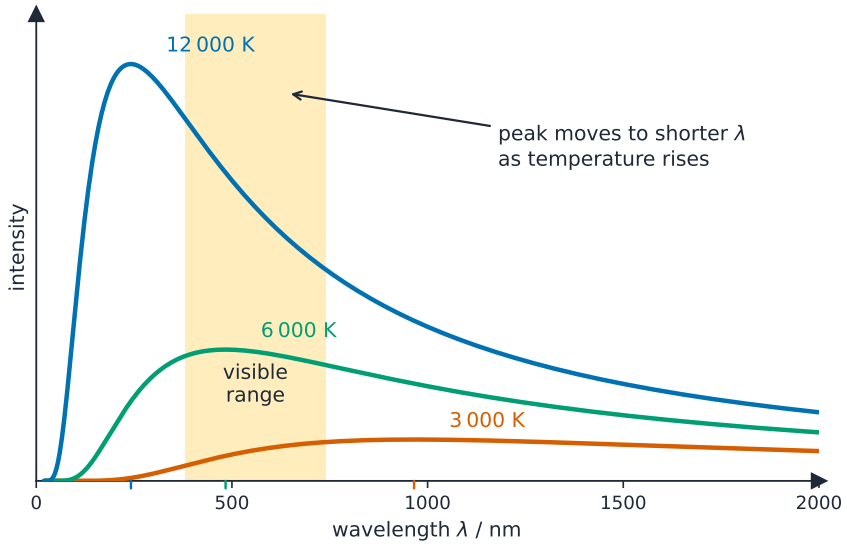
Blackbody Radiation

A **blackbody** 黑体 emits a continuous spectrum that depends only on its temperature. Hotter objects glow brighter and peak at **shorter** wavelengths (red-hot to white-hot to blue-hot). Explaining this spectrum required quantized energy – a founding problem of quantum theory.

Every object turns some of its internal thermal energy into electromagnetic radiation. A **blackbody** is the ideal case: it absorbs all radiation that falls on it, and in thermal equilibrium with its surroundings it emits energy at the same rate as it absorbs it. Classical physics predicted far too much ultraviolet light; Planck fixed this by assuming energy is emitted in packets of hf. Two rules describe the spectrum:

- **Wien's law** 维恩定律: the peak wavelength falls as temperature rises, $\lambda_{\max} T = 2.90 \times 10^{-3}$ m K.
- **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律: the power emitted is proportional to the surface area and to T^4, $P = \sigma A T^4$.

> **Worked example.** The Sun's surface is about 5800 K: $\lambda_{\max} = \dfrac{2.90 \times 10^{-3}}{5800} = 5.0 \times 10^{-7}$ m = 500 nm, in the middle of the visible range. A star with twice the temperature and the same size emits $2^4 = 16$ times the power, peaking at 250 nm (ultraviolet).



A hotter black body radiates more, and its peak shifts to shorter wavelengths

The Photoelectric Effect

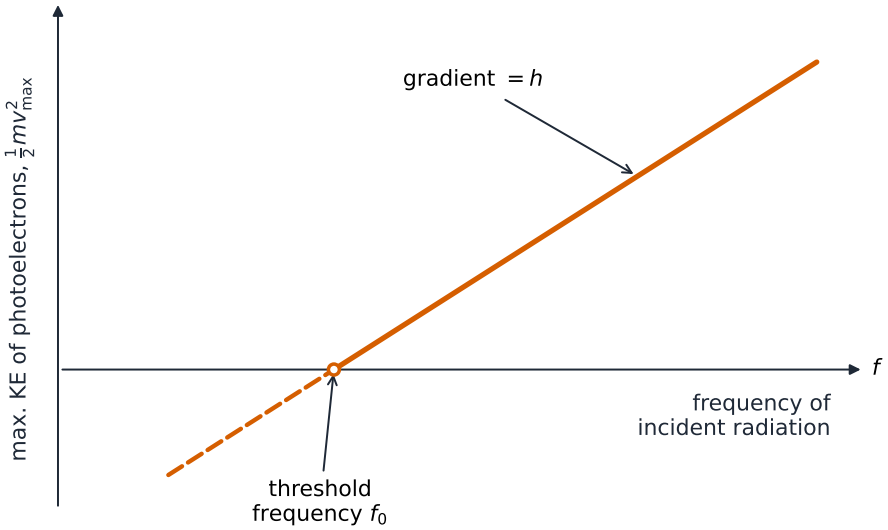
Shining light on a metal can eject electrons – the **photoelectric effect** 光电效应. Key facts (which only the photon picture explains): electrons come out only if the photon's frequency exceeds a threshold, no matter how bright a dimmer, lower-frequency light is. Energy conservation gives

$$K_{\max} = hf - \phi,$$

where ϕ is the metal's **work function** 逸出功 (the energy to free an electron). The lowest frequency that frees electrons is the **threshold frequency** 截止频率 $f_0 = \phi/h$.

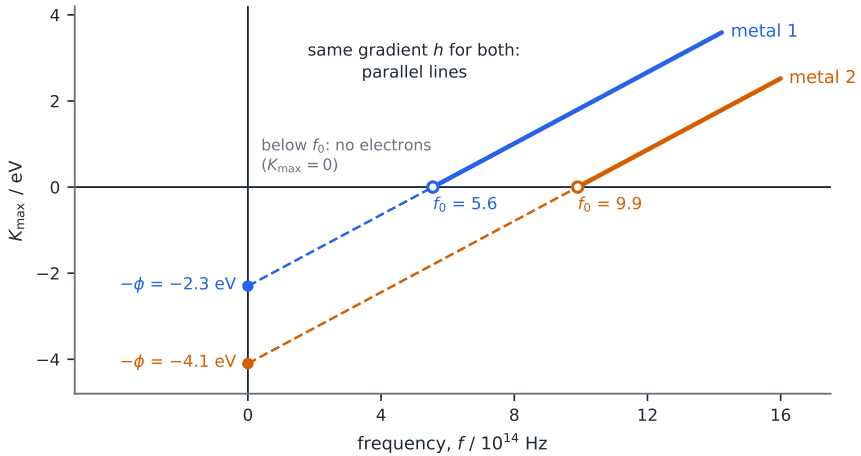
Why it proves photons exist. A wave model says brighter light should, given time, free electrons at any frequency. Experiments show the opposite. Below f_0, no electrons come out however bright the light. Above f_0, electrons come out at once even in dim light. Their maximum kinetic energy depends only on the frequency, never on the brightness. Each electron absorbs one photon, so only the energy of one photon matters.

Measuring it. In the usual set-up, light falls on a metal plate in a vacuum tube, and a variable voltage pushes back on the electrons. The voltage that just stops the fastest electrons is the **stopping potential** 遏止电压 ΔV_s, and $K_{\max} = e\,\Delta V_s$.



The maximum kinetic energy of photoelectrons rises linearly with frequency

Reading the graph. $K_{\max} = hf - \phi$ is a straight line. Its **gradient** is h, the same for every metal. It crosses the frequency axis at f_0. Extended back, it crosses the $K_{\max}$ axis at $-\phi$. Below f_0 no photoelectrons are emitted, so there is no measured $K_{\max}$ and no data line along the axis. A metal with a larger work function gives a **parallel** line shifted to the right.



Every metal gives a line of gradient h; a larger work function shifts the line to the right

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Worked example. Light of frequency 8.0×10^{14} Hz falls on a metal with work function $\phi = 3.0 \times 10^{-19}$ J. The most energetic electrons come off with

$$K_{\max} = hf - \phi = (6.63 \times 10^{-34} \times 8.0 \times 10^{14}) - 3.0 \times 10^{-19} = 5.3 \times 10^{-19} - 3.0 \times 10^{-19} = 2.3 \times 10^{-19} \text{ J}.$$

Below the threshold frequency ϕ/h, $K_{\max}$ would be negative – meaning no electrons escape at all, however bright the light.

Worked example (the electroscope). A negatively charged electroscope has its leaves spread apart. Ultraviolet light on its metal sphere makes the leaves fall: each UV photon has more energy than the work function, so it frees electrons, the electroscope loses negative charge, and the leaves repel less. Green light does nothing, because each green photon has less energy than ϕ. Making the green light brighter gives more photons, each with the **same** small energy, so still no electron escapes and the leaves still do not move.

Compton Scattering

In **Compton scattering** 康普顿散射, a photon collides with an electron like two particles, transferring some energy and momentum. The scattered photon comes out with **less energy** (longer wavelength). This is direct evidence that photons carry momentum and behave as particles.

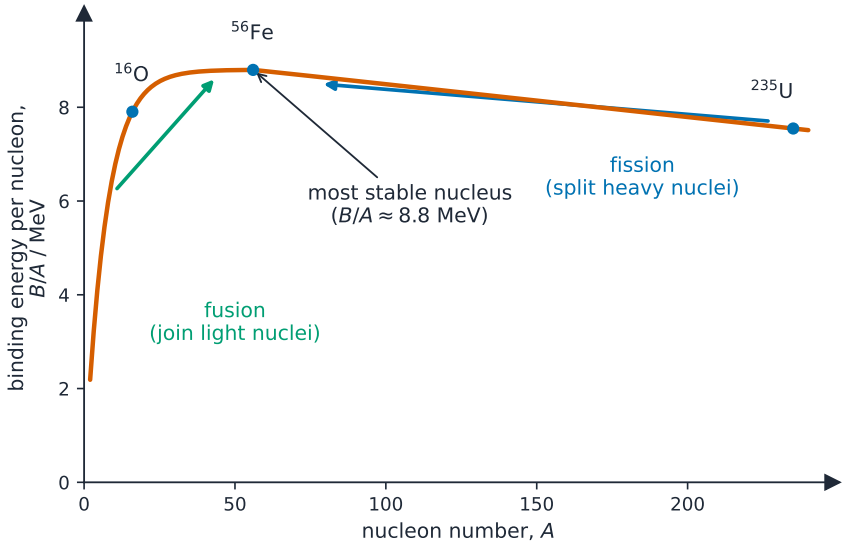
To explain it, treat the photon as a particle with momentum $p = h/\lambda$ and apply conservation of energy and conservation of momentum to the collision. The electron recoils and gains kinetic energy, so the photon loses energy: its frequency falls, its wavelength grows and its momentum falls. The larger the angle the photon is scattered through, the more energy it gives up, so the larger the increase in wavelength. A head-on bounce straight back gives the largest change. A wave model cannot explain why the change depends on angle but not on brightness.

Fission, Fusion, and Nuclear Decay

The **nucleus** 原子核 stores enormous energy. **Fission** 裂变 splits a heavy nucleus into lighter ones, releasing energy (nuclear reactors, bombs). **Fusion** 聚变 joins light nuclei into a heavier one (the Sun's power). In energy-releasing fission or fusion, the final products have slightly **less total rest mass**; the difference supplies energy according to $E = mc^2$. Not every possible fission or fusion reaction releases energy.

Nucleons are held together by the **strong force** 强相互作用, which is attractive and much stronger than the electric repulsion between protons, but acts only over nuclear distances. Every nuclear reaction conserves nucleon number, charge, energy (including rest energy, $E = mc^2$) and momentum. The released energy appears as kinetic energy of the products and as gamma photons. Some heavy nuclei split on their own (spontaneous fission). Absorbing a neutron can also induce fission, as in uranium-235 reactors; uranium-235 can undergo spontaneous fission too, but extremely rarely.

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Binding energy per nucleon peaks near iron, so both fusion and fission can release energy

Worked example. Even a tiny mass converts to a huge energy. If a nuclear reaction loses 1.0×10^{-3} kg of mass, it releases $E = mc^2 = 1.0 \times 10^{-3} \times (3.0 \times 10^8)^2 = 9.0 \times 10^{13}$ J – roughly the energy of 20 000 tonnes of TNT.

Worked example (annihilation). An electron moving with kinetic energy K meets a positron at rest, and both are destroyed, producing photons. By conservation of energy, the photons share

$$E = 2mc^2 + K = 2 \times 8.2 \times 10^{-14} \text{ J} + K = 1.64 \times 10^{-13} \text{ J} + K.$$

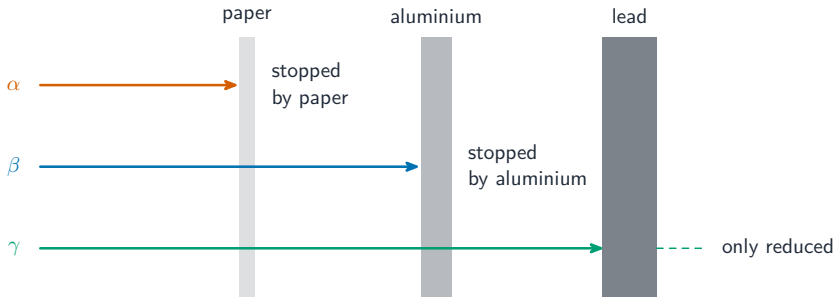
For a slow electron K is tiny next to $2mc^2$. Two photons, not one, are needed when the total momentum is close to zero, so that momentum is conserved.



A nuclear power station: fission of heavy nuclei releases the energy that drives the plant

Types of Radioactive Decay

Unstable nuclei undergo **radioactive decay** 放射性衰变, emitting:



The penetrating power of alpha, beta, and gamma radiation

- **Alpha decay** 阿尔法衰变 (α): a helium nucleus – mass number drops by 4.
- **Beta decay** 贝塔衰变 (β) comes in two kinds. In **beta-minus** (β^-) a neutron becomes a proton, emitting an electron and an **antineutrino** 反中微子: $n \to p + e^- + \bar{\nu}$. In **beta-plus** ($\beta^+$), inside a nucleus with enough available decay energy, a proton becomes a neutron, emitting a **positron** 正电子 and a **neutrino** 中微子: $p \to n + e^+ + \nu$.
- **Gamma decay** 伽马衰变 (γ): a high-energy photon – the nucleus sheds excess energy.

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The neutrino and antineutrino (symbols ν, $\bar{\nu}$) are tiny, chargeless, almost-massless particles that carry off energy and keep beta decay balanced. Decay is random, but a sample halves every **half-life** 半衰期: after each half-life, half of the remaining nuclei have decayed. Every decay equation conserves three quantities: **nucleon number** (mass number), **charge**, and **lepton number** 轻子数 – an electron or neutrino counts as $+1$ lepton, a positron or antineutrino as -1, and the totals must match on both sides.

Decay particles. An alpha particle is a helium-4 nucleus: two protons and two neutrons (α or He^{2+}). A positron has the electron's mass but a positive charge (e^+ or β^+). Neutrinos and antineutrinos have no charge and almost no mass; they feel only the weak force and gravity, so they pass through matter almost without interacting. Gamma decay often follows an alpha or beta decay: the new nucleus is left excited, and it drops to a lower energy state by emitting a photon. Which decay a nucleus undergoes depends on its isotope: a nucleus with too many neutrons tends toward β^-, one with too few toward β^+.

> **Worked example (balancing).** Carbon-14 decays by β^-:
>
> $$^{14}_{6}\mathrm{C} \to {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}e + \bar{\nu}.$$
>
> Nucleons: $14 = 14 + 0$. Charge: $6 = 7 - 1$. Leptons: $0 = (+1) + (-1)$.

The decay constant. No one can predict when one nucleus will decay; decay is random, but large numbers follow a rule. The **decay constant** 衰变常数 λ is the probability per second that a nucleus decays, and

$$\lambda = \frac{\ln 2}{t_{1/2}}, \qquad N = N_0 e^{-\lambda t}.$$

Half-lives range from fractions of a second to billions of years.

> **Worked example.** A radioactive sample has a half-life of 8.0 days. What fraction is left after 24 days? That is $24/8.0 = 3$ half-lives, so the fraction remaining is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ – about 12.5%. In an alpha decay of uranium-238 ($^{238}_{92}\mathrm{U}$), the daughter has mass number $238 - 4 = 234$ and atomic number $92 - 2 = 90$: thorium-234.

> **Worked example (carbon dating).** Carbon-14 has $t_{1/2} = 5730$ y. A bone has 25% of the carbon-14 of living bone. That is $\left(\frac{1}{2}\right)^2$, two half-lives, so it is $2 \times 5730 = 11\,460$ y old. With the decay constant, for any fraction: $\lambda = \dfrac{0.693}{5730} = 1.21 \times 10^{-4}\ \mathrm{y}^{-1}$ and $t = \dfrac{\ln(N_0/N)}{\lambda} = \dfrac{\ln 4}{1.21 \times 10^{-4}} = 11\,500$ y.

Practice

Answer each before reading the answers below. Take $h = 6.63 \times 10^{-34}$ J s, $c = 3.0 \times 10^{8}$ m/s and 1 eV $= 1.6 \times 10^{-19}$ J.

1. Find the energy of a photon of blue light of wavelength 450 nm, in joules and in electronvolts.
2. That light falls on a metal with work function 2.3 eV. Find the maximum kinetic energy of the ejected electrons.
3. In hydrogen the $n = 3$ level is at -1.51 eV and the $n = 2$ level at -3.40 eV. Find the energy and the wavelength of the photon emitted when an electron drops from $n = 3$ to $n = 2$.
4. The activity of a sample falls from 1600 Bq to 100 Bq in 20 hours. Find its half-life.
5. A student says: "Brighter light ejects photoelectrons with more kinetic energy." Correct the student.
6. A proton and an electron move at the same speed. Which has the longer de Broglie wavelength? Explain.
7. A metal has a threshold frequency of 6.0×10^{14} Hz. Find its work function in eV, and the stopping potential when light of 9.0×10^{14} Hz falls on it.
8. Complete the decay: ${}^{22}_{11}\mathrm{Na} \to {}^{22}_{10}\mathrm{Ne} + \,?\, + \,?$
9. A sample has a half-life of 3.0 h. Find its decay constant, and the fraction left after 5.0 h.

Answers

1. $E = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{450 \times 10^{-9}} = 4.4 \times 10^{-19}$ J, which is $\dfrac{4.42 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.8$ eV.
2. $K_{\max} = hf - \phi = 2.76 - 2.30 = 0.46$ eV (about 7.4×10^{-20} J).
3. $E = -1.51 - (-3.40) = 1.89$ eV $= 3.0 \times 10^{-19}$ J. Then $\lambda = \dfrac{hc}{E} = \dfrac{1.99 \times 10^{-25}}{3.02 \times 10^{-19}} = 6.6 \times 10^{-7}$ m, about 660 nm: the red line of hydrogen.
4. $1600 \to 800 \to 400 \to 200 \to 100$ is 4 half-lives in 20 h, so the half-life is 5.0 h.
5. Brighter light at the same frequency means **more photons** per second, so more electrons come off each second, but each photon still carries hf. The maximum kinetic energy $hf - \phi$ depends only on the frequency. Raise the frequency, not the brightness, to get faster electrons.
6. The electron. At the same speed it has a much smaller mass, so a smaller momentum mv, and $\lambda = h/(mv)$ is longer.
7. $\phi = hf_0 = 4.14 \times 10^{-15} \times 6.0 \times 10^{14} = 2.5$ eV. $K_{\max} = 4.14 \times 10^{-15} \times 9.0 \times 10^{14} - 2.5 = 3.7 - 2.5 = 1.2$ eV, so $\Delta V_s = 1.2$ V.
8. Z falls by 1 with A unchanged, so a proton became a neutron: β^+ decay. The missing particles are a positron ${}^{0}_{+1}e$ and a neutrino ν (lepton number $0 = -1 + 1$).
9. $\lambda = \dfrac{0.693}{3.0} = 0.23\ \mathrm{h}^{-1}$. $\dfrac{N}{N_0} = e^{-0.231 \times 5.0} = e^{-1.16} = 0.31$.

Exam tips

- Photon energy is $E = hf$; below the **threshold frequency** no electrons are emitted **however bright** the light (the photoelectric effect).
- Use $K_{\max} = hf - \phi$ for the fastest photoelectrons (ϕ = work function).
- Electrons jump between **discrete energy levels** by absorbing/emitting a photon whose energy equals the level gap – the source of line spectra.
- In fission and fusion the small **mass lost** becomes energy via $E = mc^2$.
- Balance decay equations by conserving **mass number, charge, and lepton number** (write the antineutrino in β^-, the neutrino in β^+), and halve the sample every **half-life** ($(\frac{1}{2})^n$ after n half-lives).

Common mistakes

- Thinking brighter light gives faster photoelectrons. Intensity changes the *number* of photons; each photon's energy, and so the maximum kinetic energy, depends only on frequency: $K_{\max} = hf - \phi$.
- Expecting electrons below the threshold frequency if the light is bright enough. Below $f_0 = \phi/h$ no electrons are emitted, however intense the light.
- Mixing joules and electronvolts in $E = hf$. Use $h = 6.63 \times 10^{-34}$ J s for joules or $h = 4.14 \times 10^{-15}$ eV s for electronvolts; 1 eV $= 1.6 \times 10^{-19}$ J.
- Saying an atom can absorb any photon. Below its ionization threshold, it absorbs only photons whose energy matches an allowed level gap. A sufficiently energetic photon can instead ionize it, with excess energy becoming the freed electron's kinetic energy.
- Saying that after two half-lives nothing is left. One quarter of the original nuclei remains; after n half-lives, $(\frac{1}{2})^n$ remains.
- Leaving out the mass in $\Delta E = \Delta m\, c^2$ units: Δm in kilograms and $c = 3.0 \times 10^8$ m/s give joules.
- Saying a larger Bohr orbit has less energy. $E = -\dfrac{ke^2}{2r}$ is negative, so a larger r means a *higher* (less negative) energy.
- Drawing the second metal's $K_{\max}$ line steeper. Every metal has the same gradient h; only the intercepts change.
- Saying Compton scattering changes the photon's speed. The photon still travels at c; its energy, momentum and frequency fall, and its wavelength grows.

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