Past-paper walk-through

Kirchhoff's Loop Rule ▪ 11.6

eXercise

Questions in this set

- 2024 Q3
- 2023 Q2
- 2019 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2024 ▪ Q3

[Figure 1: A circuit diagram. An ideal battery of emf $\mathcal{E}$ is connected on the left. From the top of the battery, a wire goes right through resistor R_1 to a node. From that node, resistor R_2 branches down in parallel back to the bottom rail. Also from that same node, a wire continues right to another node, from which resistors R_3 (upper) and R_4 (lower) branch down in parallel, both returning to the bottom rail, which connects back to the battery.]

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(a)(i) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and R only.

Derive an expression for the current I_1 in Resistor R_1.

(a)(ii) **Derive** an expression for the current I_3 in Resistor R_3.

Question 3

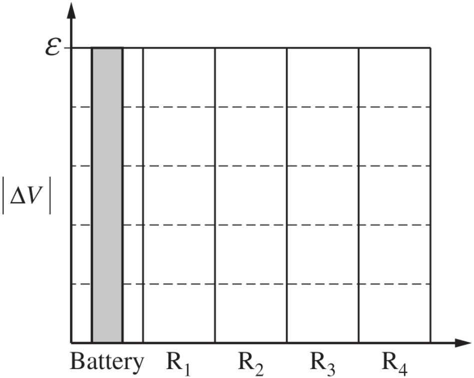


Figure 2

Question 3

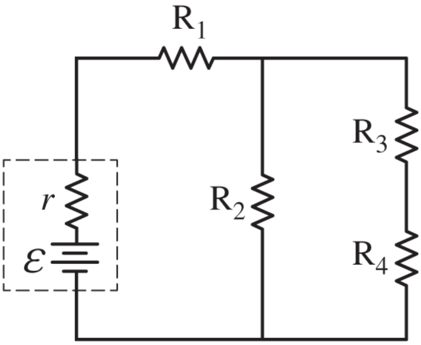


Figure 3

Question 3

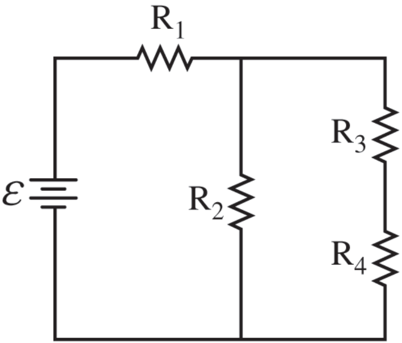


Figure 1

Question 3

2024 ▪ Q3

[Figure 1: A circuit diagram. An ideal battery of emf $\mathcal{E}$ is connected on the left. From the top of the battery, a wire goes right through resistor R_1 to a node. From that node, resistor R_2 branches down in parallel back to the bottom rail. Also from that same node, a wire continues right to another node, from which resistors R_3 (upper) and R_4 (lower) branch down in parallel, both returning to the bottom rail, which connects back to the battery.]

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(b) [Figure 2: A partially completed bar chart. The vertical axis is labeled $|\Delta V|$ with a horizontal dashed reference line marked $\mathcal{E}$ near the top. The horizontal axis lists categories "Battery", R_1, R_2, R_3, R_4. A single shaded bar is drawn above "Battery" reaching up to the $\mathcal{E}$ line; the bars for R_1, R_2, R_3, R_4 are blank for the student to draw.]

Question 3

The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.
- In Figure 2, **draw** a bar to represent $|\Delta V|$ across **each** resistor, relative to the emf $\mathcal{E}$ of the ideal battery. - The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a "0" in that column.
A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed as $P = \dfrac{3\mathcal{E}^2}{5R}$.

Question 3

2024 ▪ Q3

[Figure 1: A circuit diagram. An ideal battery of emf $\mathcal{E}$ is connected on the left. From the top of the battery, a wire goes right through resistor R_1 to a node. From that node, resistor R_2 branches down in parallel back to the bottom rail. Also from that same node, a wire continues right to another node, from which resistors R_3 (upper) and R_4 (lower) branch down in parallel, both returning to the bottom rail, which connects back to the battery.]

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(c) State whether the expression for P is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

Question 3

2024 ▪ Q3

[Figure 1: A circuit diagram. An ideal battery of emf $\mathcal{E}$ is connected on the left. From the top of the battery, a wire goes right through resistor R_1 to a node. From that node, resistor R_2 branches down in parallel back to the bottom rail. Also from that same node, a wire continues right to another node, from which resistors R_3 (upper) and R_4 (lower) branch down in parallel, both returning to the bottom rail, which connects back to the battery.]

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(d) [Figure 3: The same circuit as Figure 1, but the ideal battery is replaced by a nonideal battery, drawn as an emf $\mathcal{E}$ in series with an internal resistance r enclosed in a dashed box. The rest of the circuit (resistors R_1, R_2, R_3, R_4 in the same configuration) is unchanged.]

Question 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor R_1 is $P_{original}$. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance r to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor R_1 in the new circuit is P_{new}.

Indicate whether P_{new} is greater than, less than, or equal to $P_{original}$.

_____ $P_{new} > P_{original}$ _____ $P_{new} < P_{original}$ _____ $P_{new} = P_{original}$

Briefly **justify** your answer.

Solution 3

(a)(i) Mark scheme

- Total resistance: the right-most branch (Resistors R_3 and R_4 in series) has $R_s = R + R = 2R$. This branch is in parallel with R_2: $\frac{1}{R_p} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$, so $R_p = \frac{2R}{3}$. This parallel combination is in series with R_1: $R_{total} = R + \frac{2R}{3} = \frac{5R}{3}$. Applying Ohm's law to the whole circuit with the ideal battery's emf $\mathcal{E}$: $I_1 = \frac{\Delta V}{R_{total}} = \frac{\mathcal{E}}{5R/3} = \frac{3\mathcal{E}}{5R}$. Scoring: 1 pt for correctly determining $R_{total} = 5R/3$; 1 pt for a multi-step derivation with correct substitution of $\mathcal{E}$ and R_{total} into Ohm's law, consistent with the first point (giving $I_1 = 3\mathcal{E}/5R$).

Solution 3

(a)(ii) Mark scheme

- I_3 is the current through the series branch R_3-R_4. Since R_2 and the $(R_3 + R_4)$ branch are in parallel (same potential difference): $I_2 R_2 = I_3(R_3 + R_4) \Rightarrow I_2 R = I_3(2R) \Rightarrow I_2 = 2I_3$. By the junction rule, $I_1 = I_2 + I_3 = 2I_3 + I_3 = 3I_3$, so $I_3 = I_1/3$. Substituting $I_1 = 3\mathcal{E}/5R$ from part (a)(i): $I_3 = \dfrac{3\mathcal{E}/5R}{3} = \dfrac{\mathcal{E}}{5R}$. (Equivalently: one-third of the total current I_1 flows through R_3, since R_2 carries twice the current of the R_3-R_4 branch.) Scoring: 1 pt for applying Kirchhoff's/Ohm's law to relate I_3 to the current in or potential difference across R_1, R_2, or R_3; 1 pt for a correct final expression from a multi-step derivation, consistent with part (a)(i), giving $I_3 = \mathcal{E}/(5R)$.

Solution 3

(b) Mark scheme

- Bar chart of $|\Delta V|$ for each element (Battery bar already shown at $\mathcal{E}$): the R_1 bar is nonzero, at $\Delta V_1 = I_1 R = (3\mathcal{E}/5R)(R) = 3\mathcal{E}/5$. Since R_2 is in parallel with the R_3-R_4 series branch, $\Delta V_2 = \Delta V_3 + \Delta V_4$, and this equals $\mathcal{E} - \Delta V_1 = 2\mathcal{E}/5$. R_3 and R_4 carry the same current and have equal resistance, so $\Delta V_3 = \Delta V_4$, each equal to half of ΔV_2, i.e. $\mathcal{E}/5$. Final bar heights: Battery $= \mathcal{E}$, $R_1 = 3\mathcal{E}/5$, $R_2 = 2\mathcal{E}/5$, $R_3 = \mathcal{E}/5$, $R_4 = \mathcal{E}/5$. Scoring: 1 pt for a bar showing ΔV_3 is nonzero and equal to ΔV_4; 1 pt for a bar showing ΔV_2 is nonzero and equal to $\Delta V_3 + \Delta V_4$; 1 pt for drawing ALL bars correctly (including correct relative heights of Battery, R_1, R_2, R_3, R_4).

Solution 3

(c) Mark scheme

- The equation $P = \dfrac{3\mathcal{E}^2}{5R}$ is CORRECT. Justification: power dissipated can be written $P = (\Delta V)^2/R$. From the bar chart in part (b), the potential difference across the (ideal) battery is $\mathcal{E}$; from part (a)(i), the total resistance is $R_{total} = 5R/3$. Therefore the total power delivered by the battery is $P = \dfrac{(\Delta V)^2}{R_{total}} = \dfrac{\mathcal{E}^2}{5R/3} = \dfrac{3\mathcal{E}^2}{5R}$, matching the given expression, so it is correct. Scoring: 1 pt for stating the equation is correct/incorrect (consistent with the student's own part (a) derivations or part (b) bar chart) with an attempted justification; 1 pt for a justification relating P to at least one of the potential

Solution 3

difference across the battery, the current in the battery, or R_1, R_2, R_3, R_4; 1 pt for a justification that relates the equation $P = 3\mathcal{E}^2/5R$ to the bar chart from (b) or the derivations from (a)(i)/(a)(ii).

Solution 3

(d) Mark scheme

- $P_{new} < P_{original}$. Justification: the emf $\mathcal{E}$ of the battery is the same in the new circuit, but the new circuit has additional internal resistance r, so its total resistance is greater than in the original circuit. A larger total resistance with the same emf means a smaller total (and branch) current, so the current in R_1 is less in the new circuit than in the original — equivalently, the potential difference across R_1 is less. Since the power dissipated in R_1 depends on $I_1^2 R$ (or $(\Delta V_1)^2/R$), a decrease in I_1 (or ΔV_1) means $P_{new} < P_{original}$. Scoring: 1 pt for correctly selecting $P_{new} < P_{original}$ with an attempted justification; 1 pt for a correct justification citing that the current in R_1 (or the potential difference across R_1) is less in the new circuit than in the original.

Question 2

2023 ▪ Q2

(12 points, suggested time 25 minutes)
Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\ \Omega$.

(a)(i) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.

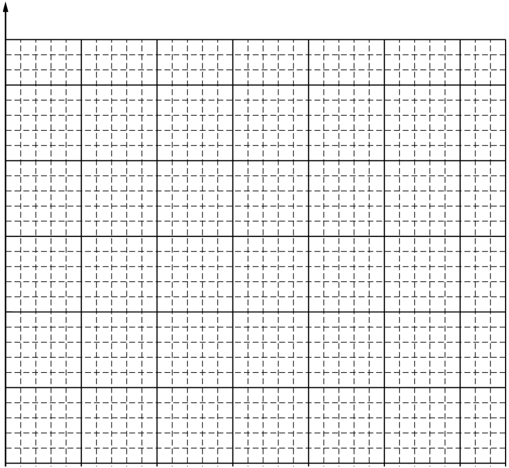
[Diagram: A circuit component box (labeled "Circuit Component") connected in series via a wire to a $500\ \Omega$ resistor, drawn with the standard zigzag resistor symbol. The circuit component is drawn as a rectangle with terminals. The student must complete the diagram with standard circuit equipment (e.g., battery, switch, meters).]

Question 2

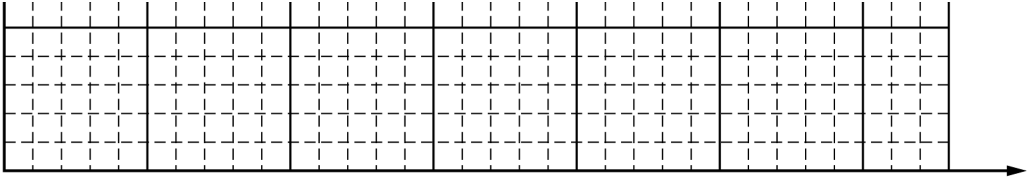
(a)(ii) Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

(a)(iii) What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

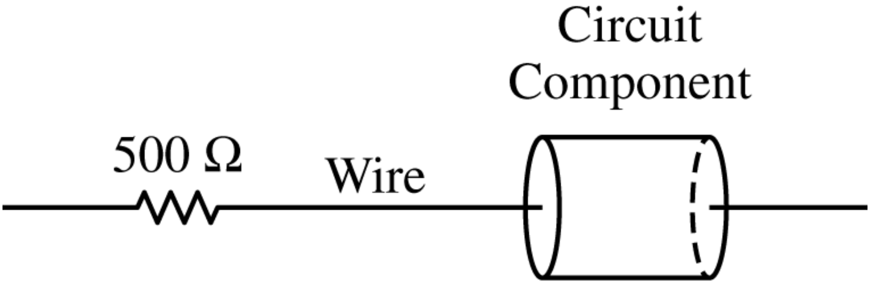
Question 2



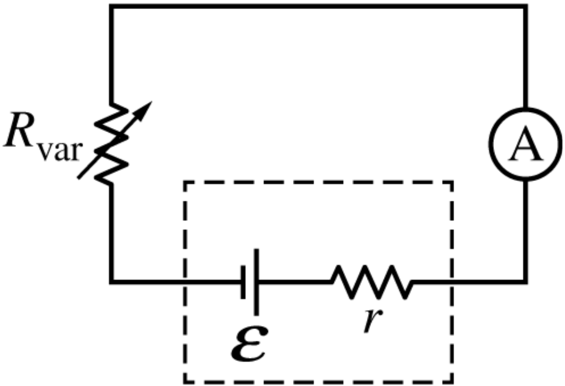
Question 2



Question 2



Question 2



Question 2

2023 ▪ Q2

(12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\ \Omega$.

(b)(i) [Figure: A circuit diagram showing a variable resistor R_{var} connected in series with an ammeter (circle with "A"), and a battery with emf $\mathcal{E}$ and internal resistance r (shown as a battery symbol inside a dashed box labeled $\mathcal{E}$, r).]

The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

Question 2

I (A)	R_{var} (Ω)
0.087	200
0.060	300
0.042	450
0.027	700
0.016	1200

Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

(b)(ii) Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: ____________________ Vertical Axis: ____________________

(b)(iii) Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data. You may use the blank columns in the table for any quantities you graph other than the given data.

[Graph: A blank grid with light dashed gridlines, with an unlabeled vertical axis (arrow pointing up) and unlabeled horizontal axis (arrow pointing right), for the student to plot and scale the chosen quantities.]

Question 2

(b)(iv) Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

Solution 2

(a)(i) Mark scheme

- A correct circuit diagram: the unknown circuit component is connected in series with the $500\,\Omega$ resistor, a wire, and a battery/power supply, forming a complete circuit that produces current through the component (Point 1: source of potential difference in a complete circuit driving current through the component). A voltmeter is connected in parallel across the component (or across the resistor) OR an ammeter is connected in series in the circuit to measure current (a lightbulb in series is an acceptable ammeter substitute) (Point 2: an appropriately connected measurement device, e.g. voltmeter or ammeter).

Solution 2

(a)(ii) Mark scheme

- Procedure: measure one of — the potential difference across the circuit component, the current in the circuit, or the potential difference across the known $500\,\Omega$ resistor (Point 1). Take this measurement at two different times: immediately after the circuit is closed, and again a long time after the circuit is closed (or take a single measurement a long time after closing), consistent with the chosen quantity (Point 2). Example: measure the current in the ammeter immediately after the circuit is closed and a long time after the circuit is closed; or measure the potential difference across the component (or across the $500\,\Omega$ resistor) immediately after closing and a long time after closing.

Solution 2

(a)(iii) Mark scheme

- Expected results if the component is an uncharged capacitor: immediately after the circuit is closed, a nonzero current is measured; a long time after the circuit is closed, the current decreases to zero (equivalently, the charge on the capacitor's plates increases over time until fully charged) (Point 1). The electric potential difference across the capacitor increases from zero (immediately after closing) up to a value equal to the battery's potential difference a long time after closing, since the fully-charged capacitor allows no further current and (by Kirchhoff's loop rule) no potential difference remains across the other circuit elements (Point 2).

Solution 2

(b)(i)

Mark scheme

- Applying Kirchhoff's loop rule around the circuit (battery of emf $\mathcal{E}$ and internal resistance r, in series with variable resistor R_{var}, current I): $0 = +\mathcal{E} - Ir - IR_{var}$.

Solution 2

(b)(ii)

Mark scheme

- Any pair of quantities that linearizes the loop equation to let students find $\mathcal{E}$ from slope or intercept, e.g.: I vs. $\dfrac{1}{r + R_{var}}$ (slope $= \mathcal{E}$); or IR_{var} vs. I (y-intercept $= \mathcal{E}$); or R_{var} vs. $\dfrac{1}{I}$.

Solution 2

(b)(iii)

Mark scheme

- A correctly constructed graph of the chosen linearized quantities: (1) both axes carry numerical values on a linear scale with appropriate labels and units (e.g. I (A) vs. $1/(r + R_{var})$ (Ω^{-1}), or IR_{var} (V) vs. I (A)); (1) the data points are plotted so they span at least half of the grid area; (1) a straight best-fit line is drawn that reasonably approximates the trend of the plotted data points.

Solution 2

(b)(iv) Mark scheme

- Using $I = \mathcal{E}\left(\frac{1}{r+R_{var}}\right) = \mathcal{E}\left(\frac{1}{30\,\Omega + R_{var}}\right)$, the slope of I vs. $1/(r+R_{var})$ equals $\mathcal{E}$: e.g. slope $= \frac{(0.08\text{ A} - 0.04\text{ A})}{(0.004\,\Omega^{-1} - 0.002\,\Omega^{-1})} \approx \mathcal{E} \approx 20\text{ V}$. Equivalently, for $IR_{var} = -Ir + \mathcal{E}$, the y-intercept of IR_{var} vs. I equals $\mathcal{E} \approx 20\text{ V}$. Full credit for any correct use of the graph (slope or intercept, as appropriate to the chosen axes) giving a value with correct units between 18.0 V and 22.0 V.

Question 2

2019 ▪ Q2

[Figure: Two circuit diagrams side by side. "Circuit 1": An ideal variable power supply V_{PS} (circle with battery symbol) connects in a loop to an ammeter A (circle) in series with the top branch, a resistor R in series with a voltmeter V_R on the right branch, all in parallel with V_{PS}. "Circuit 2": Same layout as Circuit 1, except the ammeter A is drawn inside a dashed box together with a zigzag resistor symbol labeled (representing internal resistance), indicating the ammeter has significant internal resistance.]

The two circuits shown above contain an ideal variable power supply, an ohmic resistor of resistance R, an ammeter A, and two voltmeters V_{PS} and V_R. In circuit 1 the ammeter has negligible resistance, and in circuit 2 the ammeter has significant internal ohmic resistance r. The potential difference of the power supply is varied, and measurements of current and potential difference are recorded.

Question 2

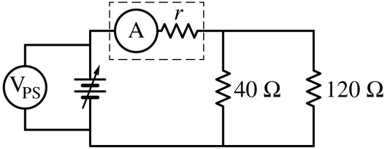
(a) The axes below can be used to graph the current measured by the ammeter as a function of the potential difference measured across the power supply. On the axes, do the following.

- Sketch a possible graph for circuit 1 and label it 1.
- Sketch a possible graph for circuit 2 and label it 2.

[Graph: y-axis labeled "Current", x-axis labeled "Power Supply Potential Difference", origin labeled *O*; blank axes with no data plotted, to be filled in by the student.]

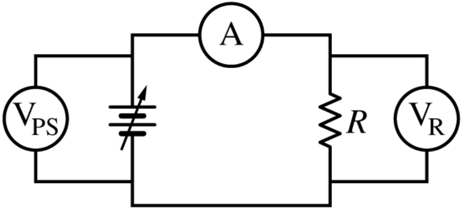
Question 2

(e) Voltmeter V_R in circuit 2 is replaced by a resistor with resistance 120 Ω to create circuit 3 shown below. Voltmeter V_{PS} still reads 3.0 V.

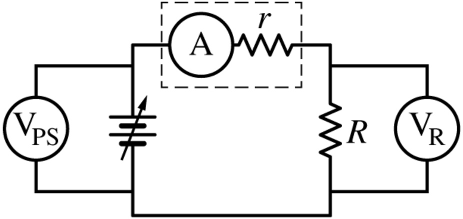


Circuit 3

Question 2



Circuit 1



Circuit 2

Question 2