

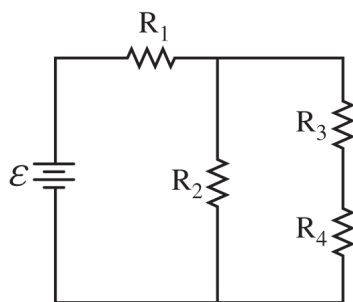
Begin your response to **QUESTION 3** on this page.

Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1 , R_2 , R_3 , and R_4 , each of resistance R , as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and R only.

i. **Derive** an expression for the current I_1 in Resistor R_1 .

ii. **Derive** an expression for the current I_3 in Resistor R_3 .

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(b) The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.

- In Figure 2, **draw** a bar to represent $|\Delta V|$ across each resistor, relative to the emf $\mathcal{E}$ of the ideal battery.
- The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a “0” in that column.

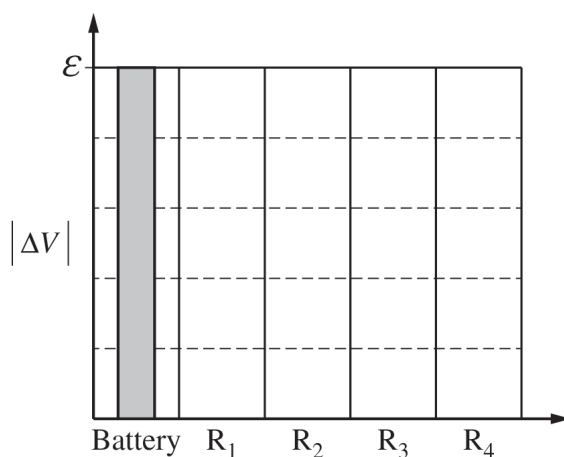


Figure 2

A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed

as $P = \frac{3\mathcal{E}^2}{5R}$.

(c) **State** whether the expression for P is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

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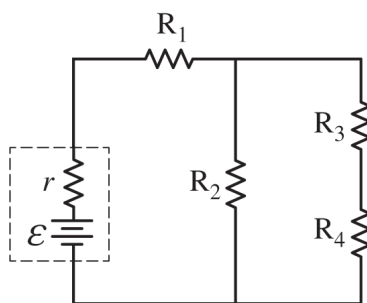
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Figure 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor R_1 is P_{original} . The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance r to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor R_1 in the new circuit is P_{new} .

(d) **Indicate** whether P_{new} is greater than, less than, or equal to P_{original} .

_____ $P_{\text{new}} > P_{\text{original}}$ _____ $P_{\text{new}} < P_{\text{original}}$ _____ $P_{\text{new}} = P_{\text{original}}$

Briefly **justify** your answer.

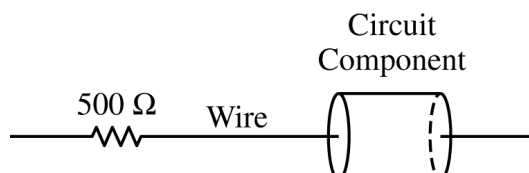
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2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\ \Omega$.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.

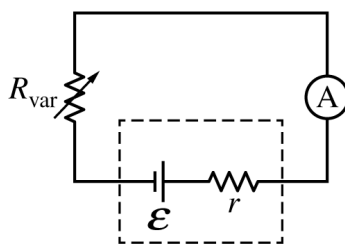


ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

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The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I , r , and R_{var} .

ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

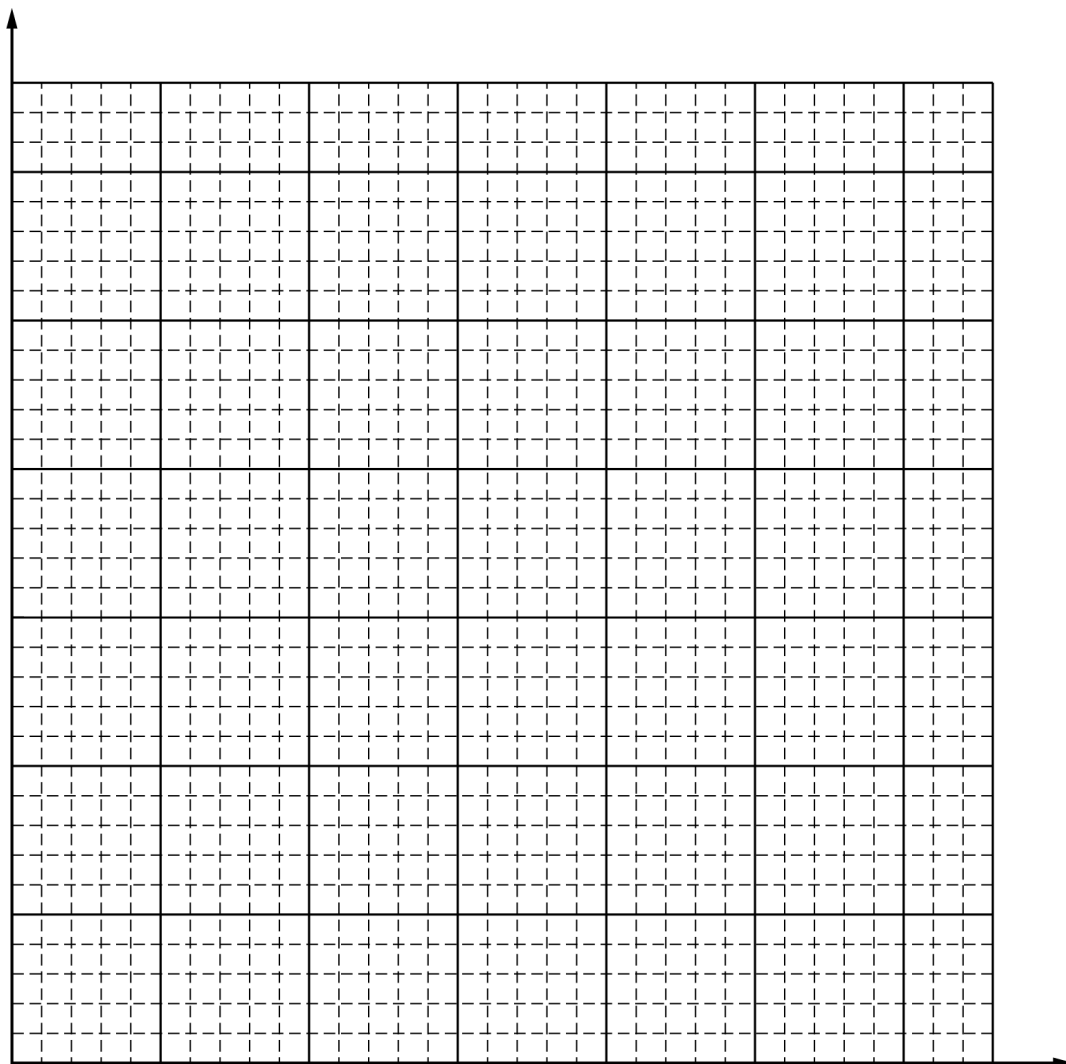
Horizontal Axis: _____

Vertical Axis: _____

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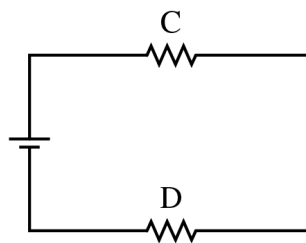
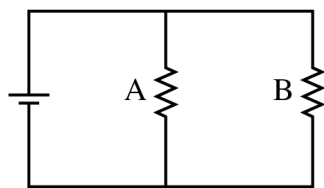
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- iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data. You may use the blank columns in the table for any quantities you graph other than the given data.



- iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

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2. (12 points, suggested time 25 minutes)

Students perform an experiment with a battery and four resistors, A, B, C, and D. The resistance of resistors A and C is $R_A = R_C = R$. The resistance of resistors B and D is $R_B = R_D = 2R$. The students create the two circuits shown above and measure the potential differences ΔV_A , ΔV_B , ΔV_C , and ΔV_D across resistors A, B, C, and D, respectively.

- (a) From greatest to least, rank the magnitudes of the potential differences across the resistors. Use “1” for the greatest magnitude, “2” for the next greatest magnitude, and so on. If any potential differences have the same magnitude, use the same number for their ranking.

____ ΔV_A ____ ΔV_B ____ ΔV_C ____ ΔV_D

Justify your answer.

In another experiment, the students have a capacitor with unknown capacitance C_U . They want to determine C_U by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The students wait until the capacitors are fully charged and then record the potential difference ΔV_{known} across the known capacitor and the potential difference ΔV_U across the unknown capacitor. Their data are shown in the table on the following page.

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Known Capacitance of Capacitors (μF)	ΔV_{known} (V)	ΔV_{U} (V)		
200	0.91	3.53		
300	0.65	3.74		
400	0.51	3.95		
500	0.42	4.06		
600	0.36	4.17		

(b)

- i. Calculate the amount of charge on the capacitor of known capacitance of $200\ \mu\text{F}$ in the students' experiment.
- ii. Briefly explain why the data in the table provide evidence that the capacitors are connected in series.
- iii. Briefly explain why connecting the capacitors in parallel would not provide enough information to determine the capacitance of the unknown capacitor if the only measuring device available is a voltmeter.

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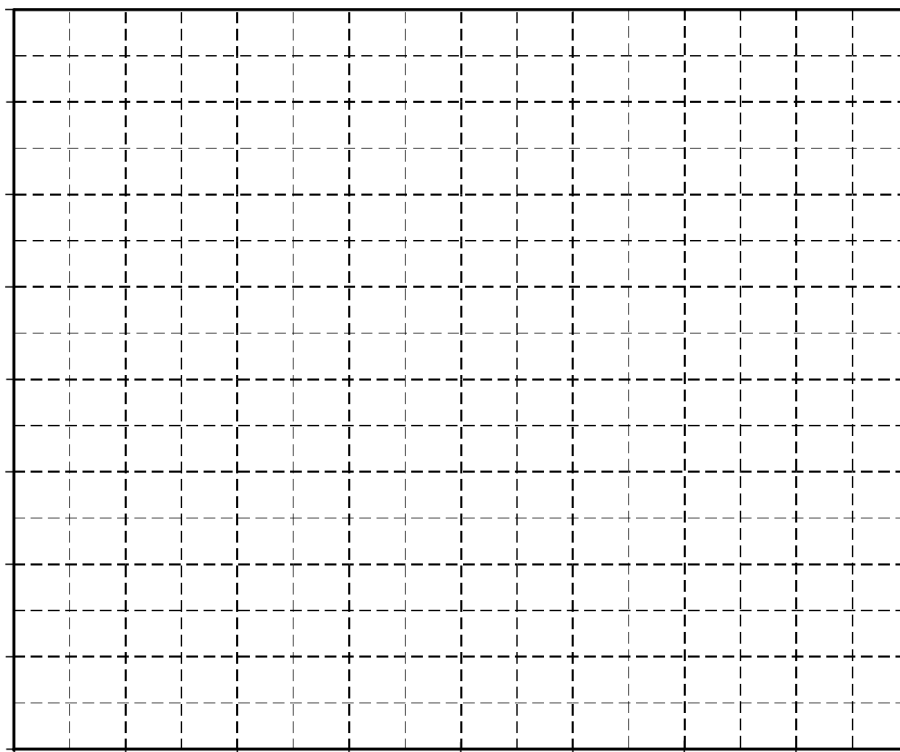
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(c) The students want to produce a linear graph of the data so that the capacitance C_U of the unknown capacitor can be determined from the slope of the best-fit line for the data.

i. Indicate two quantities that could be plotted to produce the desired graph. Use the empty columns of the data table in part (b) to record any values that you need to calculate.

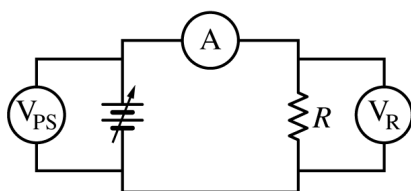
Vertical axis _____ Horizontal axis _____

ii. Label the axes below and provide an appropriate scale with units. Plot the data points for the quantities indicated in part (c)(i) on the axes and draw a best-fit line.

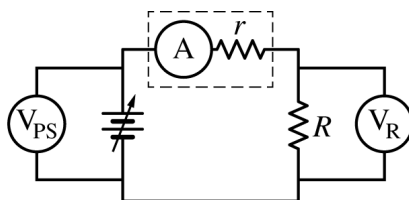


iii. Using your best-fit line, determine the capacitance of capacitor C_U .

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Circuit 1

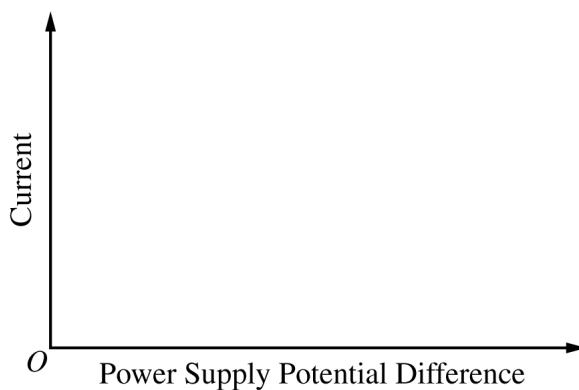


Circuit 2

2. (12 points, suggested time 25 minutes)

The two circuits shown above contain an ideal variable power supply, an ohmic resistor of resistance R , an ammeter A , and two voltmeters V_{PS} and V_R . In circuit 1 the ammeter has negligible resistance, and in circuit 2 the ammeter has significant internal ohmic resistance r . The potential difference of the power supply is varied, and measurements of current and potential difference are recorded.

- (a) The axes below can be used to graph the current measured by the ammeter as a function of the potential difference measured across the power supply. On the axes, do the following.
- Sketch a possible graph for circuit 1 and label it 1.
 - Sketch a possible graph for circuit 2 and label it 2.

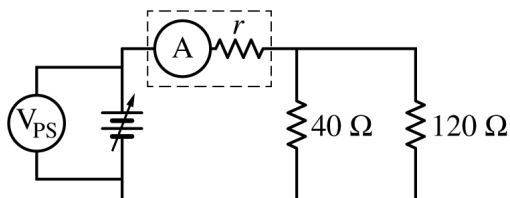


- (b) Let ΔV_{PS} be the potential difference measured by voltmeter V_{PS} across the power supply, and let I be the current measured by the ammeter A . For each circuit, write an equation that satisfies conservation of energy, in terms of ΔV_{PS} , I , R , and r , as appropriate.

Circuit 1

Circuit 2

- (c) Explain how your equations in part (b) account for any differences between graphs 1 and 2 in part (a).
- (d) In circuit 2, $R = 40\ \Omega$. When voltmeter V_{PS} reads 3.0 V, voltmeter V_R reads 2.5 V. Calculate the internal resistance r of the ammeter.
- (e) Voltmeter V_R in circuit 2 is replaced by a resistor with resistance $120\ \Omega$ to create circuit 3 shown below. Voltmeter V_{PS} still reads 3.0 V.



Circuit 3

- Calculate the equivalent resistance R_{eq} of the circuit.
- Calculate the current in each of the resistors that are in parallel.

2. (12 points, suggested time 25 minutes)

A group of students is given several long, thick, cylindrical conducting rods of the same unknown material with various lengths and diameters and asked to experimentally determine the resistivity of the material using a graph. The available equipment includes a voltmeter, an ammeter, connecting wires, a variable-output DC power supply, and a metric ruler.

(a)

- i. Describe a procedure the students could use to collect the data needed to create the graph, including the measurements to be taken and a labeled diagram of the circuit to be used. Include enough detail that another student could follow the procedure and obtain similar data.

Draw a labeled diagram here.

Write your procedure here.

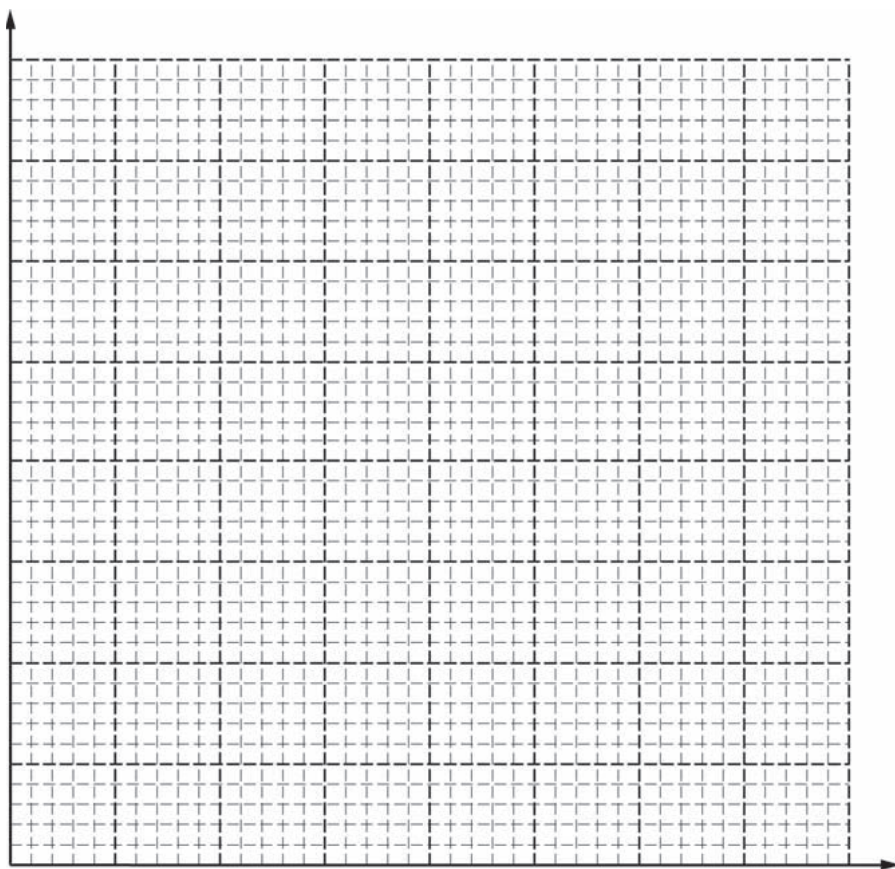
- ii. Describe how the data could be graphed in a way that is useful for determining the resistivity of the material. Describe how the graph could be analyzed to calculate the resistivity.

The students are now given a rectangular rod of the material, as shown below, whose dimensions are not known. The students are asked to experimentally determine the resistance of the rod. They obtain the data in the table below for the potential difference ΔV across the rod and the current I in it.



ΔV (V)	6.0	5.0	3.5	2.5	2.0	1.5
I (A)	0.078	0.070	0.044	0.036	0.027	0.018

- (b) On the axes below, plot the data so that the resistance of the rectangular rod can be determined from a best-fit line. Label and scale the axes. Use the best-fit line to determine the resistance of the rod, clearly showing your calculations.



- (c) After completing their calculations, the students begin to consider the factors that might have produced uncertainties in their results.
- The students realize that they did not take into account the internal resistance of the power supply. Briefly describe how this would affect their value of the resistance of the rectangular rod. Explain your reasoning.
 - The students realize that they did not take into account a possible change in the temperature of the cylindrical rods. Should the students be concerned about this? Explain why or why not.