Past-paper walk-through

Reflection ▪ 13.1

eXercise

Questions in this set

- 2023 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

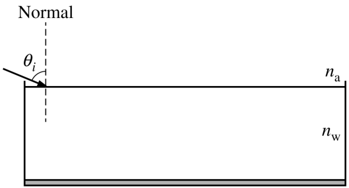
2023 ▪ Q1

(10 points, suggested time 20 minutes)
A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.
[Figure 1: A rectangular tank of water with a mirrored bottom, labeled "Mirror" along the bottom. A dashed vertical line labeled "Normal" is drawn at the point where a beam enters the top surface of the water at angle θ_i from the normal. The region above the water surface is labeled n_a; the region inside the tank (the water) is labeled n_w.]
(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.

Question 1

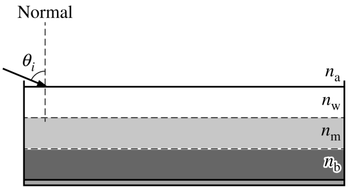
[Diagram: A blank rectangular tank with a mirrored bottom, same as Figure 1, showing "Normal" (dashed vertical line) and incident angle θ_i at the entry point, with n_a labeled above the water surface and n_w labeled inside the tank, for the student to sketch the beam's path.]

Question 1



Original Tank

Model A



Model B

Question 1

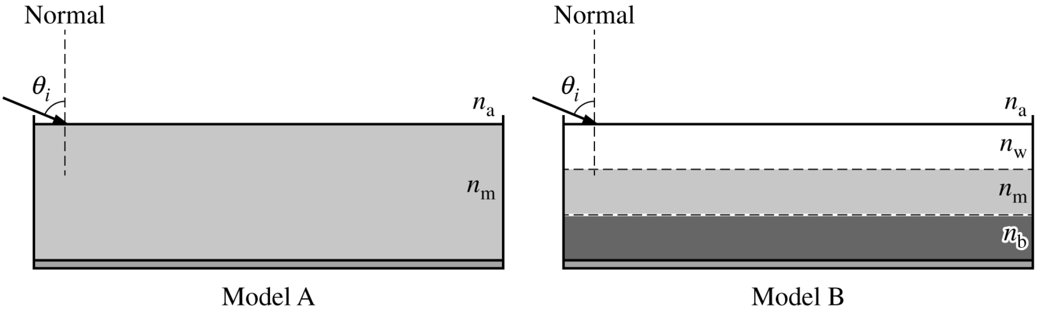


Figure 2

Question 1

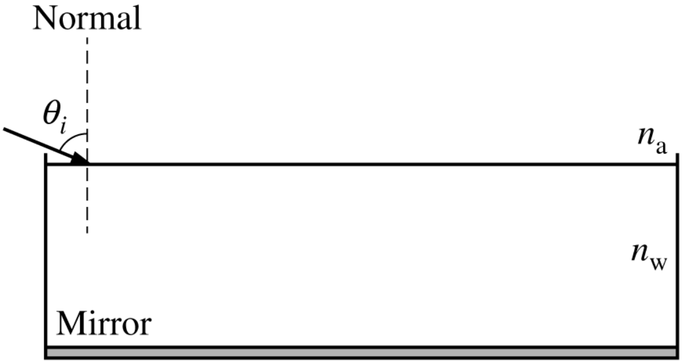


Figure 1

Question 1

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(10 points, suggested time 20 minutes)
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[Figure 1: A rectangular tank of water with a mirrored bottom, labeled "Mirror" along the bottom. A dashed vertical line labeled "Normal" is drawn at the point where a beam enters the top surface of the water at angle θ_i from the normal. The region above the water surface is labeled n_a; the region inside the tank (the water) is labeled n_w.]

(b) Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with the water. The models are shown in Figure 2.

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Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.
Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar in the water as in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).
[Figure 2: Two diagrams side by side, each a rectangular tank with a mirrored bottom and a valve/beam entering at angle θ_i from the "Normal." Model A: a single shaded region labeled n_m inside the tank below the surface labeled n_a (air above), with the beam entering at θ_i. Model B: three horizontal layers inside the tank, from top to bottom labeled n_w (top layer, unshaded), n_m (middle layer, lightly shaded), and n_b

Question 1

(bottom layer, densely shaded/hatched), with air (n_a) above and the beam entering at θ_i.]

Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

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(10 points, suggested time 20 minutes)

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[Figure 1: A rectangular tank of water with a mirrored bottom, labeled "Mirror" along the bottom. A dashed vertical line labeled "Normal" is drawn at the point where a beam enters the top surface of the water at angle θ_i from the normal. The region above the water surface is labeled n_a; the region inside the tank (the water) is labeled n_w.]

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(c)(i) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		**Model B**	
θ_i	Incident angle of the beam in air	θ_i	Incident angle of the beam in air
θ_1	Angle the beam makes with the normal in the mixture in Model A	θ_2	Angle the beam makes with the normal in the top layer in Model B
		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

Question 1

Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

(c)(ii) Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

______ θ_1 ______ θ_2 ______ θ_3 ______ θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

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(10 points, suggested time 20 minutes)
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[Figure 1: A rectangular tank of water with a mirrored bottom, labeled "Mirror" along the bottom. A dashed vertical line labeled "Normal" is drawn at the point where a beam enters the top surface of the water at angle θ_i from the normal. The region above the water surface is labeled n_a; the region inside the tank (the water) is labeled n_w.]

(d) [Figure: Three diagrams. "Original Tank" — a rectangular water tank with a mirrored bottom, beam entering at θ_i from the Normal, region above labeled n_a, water labeled n_w. "Model A" — same tank with a single mixture layer labeled n_m. "Model B" — same tank with three layers labeled n_w (top), n_m (middle), n_b (bottom).]

Question 1

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.
Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

Solution 1

(a) Mark scheme

- Sketch: the beam refracts at the water surface toward the normal (since $n_w > n_a$), so it travels in a straight line from the entry point down to the mirrored bottom at an angle from the normal that is LESS than θ_i. It reflects off the mirror and travels back up along a path that is the mirror image of the incoming path, symmetric about the vertical line through the point of reflection on the mirror, exiting the water surface (parallel to, but horizontally displaced from, the entry ray). Point 1: straight-line path from entry point to the bottom with refraction angle $< \theta_i$. Point 2: continuous path symmetric about the vertical axis through the point where the beam hits the mirror.

Solution 1

(b)

Mark scheme

- As light travels from one medium into a medium with a higher index of refraction (the sugar mixture, $n_m > n_w$), the speed of light decreases while the frequency of the light remains the same. Therefore, by $\lambda = v/f$, the wavelength of the light decreases. Point earned for indicating the wavelength decreases with no incorrect statements.

Solution 1

(c)(i)

Mark scheme

- Applying Snell's law across each boundary and combining (since $n_1 \sin\theta_1 = n_2 \sin\theta_2$ at every interface): $n_a \sin\theta_i = n_w \sin\theta_2 = n_m \sin\theta_3 = n_b \sin\theta_4$. Using only the air and bottom-layer boundary: $n_a \sin\theta_i = n_b \sin\theta_4$, so $\sin\theta_4 = \dfrac{n_a}{n_b}\sin\theta_i$, i.e. $\theta_4 = \sin^{-1}\left(\dfrac{n_a}{n_b}\sin\theta_i\right)$. Full credit for a correct Snell's-law application across the two media boundaries (air to bottom layer directly is acceptable).

Solution 1

(c)(ii) Mark scheme

- Ranking from greatest (1) to least (4): θ_2 is rank 1 (greatest), θ_1 and θ_3 are tied at rank 2, θ_4 is rank 3 (least). Explanation: θ_2 has the greatest value because water (top layer, Model B) has the lowest index of refraction of the layers. θ_1 and θ_3 are equal because each is in a layer with the same index of refraction (n_m), but the angles are smaller than θ_2 because the index of refraction is larger in this layer. θ_4 has the smallest value because the bottom layer (n_b) has the highest index of refraction (larger index of refraction bends the ray more toward the normal, i.e. smaller angle). Four points: (1) θ_4 alone is the smallest angle; (1) θ_2 alone is the largest angle; (1) $\theta_1 = \theta_3$; (1) explanation correctly relates index of refraction to angle size (higher index $\rightarrow$ smaller angle from normal).

Solution 1

(d) Mark scheme

- Both horizontal distances d_A and d_B are less than d_w (the original all-water horizontal distance). The light rays for all scenarios enter from air, but in Models A and B the light rays enter a medium (or media) with an index of refraction greater than that of water. Therefore the light rays bend more toward the normal in Models A and B than in the original all-water tank. Bending more toward the normal results in a shorter horizontal distance traveled before the beam reaches the mirrored bottom and exits. Two points: (1) both $d_A, d_B < d_w$ with an attempt at a relevant explanation; (1) correctly indicating the horizontal distance decreases as refraction toward the normal increases.