# Past-paper walk-through

## Quantum Theory and Wave-Particle Duality ▪ 15.1

eXercise

# Questions in this set

- 2026 Q2
- 2024 Q1
- 2022 Q3
- 2021 Q4

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 2

2026 ▪ Q2

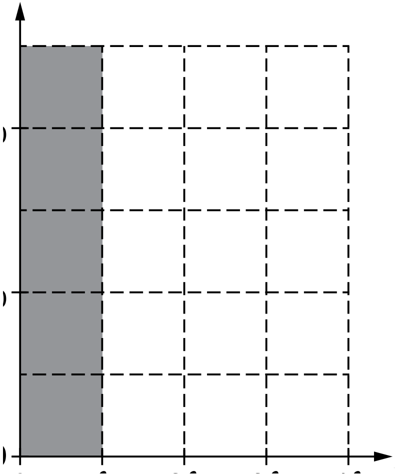
Figure 1 represents the energy levels and their corresponding states for a hypothetical atom.

[Figure 1: An energy-level diagram with two columns, "Energy" and "State". Three horizontal levels are drawn: the topmost level is labelled $-2E_0$ on the left and $n = 3$ on the right; the middle level is labelled $-3E_0$ on the left and $n = 2$ on the right; the bottom level (drawn noticeably lower, with extra vertical spacing below $n = 2$) is labelled $-5E_0$ on the left and $n = 1$ on the right.]

**(a)** Part A

On Figure 1, **draw** arrows to represent all possible atomic transitions that could result in the emission of a photon.

# Question 2



# Question 2

2026 ▪ Q2

Figure 1 represents the energy levels and their corresponding states for a hypothetical atom. [Figure 1: An energy-level diagram with two columns, "Energy" and "State". Three horizontal levels are drawn: the topmost level is labelled $-2E_0$ on the left and $n = 3$ on the right; the middle level is labelled $-3E_0$ on the left and $n = 2$ on the right; the bottom level (drawn noticeably lower, with extra vertical spacing below $n = 2$) is labelled $-5E_0$ on the left and $n = 1$ on the right.]

**(b)** Part B

**Derive** an expression for the wavelength of the highest-energy photon that can be emitted from the atom. Express your answer in terms of $E_0$ and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

The following information applies to parts C and D.

# Question 2

A device can emit monochromatic electromagnetic radiation. The wavelength $\lambda$ of the radiation can be varied over a continuous range of wavelengths.

# Question 2

2026 ▪ Q2

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**(c)** Part C

On Figure 2, **sketch** the energy $E$ of emitted photons from the device as a function of $\lambda$ for $\lambda_0 < \lambda < 4\lambda_0$. Photons with wavelength $\lambda_0$ have energy $4E_0$. Sketches made in the shaded region will not be scored.

[Figure 2: A set of axes with vertical axis $E$ labelled with gridlines at $2E_0$ and $4E_0$, and horizontal axis $\lambda$ labelled with gridlines at $0$, $\lambda_0$, $2\lambda_0$, $3\lambda_0$, $4\lambda_0$. The region from $\lambda = 0$

# Question 2

to $\lambda = \lambda_0$ is shaded gray (sketches there are not scored); dashed gridlines mark the intersections at $(\lambda_0, 4E_0)$ and other grid points across the unshaded region from $\lambda_0$ to $4\lambda_0$. No curve is drawn — to be sketched by the student.]

# Question 2

2026 ▪ Q2

Figure 1 represents the energy levels and their corresponding states for a hypothetical atom. [Figure 1: An energy-level diagram with two columns, "Energy" and "State". Three horizontal levels are drawn: the topmost level is labelled $-2E_0$ on the left and $n = 3$ on the right; the middle level is labelled $-3E_0$ on the left and $n = 2$ on the right; the bottom level (drawn noticeably lower, with extra vertical spacing below $n = 2$) is labelled $-5E_0$ on the left and $n = 1$ on the right.]

**(d)** Part D

A student claims that, based on the energy states shown in Figure 1, the atom can emit a photon of wavelength $\lambda_0$ and energy $4E_0$ like the device described in part C. **Indicate** whether the claim is correct or incorrect. Briefly **justify** your answer by referencing the representation in part A.

# Question 1

2024 ▪ Q1

In each trial of a photoelectric experiment, a scientist uses a device to shine light of a single frequency on two different metals, 1 and 2. The device can emit light with frequency $f_A$, $f_B$, or $f_C$. Each frequency of light is used to test both metals.
The scientist determines the minimum de Broglie wavelength $\lambda_e$ of the electrons ejected from the metal in each trial of the experiment. The following table summarizes the results of the experiment. For each trial, the scientist analyzes only the electrons with the minimum de Broglie wavelength.

| Trial | Frequency of Light | Metal Tested | $\lambda_e$ ($\times 10^{-10}$ m) |
|---|---|---|---|
| 1 | $f_A$ | Metal 1 | 6.9 |
| 2 | $f_A$ | Metal 2 | 9.4 |
| 3 | $f_B$ | Metal 1 | No electrons ejected |
| 4 | $f_B$ | Metal 2 | No electrons ejected |
| 5 | $f_C$ | Metal 1 | 5.3 |
| 6 | $f_C$ | Metal 2 | 6.3 |

**(a)** In a coherent, paragraph-length response, **indicate** which frequency, $f_A$, $f_B$, or $f_C$, is greatest and which frequency is least. **Justify** your answer using physics principles.

# Question 1

**(b) Calculate** the maximum kinetic energy of the electrons ejected from Metal 1 in Trial 1. Assume that the momentum $p$ of an ejected electron can be described by the classical definition $p = mv$.
**(c) Indicate** whether the work function of Metal 1 is greater than, less than, or equal to the work function of Metal 2. **Justify** your answer by referring to the table of results.

# Solution 1

## (a) Mark scheme

- $f_C$ is greatest and $f_B$ is least. Reasoning: an electron is ejected only if the incident photon's energy ($E = hf$) exceeds the metal's work function. Because no electrons were ejected using $f_B$, its photon energy (and therefore frequency) must be the least of the three. A photon with greater frequency gives an ejected electron more kinetic energy, and the kinetic energy of an electron is inversely related to its de Broglie wavelength ($\lambda = h/p$, $K = p^2/2m$, so larger $\lambda$ means smaller $K$). Because the de Broglie wavelength of electrons ejected by light of frequency $f_A$ is greater than that of electrons ejected by light of frequency $f_C$, $f_A$ must be less than $f_C$. Therefore $f_C$ is the greatest frequency, and $f_B$ (which ejects no electrons at all) is the least. Scoring: 1 pt for correctly ranking $f_B$ least and $f_C$ greatest; 1 pt for

# Solution 1

correctly relating photon frequency to photon energy ($E = hf$); 1 pt for indicating that an ejected electron's kinetic energy/speed is inversely related to its de Broglie wavelength; 1 pt for stating that either (i) greater photon energy/frequency gives greater ejected-electron kinetic energy, or (ii) the lowest photon energy/frequency is below the work function/threshold frequency so no electron is ejected; 1 pt for an overall logical, relevant, internally consistent paragraph-length argument.

# Solution 1

## (b) Mark scheme

- $K = 5 \times 10^{-19}$ J (equivalently about $3$ eV). Derivation: the electron's de Broglie wavelength is $\lambda_e = h/p = h/(mv)$, so $v = h/(m\lambda_e)$. Substituting into kinetic energy: $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{m\lambda_e}\right)^2 = \frac{h^2}{2m\lambda_e^2}$. Using $h = 6.63 \times 10^{-34}$ J · s, $m = 9.11 \times 10^{-31}$ kg, and the Trial 1 / Metal 1 de Broglie wavelength $\lambda_e = 6.9 \times 10^{-10}$ m from the data table: $K = \dfrac{(6.63 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(6.9 \times 10^{-10})^2} = 5 \times 10^{-19}$ J. Scoring: 1 pt for either $\lambda_e = h/(mv)$ or $K = p^2/2m$; 1 pt for correctly substituting a $\lambda_e$-based (or

# Solution 1

numeric) expression for speed/momentum into the kinetic-energy equation; 1 pt for the correct final numeric answer with units ($5 \times 10^{-19}$ J or 3 eV).

# Solution 1

## (c) Mark scheme

- The work function of Metal 1 is LESS than the work function of Metal 2. Reasoning: when light of the same frequency (same photon energy $hf$) is incident on both metals, the electron ejected from Metal 1 has a smaller de Broglie wavelength than the electron ejected from Metal 2, so the electron ejected from Metal 1 has more kinetic energy. The work function equals the difference between the photon energy and the maximum kinetic energy ($\phi = hf - K_{max}$). Since the photon energy is the same for both metals but $K_{max}$ is larger for Metal 1, the difference (and thus the work function) is smaller for Metal 1. Scoring: 1 pt for correctly indicating Metal 1's work function is less than Metal 2's, with an attempted justification; 1 pt for citing at least two of: the correct relation

# Solution 1

between work function and $hf - K_{max}$; that the incident photon frequency/energy is the same for both metals; that de Broglie wavelength is inversely related to the ejected electron's energy.

# Question 3

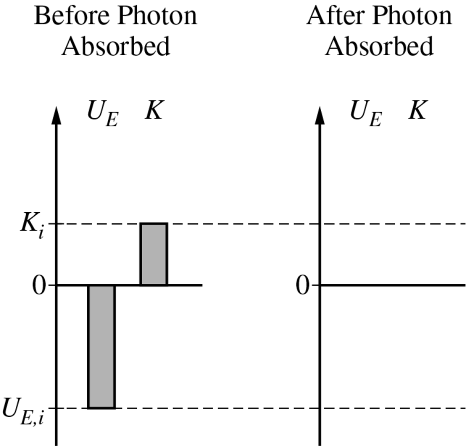
2022 ▪ Q3

[Diagram: A circle representing the electron's circular orbit of radius $r$ around a stationary proton at the center. The electron, labeled with a minus sign, is on the circle at upper left, with an arrow pointing along the circle indicating its direction of motion; a line labeled $r$ connects the electron to the central proton, labeled with a plus sign. Note: Figure not drawn to scale.]

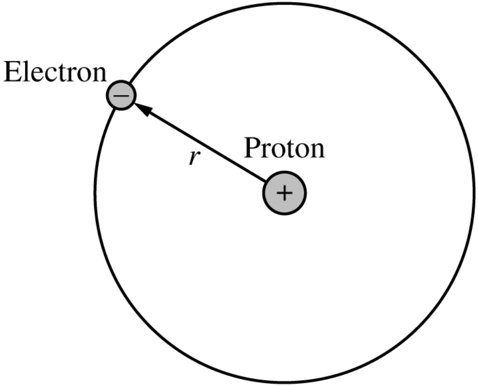
A hydrogen atom can be modeled as an electron in a circular orbit of radius $r$ about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

**(a)** Derive an equation for the speed $v$ of the electron in terms of $r$ and physical constants, as appropriate.

# Question 3


Before Photon Absorbed
After Photon Absorbed
$U_E$
$K$
$K_i$
0
$U_{E,i}$
$U_E$
$K$
0

# Question 3




Note: Figure not drawn to scale.

# Question 3

2022 ▪ Q3

[Diagram: A circle representing the electron's circular orbit of radius $r$ around a stationary proton at the center. The electron, labeled with a minus sign, is on the circle at upper left, with an arrow pointing along the circle indicating its direction of motion; a line labeled $r$ connects the electron to the central proton, labeled with a plus sign. Note: Figure not drawn to scale.]

A hydrogen atom can be modeled as an electron in a circular orbit of radius $r$ about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

**(b)** Derive an equation for the total energy of the atom in terms of $r$ and physical constants, as appropriate.

# Question 3

2022 ▪ Q3

[Diagram: A circle representing the electron's circular orbit of radius $r$ around a stationary proton at the center. The electron, labeled with a minus sign, is on the circle at upper left, with an arrow pointing along the circle indicating its direction of motion; a line labeled $r$ connects the electron to the central proton, labeled with a plus sign. Note: Figure not drawn to scale.]

A hydrogen atom can be modeled as an electron in a circular orbit of radius $r$ about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

**(c)** When the hydrogen atom absorbs a photon, the electron moves to an orbit with a larger radius and the total energy of the atom increases. Is your equation for the energy derived in part (b) consistent with this description of the model of a hydrogen atom absorbing a photon? Explain why the equation is or is not consistent.

# Question 3

2022 ▪ Q3

[Diagram: A circle representing the electron's circular orbit of radius $r$ around a stationary proton at the center. The electron, labeled with a minus sign, is on the circle at upper left, with an arrow pointing along the circle indicating its direction of motion; a line labeled $r$ connects the electron to the central proton, labeled with a plus sign. Note: Figure not drawn to scale.]

A hydrogen atom can be modeled as an electron in a circular orbit of radius $r$ about a stationary proton, as shown above. The gravitational force between the proton and electron is negligible compared to the electrostatic force between them.

**(d)(i)** Experiments show that a hydrogen atom can absorb a photon of frequency $3.2 \times 10^{15}$ Hz.

Calculate the energy of a photon with this frequency.

**(d)(ii)** A student claims that when a hydrogen atom absorbs a photon at this frequency, the energy could be converted into mass, adding an electron to the atom.

# Question 3

Calculate the amount of energy needed to create a particle with the mass of an electron and determine whether or not there is sufficient energy gained by the atom to add another electron.

**(d)(iii)** [Diagram: Two bar charts side by side, titled "Before Photon Absorbed" and "After Photon Absorbed." Each chart has a vertical axis with two bars labeled $U_E$ and $K$. In the left ("Before") chart, a dashed horizontal line marks a positive level $K_i$ and a dashed horizontal line below zero marks a negative level $U_{E,i}$; the $U_E$ bar extends from 0 down to $U_{E,i}$ (negative), and the $K$ bar extends from 0 up to $K_i$ (positive). The right ("After") chart shows only the zero line, with both bars blank for the student to draw.] The left bar chart in the figure above is complete and represents the initial electric potential energy $U_{E,i}$ of the atom and the initial kinetic energy $K_i$ of the electron before the photon is absorbed. In the space provided on the right, draw a bar chart to

# Question 3

represent a possible final electric potential energy of the atom and final kinetic energy of the electron.

# Solution 3

## (a) Mark scheme

- (1 point) Set the electrostatic (Coulomb) force equal to the net (centripetal) force on the electron: $\Sigma F = ma \Rightarrow F_E = F_C \Rightarrow \frac{kq^2}{r^2} = \frac{mv^2}{r}$ (an incorrect mass label is still acceptable for this point). (1 point) Use these expressions, with mass and charge consistently referring to the electron ($m_e$, $e$; $q_e/q_p$ notation also acceptable), to solve for speed: $\frac{ke^2}{r^2} = \frac{m_e v^2}{r} \Rightarrow v^2 = \frac{ke^2}{m_e r} \Rightarrow v = \sqrt{\frac{ke^2}{m_e r}}$.

# Solution 3

## (b) Mark scheme

- (1 point) Electric potential energy, using charges consistent with part (a): $U = -\dfrac{ke^2}{r}$. (1 point) Kinetic energy of the electron, substituting the speed from part (a) to eliminate $v$: $K = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e\left(\dfrac{ke^2}{m_e r}\right) = \frac{1}{2}\dfrac{ke^2}{r}$. (1 point) Indicate the total energy of the atom is the sum of potential and kinetic energy: $E = U + K = -\dfrac{ke^2}{r} + \frac{1}{2}\dfrac{ke^2}{r} = -\dfrac{ke^2}{2r}$.

# Solution 3

## (c) Mark scheme

- (1 point) Correctly indicate that the equation from part (b), $E = -\frac{ke^2}{2r}$, is consistent with the given description, with an explanation that references that equation. (1 point) Correctly address the functional dependence: as the orbital radius $r$ increases, $E$ becomes less negative — i.e. the total energy increases — which matches the description that absorbing a photon increases both the orbit radius and the atom's total energy. Model response: "The equation from part (b) indicates that as the radius increases, the total energy of the atom becomes less negative, which is an increase in the total energy. This is consistent with the given description of the atom absorbing a photon."

# Solution 3

**(d)(i)**

**Mark scheme**

- $E = hf = (6.63 \times 10^{-34}\ \mathsf{J \cdot s})(3.2 \times 10^{15}\ \mathsf{Hz}) = 2.12 \times 10^{-18}\ \mathsf{J}$.

# Solution 3

## (d)(ii)

**Mark scheme**

- (1 point) Mass-energy of an electron: $E = mc^2 = (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})^2 = 8.20 \times 10^{-14}$ J. (1 point) Compare this to the photon energy from part (d)(i) ($2.12 \times 10^{-18}$ J), in matching units (J to J, or eV to eV), and correctly conclude the photon energy is negligible compared to the electron's mass-energy — so there is NOT enough energy to create a new electron (the student's claim is incorrect). An answer consistent with the student's own (d)(i)/(d)(ii) numbers also earns this point.

# Solution 3

## (d)(iii)

**Mark scheme**

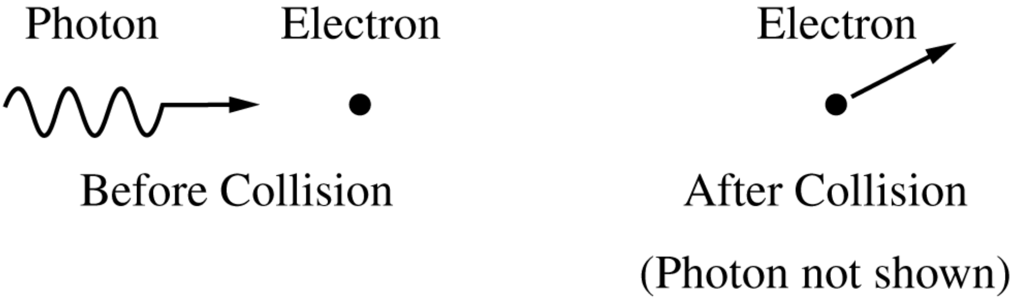
- On the "After Photon Absorbed" bar chart relative to "Before": (1 point) the electric potential energy bar $U_E$ is drawn smaller in magnitude but still negative (shorter than $U_{E,i}$, still below zero). (1 point) the kinetic energy bar $K$ is drawn smaller in magnitude but still positive (shorter than $K_i$, still above zero). This reflects that as $r$ increases, both $|U| = ke^2/r$ and $K = \frac{1}{2}ke^2/r$ decrease in magnitude while $U$ stays negative and $K$ stays positive.

# Question 4

2021 ▪ Q4

Light and matter can be modeled as waves or as particles. Some phenomena can be explained using the wave model, and others can be explained using the particle model.
**(a)** Calculate the speed, in m/s, of an electron that has a wavelength of 5.0 nm.

# Question 4


Photon
Electron
Before Collision
Electron
After Collision
(Photon not shown)

# Question 4

2021 ▪ Q4

Light and matter can be modeled as waves or as particles. Some phenomena can be explained using the wave model, and others can be explained using the particle model.

**(b)** The electron is moving with the speed calculated in part (a) when it collides with a positron that is at rest. A positron is a particle identical to an electron except that its charge is positive. The two particles annihilate each other, producing photons. Calculate the total energy of the photons.

# Question 4

2021 ▪ Q4

Light and matter can be modeled as waves or as particles. Some phenomena can be explained using the wave model, and others can be explained using the particle model.

**(c)** [Diagram: on the left, a wavy arrow labeled "Photon" moving right toward a dot labeled "Electron", captioned "Before Collision". On the right, a dot labeled "Electron" with a straight arrow pointing up and to the right, captioned "After Collision (Photon not shown)".]

A photon approaches an electron at rest, as shown above on the left, and collides elastically with the electron. After the collision, the electron moves toward the top of the page and to the right, as shown above on the right, at a known speed and angle. In a coherent, paragraph-length response, indicate a possible direction for the photon that exists after the collision and its frequency compared to that of the original photon.

# Question 4

Describe the application of physics principles that can be used to determine the direction of motion and frequency of the photon that exists after the collision.

# Solution 4

## (a)

**Mark scheme**

- Using the de Broglie relation $\lambda$ = h/p = h/(mv), solve for speed: v = h/(m$\lambda$) = (6.63×10^-34 J · s) / [(9.11×10^-31 kg)(5.0×10^-9 m)] = 1.5×10^5 m/s. Award 1 point for the correct relationship ($\lambda$=h/mv, rearranged for v) and 1 point for correctly substituting the given values (m_e = 9.11×10^-31 kg, $\lambda$ = 5.0×10^-9 m, h = 6.63×10^-34 J · s).

# Solution 4

## (b) Mark scheme

- Using $E=mc^2$ to convert the rest masses of the electron and positron to energy (1 point), and including the kinetic energy of the moving electron (1 point), with a speed consistent with part (a) and correct substitutions (1 point): E_tot = 2(9.11×10^-31 kg)(3×10^8 m/s)^2 + (1/2)(9.11×10^-31 kg)(1.5×10^5 m/s)^2 = 1.6×10^-13 J. (The at-rest positron contributes only its rest energy $mc^2$; the moving electron contributes its rest energy $mc^2$ plus its kinetic energy $(1/2)mv^2$, using v from part (a); both particles' total energy converts entirely into photon energy upon annihilation.)

# Solution 4

## (c) Mark scheme

- A complete paragraph argument should establish: (1) since the electron moves up and to the right after the collision, to conserve momentum in the vertical direction the outgoing photon must carry a component of momentum (or velocity) toward the BOTTOM of the page (1 point); (2) momentum must be conserved separately in both the horizontal and vertical directions (1 point); (3) energy must also be conserved in the collision (1 point); (4) because some of the photon's energy is transferred to the electron (which gains kinetic energy), the outgoing photon has LESS energy than the incoming photon and therefore a LOWER frequency, via $E=hf$ (1 point); (5) the response overall is logical, relevant, internally consistent, addresses the question asked, and follows the paragraph-response format

# Solution 4

requirements (1 point). Example: 'In order to conserve momentum in the vertical direction, the photon must have a component of its momentum toward the bottom of the page. If the horizontal component of the momentum of the electron after the collision is less than the initial momentum of the photon, then the photon must move toward the right after the interaction. In order to conserve energy, the frequency of the photon after the collision is less than what it was before the collision because it gave some of its energy to the electron.'