Past-paper walk-through

Capitors ▪ 10.6

eXercise

Questions in this set

- 2025 Q3
- 2023 Q2
- 2022 Q2
- 2016 Q4
- 2015 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2025 ▪ Q3

In Experiment 1, a student is given a resistor of unknown resistance and an air-filled parallel-plate capacitor of unknown capacitance. The student is asked to predict the expected time constant τ of a circuit if these two circuit elements were connected in series with a battery. The student has access to a battery of known emf, a switch, an ammeter, a ruler, and wires, as shown in Figure 1. The plates of the capacitor are square, and the separation between the plates is small compared to the dimensions of the plates. The capacitor is initially uncharged. Assume that the dielectric constant of air is 1.

[Figure 1: a row of labeled equipment icons — Resistor, Capacitor (parallel plates), Battery, Switch, Ammeter, Ruler, and a bundle of Wires.]

Note: Figure not drawn to scale.

Question 3

(a) Describe a procedure for collecting data that would allow the student to determine the expected time constant τ. In your description, include the measurements to be made. Include any steps necessary to reduce experimental uncertainty.

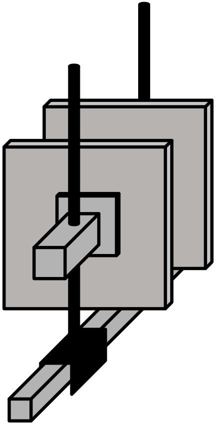
Question 3

Question 3

Question 3

$\lvert\Delta V\rvert$ (V)	q $\left(\times 10^{-10}\ \mathrm{C}\right)$
3.0	2.4
5.0	4.2
7.2	5.6
8.0	6.6
10.0	8.0

Question 3



Question 3

2025 ▪ Q3

In Experiment 1, a student is given a resistor of unknown resistance and an air-filled parallel-plate capacitor of unknown capacitance. The student is asked to predict the expected time constant τ of a circuit if these two circuit elements were connected in series with a battery. The student has access to a battery of known emf, a switch, an ammeter, a ruler, and wires, as shown in Figure 1. The plates of the capacitor are square, and the separation between the plates is small compared to the dimensions of the plates. The capacitor is initially uncharged. Assume that the dielectric constant of air is 1.

[Figure 1: a row of labeled equipment icons — Resistor, Capacitor (parallel plates), Battery, Switch, Ammeter, Ruler, and a bundle of Wires.]

Note: Figure not drawn to scale.

(b) Describe how the collected data could be analyzed to determine τ. Include references to appropriate equations and to relationships between measured and known quantities.

Question 3

2025 ▪ Q3

In Experiment 1, a student is given a resistor of unknown resistance and an air-filled parallel-plate capacitor of unknown capacitance. The student is asked to predict the expected time constant τ of a circuit if these two circuit elements were connected in series with a battery. The student has access to a battery of known emf, a switch, an ammeter, a ruler, and wires, as shown in Figure 1. The plates of the capacitor are square, and the separation between the plates is small compared to the dimensions of the plates. The capacitor is initially uncharged. Assume that the dielectric constant of air is 1.

[Figure 1: a row of labeled equipment icons — Resistor, Capacitor (parallel plates), Battery, Switch, Ammeter, Ruler, and a bundle of Wires.]

Note: Figure not drawn to scale.

(c)(i) In Experiment 2, the student is asked to determine the capacitance C of a new parallel-plate capacitor. For each trial, the absolute value $|\Delta V|$ of the potential

Question 3

difference across the capacitor is varied and the charge q stored on one plate of the fully charged capacitor is measured. Table 1 contains the data collected.

Table 1

$\lvert \Delta V \rvert$ (V)	q $(\times 10^{-10}$ C)
3.0	2.4
5.0	4.2
7.2	5.6
8.0	6.6
10.0	8.0

Indicate two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine C.

Vertical axis:

Horizontalaxis:

(c)(ii) On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.

Question 3

- **Plot** the points you recorded in Table 2.

[Grid: a blank square-ruled graphing grid, approximately 30 by 40 small squares, provided for the student to label axes and plot points.]
(c)(iii) Draw a best-fit line for the data graphed in part C (ii).

Question 3

2025 ▪ Q3

In Experiment 1, a student is given a resistor of unknown resistance and an air-filled parallel-plate capacitor of unknown capacitance. The student is asked to predict the expected time constant τ of a circuit if these two circuit elements were connected in series with a battery. The student has access to a battery of known emf, a switch, an ammeter, a ruler, and wires, as shown in Figure 1. The plates of the capacitor are square, and the separation between the plates is small compared to the dimensions of the plates. The capacitor is initially uncharged. Assume that the dielectric constant of air is 1.
[Figure 1: a row of labeled equipment icons — Resistor, Capacitor (parallel plates), Battery, Switch, Ammeter, Ruler, and a bundle of Wires.]

Note: Figure not drawn to scale.

(d) Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for capacitance C.

Solution 3

(a) Mark scheme

- Procedure must include measuring BOTH: (1) the dimensions of the capacitor – e.g. measure the length of one side of a capacitor plate (for the area A) and the separation distance between the plates (d); and (2) a current – e.g. construct a circuit with the battery, resistor, and capacitor, and measure the (initial) current in the resistor. The procedure must also indicate an appropriate method to reduce experimental uncertainty, e.g. repeat the measurement/procedure multiple times (discharge the capacitor and repeat the current measurement) and average the results, or take repeated trials of the dimension measurements.

Solution 3

(b) Mark scheme

- tau = Req*Ceq can be used to determine tau. Req is found from a correct relationship between resistance, the battery's emf, and the current: Req = emf/I, where I is the (initial) current in the resistor when connected to the battery. Ceq is found from a correct relationship between capacitance and the capacitor's geometry: Ceq = epsilon0*A/d, where A is the area of a capacitor plate (the square of the plate's side length) and d is the separation between the plates. Full credit requires: (1) stating tau = Req*Ceq is used to find tau; (2) both a correct R-relationship (Req = emf/I) and a correct C-relationship (Ceq = epsilon0*A/d).

Solution 3

(c)(i)

Mark scheme

- Indicate a pair of quantities whose functional relationship is linear (based on q = C*|deltaV|), e.g.: plot q as a function of |deltaV| (slope = C); or |deltaV| as a function of q (slope = 1/C); or 1/q as a function of 1/|deltaV|; or 1/|deltaV| as a function of 1/q. Any response that correctly identifies the functional dependence between the varied quantities earns credit, regardless of numerical/physical-constant coefficients or which axis each quantity is graphed on.

Solution 3

(c)(ii)

Mark scheme

- Label both axes, including units, with a linear (evenly spaced) scale – e.g. vertical axis q (x10^-10 C), horizontal axis |deltaV| (V). Plot the five data points from Table 1, converted to the chosen quantities from part C(i): (|deltaV|, q) = (3.0 V, 2.4e-10 C), (5.0 V, 4.2e-10 C), (7.2 V, 5.6e-10 C), (8.0 V, 6.6e-10 C), (10.0 V, 8.0e-10 C). The plotted points must be consistent with the quantities chosen in part C(i) (or with the table / the chosen axes).

Solution 3

(c)(iii)

Mark scheme

- Draw a single straight best-fit (trend) line through the plotted data points, approximating the overall linear trend of the five points – not a line connecting the dots point-to-point, and not required to pass through the origin or through every point.

Solution 3

(d) Mark scheme

- Using two well-separated points on the best-fit line (not raw data points) to compute the slope: slope = delta_q/delta_|deltaV| = (6.4e-10 C - 2.0e-10 C)/(8.0 V - 2.4 V) = 7.9e-11 C/V. Since q = C*|deltaV|, the slope of q vs |deltaV| equals C, so C = slope = 7.9e-11 F, approximately 8e-11 F. Full credit requires (1) correctly relating the slope of the graph to C (e.g. delta_q/delta_|deltaV| = slope = C, or the appropriate reciprocal relation for the axes chosen in C(i)), and (2) a numeric value for C approximately equal to 8e-10^-11 F, accepted in the range 7e-11 F <= C <= 9e-11 F, read from the student's own best-fit line.

Question 2

2023 ▪ Q2

(12 points, suggested time 25 minutes)
Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\ \Omega$.

(a)(i) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.

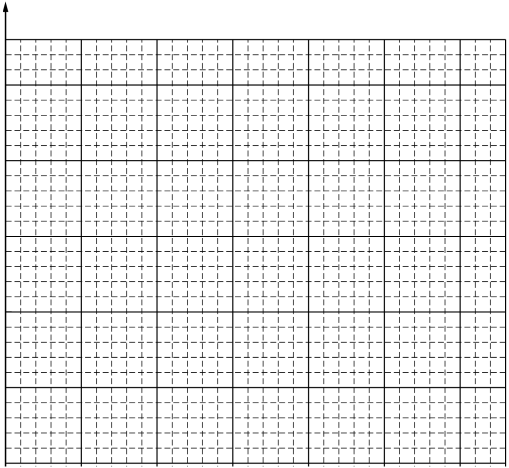
[Diagram: A circuit component box (labeled "Circuit Component") connected in series via a wire to a $500\ \Omega$ resistor, drawn with the standard zigzag resistor symbol. The circuit component is drawn as a rectangle with terminals. The student must complete the diagram with standard circuit equipment (e.g., battery, switch, meters).]

Question 2

(a)(ii) Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

(a)(iii) What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

Question 2



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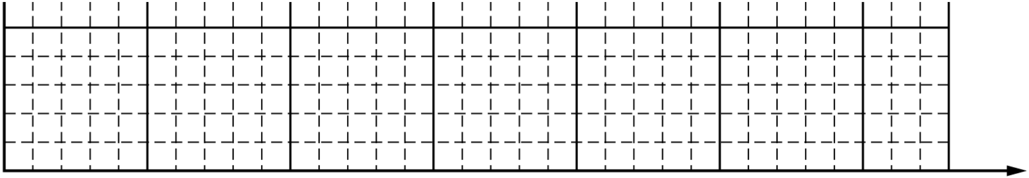

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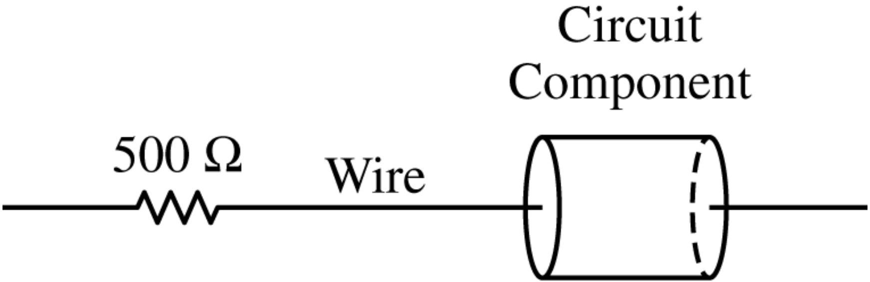
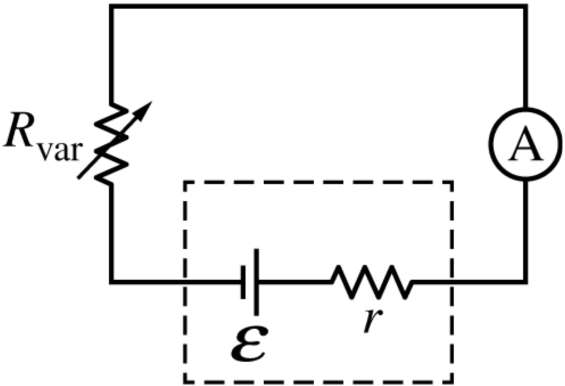


Question 2



Question 2

2023 ▪ Q2

(12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance $500\ \Omega$.

(b)(i) [Figure: A circuit diagram showing a variable resistor R_{var} connected in series with an ammeter (circle with "A"), and a battery with emf $\mathcal{E}$ and internal resistance r (shown as a battery symbol inside a dashed box labeled $\mathcal{E}$, r).]

The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

Question 2

I (A)	R_{var} (Ω)
0.087	200
0.060	300
0.042	450
0.027	700
0.016	1200

Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

(b)(ii) Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: ______________________ Vertical Axis: ______________________

(b)(iii) Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data. You may use the blank columns in the table for any quantities you graph other than the given data.

[Graph: A blank grid with light dashed gridlines, with an unlabeled vertical axis (arrow pointing up) and unlabeled horizontal axis (arrow pointing right), for the student to plot and scale the chosen quantities.]

Question 2

(b)(iv) Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

Solution 2

(a)(i) Mark scheme

- A correct circuit diagram: the unknown circuit component is connected in series with the $500\,\Omega$ resistor, a wire, and a battery/power supply, forming a complete circuit that produces current through the component (Point 1: source of potential difference in a complete circuit driving current through the component). A voltmeter is connected in parallel across the component (or across the resistor) OR an ammeter is connected in series in the circuit to measure current (a lightbulb in series is an acceptable ammeter substitute) (Point 2: an appropriately connected measurement device, e.g. voltmeter or ammeter).

Solution 2

(a)(ii) Mark scheme

- Procedure: measure one of — the potential difference across the circuit component, the current in the circuit, or the potential difference across the known $500\,\Omega$ resistor (Point 1). Take this measurement at two different times: immediately after the circuit is closed, and again a long time after the circuit is closed (or take a single measurement a long time after closing), consistent with the chosen quantity (Point 2). Example: measure the current in the ammeter immediately after the circuit is closed and a long time after the circuit is closed; or measure the potential difference across the component (or across the $500\,\Omega$ resistor) immediately after closing and a long time after closing.

Solution 2

(a)(iii) Mark scheme

- Expected results if the component is an uncharged capacitor: immediately after the circuit is closed, a nonzero current is measured; a long time after the circuit is closed, the current decreases to zero (equivalently, the charge on the capacitor's plates increases over time until fully charged) (Point 1). The electric potential difference across the capacitor increases from zero (immediately after closing) up to a value equal to the battery's potential difference a long time after closing, since the fully-charged capacitor allows no further current and (by Kirchhoff's loop rule) no potential difference remains across the other circuit elements (Point 2).

Solution 2

(b)(i)

Mark scheme

- Applying Kirchhoff's loop rule around the circuit (battery of emf $\mathcal{E}$ and internal resistance r, in series with variable resistor R_{var}, current I): $0 = +\mathcal{E} - Ir - IR_{var}$.

Solution 2

(b)(ii)

Mark scheme

- Any pair of quantities that linearizes the loop equation to let students find $\mathcal{E}$ from slope or intercept, e.g.: I vs. $\dfrac{1}{r+R_{var}}$ (slope $= \mathcal{E}$); or IR_{var} vs. I (y-intercept $= \mathcal{E}$); or R_{var} vs. $\dfrac{1}{I}$.

Solution 2

(b)(iii)

Mark scheme

- A correctly constructed graph of the chosen linearized quantities: (1) both axes carry numerical values on a linear scale with appropriate labels and units (e.g. I (A) vs. $1/(r + R_{var})$ (Ω^{-1}), or IR_{var} (V) vs. I (A)); (1) the data points are plotted so they span at least half of the grid area; (1) a straight best-fit line is drawn that reasonably approximates the trend of the plotted data points.

Solution 2

(b)(iv) Mark scheme

- Using $I = \mathcal{E}\left(\frac{1}{r + R_{var}}\right) = \mathcal{E}\left(\frac{1}{30\,\Omega + R_{var}}\right)$, the slope of I vs. $1/(r + R_{var})$ equals $\mathcal{E}$: e.g. slope $= \frac{(0.08\text{ A} - 0.04\text{ A})}{(0.004\,\Omega^{-1} - 0.002\,\Omega^{-1})} \approx \mathcal{E} \approx 20$ V. Equivalently, for $IR_{var} = -Ir + \mathcal{E}$, the y-intercept of IR_{var} vs. I equals $\mathcal{E} \approx 20$ V. Full credit for any correct use of the graph (slope or intercept, as appropriate to the chosen axes) giving a value with correct units between 18.0 V and 22.0 V.

Question 2

2022 ▪ Q2

[Diagram: Two circuits, each drawn with a battery symbol on the left. The first circuit has a battery connected to two resistors in parallel branches, labeled A and B (both drawn as resistor zig-zag symbols). The second circuit has a battery connected to two resistors in series, labeled C (top) and D (bottom, both zig-zag resistor symbols).]
Students perform an experiment with a battery and four resistors, A, B, C, and D. The resistance of resistors A and C is $R_A = R_C = R$. The resistance of resistors B and D is $R_B = R_D = 2R$. The students create the two circuits shown above and measure the potential differences ΔV_A, ΔV_B, ΔV_C, and ΔV_D across resistors A, B, C, and D, respectively.
(a) From greatest to least, rank the magnitudes of the potential differences across the resistors. Use "1" for the greatest magnitude, "2" for the next greatest magnitude, and

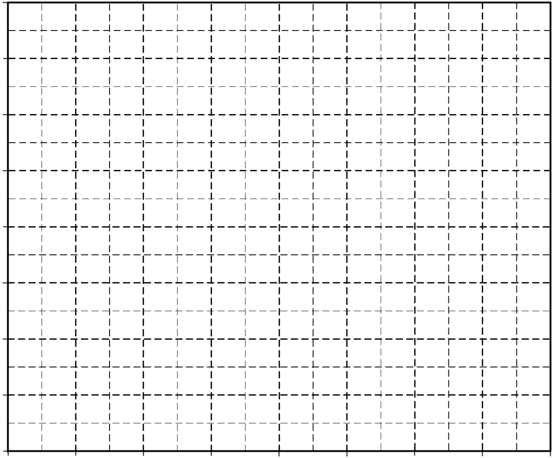
Question 2

so on. If any potential differences have the same magnitude, use the same number for their ranking.

ΔV_A ΔV_B ΔV_C ΔV_D

Justify your answer.

Question 2



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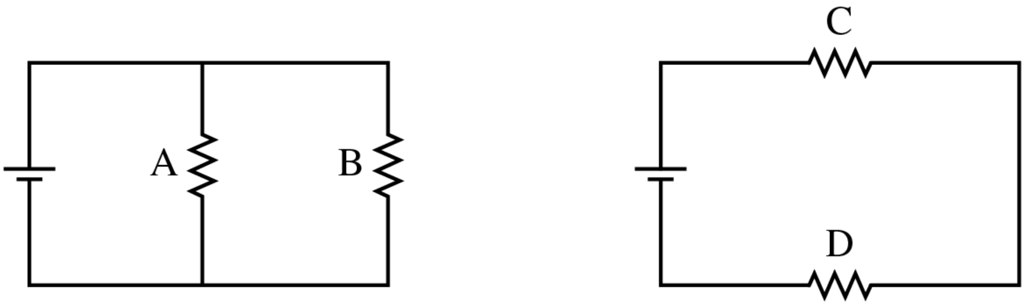

Question 2

2022 ▪ Q2

[Diagram: Two circuits, each drawn with a battery symbol on the left. The first circuit has a battery connected to two resistors in parallel branches, labeled A and B (both drawn as resistor zig-zag symbols). The second circuit has a battery connected to two resistors in series, labeled C (top) and D (bottom, both zig-zag resistor symbols).]

Students perform an experiment with a battery and four resistors, A, B, C, and D. The resistance of resistors A and C is $R_A = R_C = R$. The resistance of resistors B and D is $R_B = R_D = 2R$. The students create the two circuits shown above and measure the potential differences ΔV_A, ΔV_B, ΔV_C, and ΔV_D across resistors A, B, C, and D, respectively.

(b)(i)(n)(t)(r)(o) In another experiment, the students have a capacitor with unknown capacitance C_U. They want to determine C_U by using a battery of potential difference 4.5 V and several other capacitors of known capacitance. They create circuits with the battery, the unknown capacitor, and one of the capacitors of known capacitance. The

Question 2

students wait until the capacitors are fully charged and then record the potential difference ΔV_{known} across the known capacitor and the potential difference ΔV_U across the unknown capacitor. Their data are shown in the table on the following page.

Known Capacitance of Capacitors (μF)	ΔV_{known} (V)	ΔV_U (V)
200	0.91	3.53
300	0.65	3.74
400	0.51	3.95
500	0.42	4.06
600	0.36	4.17

(b)(i) Calculate the amount of charge on the capacitor of known capacitance of 200 μF in the students' experiment.

(b)(ii) Briefly explain why the data in the table provide evidence that the capacitors are connected in series.

(b)(iii) Briefly explain why connecting the capacitors in parallel would not provide enough information to determine the capacitance of the unknown capacitor if the only measuring device available is a voltmeter.

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2022 ▪ Q2

[Diagram: Two circuits, each drawn with a battery symbol on the left. The first circuit has a battery connected to two resistors in parallel branches, labeled A and B (both drawn as resistor zig-zag symbols). The second circuit has a battery connected to two resistors in series, labeled C (top) and D (bottom, both zig-zag resistor symbols).]

Students perform an experiment with a battery and four resistors, A, B, C, and D. The resistance of resistors A and C is $R_A = R_C = R$. The resistance of resistors B and D is $R_B = R_D = 2R$. The students create the two circuits shown above and measure the potential differences ΔV_A, ΔV_B, ΔV_C, and ΔV_D across resistors A, B, C, and D, respectively.

(c)(i) The students want to produce a linear graph of the data so that the capacitance C_U of the unknown capacitor can be determined from the slope of the best-fit line for the data.

Question 2

Indicate two quantities that could be plotted to produce the desired graph. Use the empty columns of the data table in part (b) to record any values that you need to calculate.

Vertical axis ____________________ Horizontal axis ____________________

(c)(ii) Label the axes below and provide an appropriate scale with units. Plot the data points for the quantities indicated in part (c)(i) on the axes and draw a best-fit line.

[A blank grid of dashed gridlines, unlabeled axes, provided for the student to label, scale, and plot the best-fit line on.]

(c)(iii) Using your best-fit line, determine the capacitance of capacitor C_U.

Solution 2

(a)

Mark scheme

- Ranking (greatest to least magnitude of potential difference): $1\ \Delta V_A$, $1\ \Delta V_B$, $3\ \Delta V_C$, $2\ \Delta V_D$ (1 point, for the correct ranking with A and B tied for greatest). (1 point) Indicate that resistors A and B, being in parallel with each other, must have the same potential difference across them. (1 point) Justify that $\Delta V_D > \Delta V_C$ because $R_D = 2R_C$ (same current through the two series/parallel branch resistors, so twice the resistance gives twice the potential difference by $V = IR$).

Solution 2

(b)(i)(n)(t)(r)(o)

Mark scheme

- Introductory/context text only — no scorable response required. It describes the experimental setup for determining the unknown capacitance C_U: a 4.5 V battery, several capacitors of known capacitance, circuits built from the battery + C_U + one known capacitor, waited until fully charged, then ΔV_{known} and ΔV_U recorded in a data table. This sets up parts (b)(i)–(b)(iii) and (c)(i)–(c)(iii).

Solution 2

(b)(i)

Mark scheme

- Using $\Delta V = Q/C$ for the known capacitor: $Q = C\Delta V = (200\ \mu\text{F})(0.91\ \text{V}) = 1.82 \times 10^{-4}$ C. Full credit requires the correct value including units.

Solution 2

(b)(ii)

Mark scheme

- State one of the following as evidence the capacitors are connected in series: the potential differences across the two capacitors are different from each other; OR the sum of the potential differences across the capacitors is constant across trials; OR the sum of the potential differences across the capacitors is approximately equal to the (constant) potential difference across the battery (4.5 V).

Solution 2

(b)(iii) Mark scheme

- Explain one of: with only a voltmeter available, a parallel arrangement gives the known and unknown capacitor the same potential difference always, so this measurement can't distinguish/determine the unknown capacitance; OR the charge on the unknown capacitor cannot be determined this way. Model response: "Both charge and potential difference across the capacitor are needed to determine C. Arranging the capacitors in parallel will mean both capacitors will have the same potential difference. However, capacitors in parallel will have differing amounts of charge, making it impossible to determine the charge, and, therefore, the capacitance of the unknown capacitor."

Solution 2

(c)(i)

Mark scheme

- Indicate two quantities whose plot is linear with slope C_U. Acceptable answers include: vertical axis Q_{known} (computed as $C_{known}\Delta V_{known}$, which equals Q_U in a series circuit) vs. horizontal axis ΔV_U (since $Q_U = C_U \Delta V_U$, slope $= C_U$); or Q_U vs. ΔV_U; or C_{known} vs. $\dfrac{\Delta V_U}{\Delta V_{known}}$.

Solution 2

(c)(ii)

Mark scheme

- (1 point) Label the vertical axis (e.g. Q_{known}, in μC) and horizontal axis (e.g. ΔV_U, in V) with appropriate units consistent with the choice in part (c)(i). (1 point) Plot the data points with valid, consistent scaling so the points span at least half of the grid's width and height. (1 point) Draw a single straight best-fit line that reasonably represents the trend of the plotted points. Example data spans roughly $\Delta V_U \approx 3.5$ to 4.18 V and $Q_{known} \approx 182$ to 216 μC, rising left to right.

Solution 2

(c)(iii)

Mark scheme

- (1 point) Use two well-separated points ON the best-fit line (not raw data points) to calculate its slope. (1 point) Correctly identify that the capacitance C_U equals this slope. Example: $C_U = \dfrac{212\ \mu\text{C} - 190\ \mu\text{C}}{4.10\ \text{V} - 3.67\ \text{V}} = 51.2\ \mu\text{F}$.

Question 4

2016 ▪ Q4

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(a)(i) Determine the current in the resistor and the potential difference across the capacitor immediately after the switch is closed.

(a)(ii) Determine the current in the resistor and the potential difference across the capacitor a long time after the switch is closed.

Question 4

2016 ▪ Q4

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(b)(i) The switch is opened, the capacitor is discharged, and a second, identical capacitor is added to the circuit in series with the other components. The switch is then closed again.

A long time after the switch is closed, the energy stored in the single capacitor in the original circuit is U_1, and the total energy stored in the two capacitors in the new circuit is U_2. Calculate the ratio U_1/U_2.

(b)(ii) The two capacitors in series are to be replaced with a single capacitor that will have the same energy U_2. Indicate a plate area and a distance between the plates for the new capacitor, compared with one of the original capacitors, that will accomplish

Question 4

this. Support your reasoning using appropriate physics principles and/or mathematical models.

Question 4

2016 ▪ Q4

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance R, a capacitor of capacitance C, a battery with potential difference $\mathcal{E}$, and a switch. Initially, the capacitor is uncharged and the switch is open.

(c) The students are then asked to design two circuits each containing a switch, a battery with a small internal resistance, a lightbulb, and a capacitor. In arrangement 1, the bulb should gradually light up after the switch is closed, becoming brightest after the switch has been closed a long time. In arrangement 2, the bulb should be brightest when the switch is first closed, getting dimmer with time, and going out completely when the switch has been closed for a long time.

Using standard symbols, draw two circuit diagrams, one showing a possible circuit for arrangement 1 and the other showing a possible circuit for arrangement 2. Justify your circuit diagrams with a paragraph-length explanation referring to the properties of

Question 4

lightbulbs and capacitors in circuits and the conservation of energy and/or the conservation of charge.

Solution 4

(a)(i)

Mark scheme

- Immediately after the switch is closed, the capacitor is uncharged, so there is no potential difference across it: V_C $=$ 0. Since the capacitor briefly behaves like a plain wire (no opposition to current) at this instant, the entire battery EMF ε appears across the resistor, giving current I $= \varepsilon/$R. So I $= \varepsilon/$R and V_C $=$ 0.

Solution 4

(a)(ii)

Mark scheme

- A long time after the switch is closed, the capacitor is fully charged and allows no more current to flow, so $I = 0$ everywhere in the series circuit. With zero current, there is no potential drop across the resistor, so the entire battery EMF appears across the capacitor: V_C $= \varepsilon$.

Solution 4

(b)(i) Mark scheme

- Original circuit: a single capacitor C fully charged to the battery EMF ε stores U1 = (1/2)Cε^2. New circuit: two identical capacitors C in series (equivalent capacitance C/2) share the battery voltage equally, so each has a potential difference of $\varepsilon/2$ across it (equivalently, the equivalent capacitance of the new circuit is half that of the original). Energy in each capacitor: (1/2)C(ε/2)^2 = Cε^2/8; total energy of both: U2 = 2 · (Cε^2/8) = Cε^2/4. Ratio: U1/U2 = (Cε^2/2)/(Cε^2/4) = 2. Full credit needs (1) a calculation indicating either that the potential difference across each new capacitor is half that of the original single capacitor, or that the new circuit's equivalent capacitance is half the original's

Solution 4

(giving U1 $=$ (1/2)Cε^2 and U2 $=$ Cε^2/4), and (2) correctly computing the ratio U1/U2 $=$ 2.

Solution 4

(b)(ii) Mark scheme

- Using $U = (1/2)CV\hat{}2$: if the new single capacitor is connected the same way as one original single capacitor (same potential difference across it as before), then for its stored energy to equal $U2 = (1/2$ of the original $U1)$, the new capacitor must have HALF the capacitance of the original single capacitor: $C\_new = C/2$. Since $C = \varepsilon 0\ A/d$, halving the capacitance while keeping the same plate separation d requires halving the plate area A. So: keep the same distance between plates as the original capacitor, but use half the plate area. (Any other area/spacing combination consistent with the ratio found in part (b)(i) and properly justified via $C = \varepsilon 0\ A/d$ is also acceptable.)

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(c) Mark scheme

- Arrangement 1 (bulb gradually brightens, becomes brightest after a long time): the battery, switch, and CAPACITOR form the main series loop, with the LIGHTBULB connected in PARALLEL with the capacitor. Arrangement 2 (bulb brightest immediately, dims and goes out): the battery, switch, LIGHTBULB, and capacitor are all connected in one SERIES loop (bulb and capacitor in series with each other). General principle: the bulb is brightest when the current through it is at its maximum, and a capacitor eventually stops current from flowing in its own branch once the potential difference across its plates equals the battery's EMF. Series circuit (arrangement 2) behavior: the same current flows through both the bulb and the capacitor; right after the switch closes the capacitor is uncharged

Solution 4

and offers no opposition (acts like a wire), so maximum current flows and the bulb is brightest; as the capacitor charges, the current decreases and the bulb dims, until the capacitor is fully charged, blocks all current, and the bulb goes out. (Equivalently in terms of potential difference: right after closing, the uncharged capacitor has zero potential difference, so the bulb has the full battery potential difference and is brightest; as the capacitor charges it takes on more of the potential difference, leaving less for the bulb, until the capacitor has the entire battery potential difference and the bulb has none.) Parallel circuit (arrangement 1) behavior: current is shared between the two branches; right after the switch closes almost all current flows through the initially-uncharged, low-resistance capacitor branch and little goes through the bulb, so the bulb starts dim; as the capacitor charges it draws less current, redirecting more into the bulb branch, until the capacitor is fully charged (open circuit) and all current flows through the bulb,

Solution 4

making it brightest. (Equivalently in terms of potential difference: both branches share the same potential difference across them at all times; the bulb starts with the same zero potential difference as the initially-uncharged capacitor and ends up with the full battery potential difference once the capacitor is fully charged, so the bulb ends up brightest.) Full credit needs (1) two correct circuit diagrams each matched to the correct arrangement, (2) indicating the bulb is brightest when current through it is maximum and that the capacitor stops current in its branch once its plate potential difference equals the battery EMF, (3) a correct description of the series-circuit (arrangement 2) behavior using either current or potential-difference reasoning, (4) a correct description of the parallel-circuit (arrangement 1) behavior using either current or potential-difference reasoning, and (5) a response written with sufficient paragraph structure.

Question 4

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[Figure: A diagram showing an "Electron Source" (small filled square) at the top, above two horizontal parallel plates (shown as pairs of horizontal line segments with a gap between the left and right portions, indicating a hole at the center of each plate). The top plate and bottom plate are separated by distance d (marked with a vertical double-headed arrow on the left). An electron travels straight down through the hole in the top plate with velocity v_0 (arrow pointing down, at the top plate's central gap), continues down through the gap between the plates, and exits through the hole in the bottom plate with velocity v_f (arrow pointing down, at the bottom plate's central gap). Note: Figure not drawn to scale.]

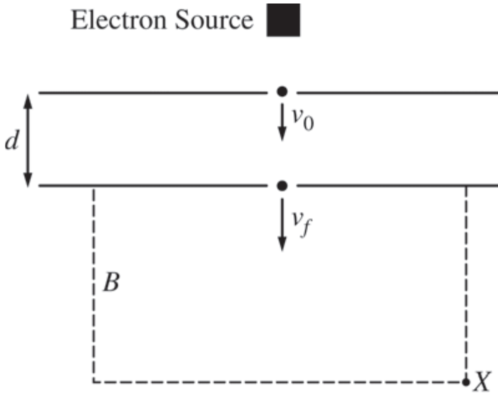
The apparatus shown in the figure above consists of two oppositely charged parallel conducting plates, each with area $A = 0.25\ \text{m}^2$, separated by a distance $d = 0.010$ m. Each plate has a hole at its center through which electrons can pass. High velocity

Question 4

electrons produced by an electron source enter the top plate with speed $v_0 = 5.40 \times 10^6$ m/s, take 1.49 ns to travel between the plates, and leave the bottom plate with speed $v_f = 8.02 \times 10^6$ m/s.

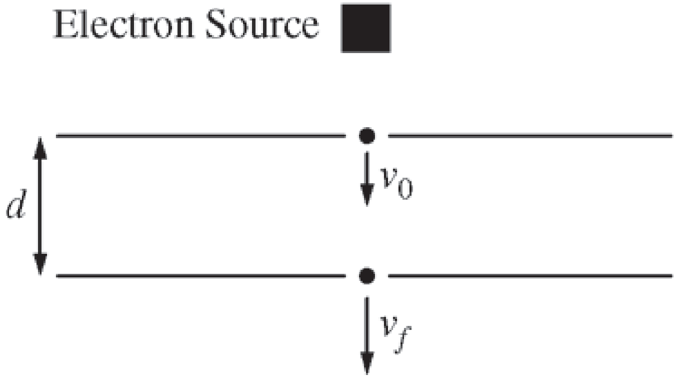
(a) Which of the plates, top or bottom, is negatively charged? Support your answer with a reference to the direction of the electric field between the plates.

Question 4



Note: Figure not drawn to scale.

Question 4



Note: Figure not drawn to scale.

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[Figure: A diagram showing an "Electron Source" (small filled square) at the top, above two horizontal parallel plates (shown as pairs of horizontal line segments with a gap between the left and right portions, indicating a hole at the center of each plate). The top plate and bottom plate are separated by distance d (marked with a vertical double-headed arrow on the left). An electron travels straight down through the hole in the top plate with velocity v_0 (arrow pointing down, at the top plate's central gap), continues down through the gap between the plates, and exits through the hole in the bottom plate with velocity v_f (arrow pointing down, at the bottom plate's central gap). Note: Figure not drawn to scale.]

The apparatus shown in the figure above consists of two oppositely charged parallel conducting plates, each with area $A = 0.25\ \mathrm{m}^2$, separated by a distance $d = 0.010$ m. Each plate has a hole at its center through which electrons can pass. High velocity electrons produced by an electron source enter the top plate with speed $v_0 = 5.40 \times 10^6$ m/s, take 1.49 ns to travel between the plates, and leave the bottom plate with speed $v_f = 8.02 \times 10^6$ m/s.

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(b) Calculate the magnitude of the electric field between the plates.

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[Figure: A diagram showing an "Electron Source" (small filled square) at the top, above two horizontal parallel plates (shown as pairs of horizontal line segments with a gap between the left and right portions, indicating a hole at the center of each plate). The top plate and bottom plate are separated by distance d (marked with a vertical double-headed arrow on the left). An electron travels straight down through the hole in the top plate with velocity v_0 (arrow pointing down, at the top plate's central gap), continues down through the gap between the plates, and exits through the hole in the bottom plate with velocity v_f (arrow pointing down, at the bottom plate's central gap). Note: Figure not drawn to scale.]

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Question 4

(c) Calculate the magnitude of the charge on each plate.

Question 4

2015 ▪ Q4

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(d)(i) The electrons leave the bottom plate and enter the region inside the dashed box shown below, which contains a uniform magnetic field of magnitude B that is perpendicular to the page. The electrons then leave the magnetic field at point X.
[Figure: Same electron-source/parallel-plate diagram as above (Electron Source square, top plate, electron traveling down at v_0, bottom plate, electron traveling down at v_f), but now below the bottom plate is a dashed-line box labeled B on its left edge, representing a region of uniform magnetic field perpendicular to the page. The bottom-right corner of the dashed box is labeled point X. Note: Figure not drawn to scale.]
On the figure above, sketch the path of the electrons from the bottom plate to point X. Explain why the path has the shape that you sketched.
(d)(ii) Indicate whether the magnetic field is directed into the page or out of the page. Briefly explain your choice.

Solution 4

(a) Mark scheme

- The top plate is negative. The electron speeds up as it travels from the top plate to the bottom plate ($v_f > v_0$: 8.02×10^6 m/s at exit vs. 5.40×10^6 m/s at entry), so the net electric force on it points downward, in its direction of motion. (1) Relate this force direction to the field direction: since the electron's charge is negative, the field must point opposite to the force on it, i.e. upward, from the bottom plate toward the top plate. (2) Relate the field direction to the plate charges: field lines point away from positive charge and toward negative charge, so a field pointing from the bottom plate toward the top plate means the bottom plate is positive and the top plate is negative. No credit for stating 'the top plate

Solution 4

is negative' without this two-step explanation (force-to-field, then field-to-charge-sign).

Solution 4

(b) Mark scheme

- Two equally-creditable 4-step derivations, both arriving at $E = 10{,}000$ N/C. KINEMATIC ROUTE: (1) find the electron's acceleration between the plates, $a = (v_f - v_i)/t$; (2) apply Newton's second law for the force needed, $F = ma = m(v_f - v_i)/t$; (3) set the electric force equal to this, $eE = F$; (4) solve for E and substitute: $E = \dfrac{m(v_f - v_i)}{et} = \dfrac{(9.11 \times 10^{-31}\ \text{kg})(8.02 \times 10^{6} - 5.40 \times 10^{6}\ \text{m/s})}{(1.6 \times 10^{-19}\ \text{C})(1.49 \times 10^{-9}\ \text{s})} = 10{,}000$ N/C. ENERGY ROUTE: (1) apply conservation of energy, $\Delta K = \Delta U$; (2) use $\Delta U = e\Delta V$, giving $\frac{1}{2}m_e(v_f^2 - v_i^2) = e\Delta V$; (3) relate potential difference to field and plate separation,

Solution 4

$\Delta V = Ed$, giving $\frac{1}{2}m_e(v_f^2 - v_i^2) = eEd$; (4) solve for E and substitute (plate separation d = 0.010 m): $E = \dfrac{m_e(v_f^2 - v_i^2)}{2ed} = \dfrac{(9.11 \times 10^{-31})[(8.02 \times 10^6)^2 - (5.40 \times 10^6)^2]}{2(1.6 \times 10^{-19}\ \mathrm{C})(0.010\ \mathrm{m})} = 10{,}000$ N/C. Each of the four labeled steps in whichever route the student uses earns one point.

Solution 4

(c)

Mark scheme

- Use the parallel-plate relation $E = Q/(\varepsilon_0 A)$, rearranged to $Q = \varepsilon_0 AE$, and correctly substitute the known values (plate area $A = 0.25$ m^2, $E = 10{,}000$ N/C from part (b)):
  $Q = (8.85 \times 10^{-12}$ C^2/N $\cdot$ m$^2)(0.25$ m$^2)(10{,}000$ N/C$) = 2.2 \times 10^{-8}$ C. This is the magnitude of the charge on each plate.

Solution 4

(d)(i)

Mark scheme

- Draw a reasonably circular arc from the point where the electron leaves the bottom plate to point X (no penalty for exactly where the path starts relative to the arrow tip) (1 point). Explain that the magnetic force on a moving charge is always perpendicular to its velocity ($\vec{F} = q\vec{v} \times \vec{B}$), so as the electron moves, this force continuously acts perpendicular to the instantaneous velocity — a centripetal-type force that continuously turns the velocity direction without changing speed, producing a curved, circular path from the bottom plate to X (1 point).

Solution 4

(d)(ii) Mark scheme

- The magnetic field points out of the page. For the electron to reach point X, the magnetic field must exert a centripetal force on the electron directed toward the top-right corner of the dashed box (curving its path toward X). Using the right-hand rule for a positive charge moving with the electron's velocity would give a force in one rotational sense; since the electron is negatively charged, the actual force (and hence the field needed to produce it) is reversed from that positive-charge result (equivalently, apply the left-hand rule directly for the negative charge). Working through the rule with the electron's actual velocity and the required centripetal direction shows the field must point out of the page. No credit for stating the field direction without this explanation; credit can be earned

Solution 4

for a correct analysis at any individual point along the path, not just the entry point.