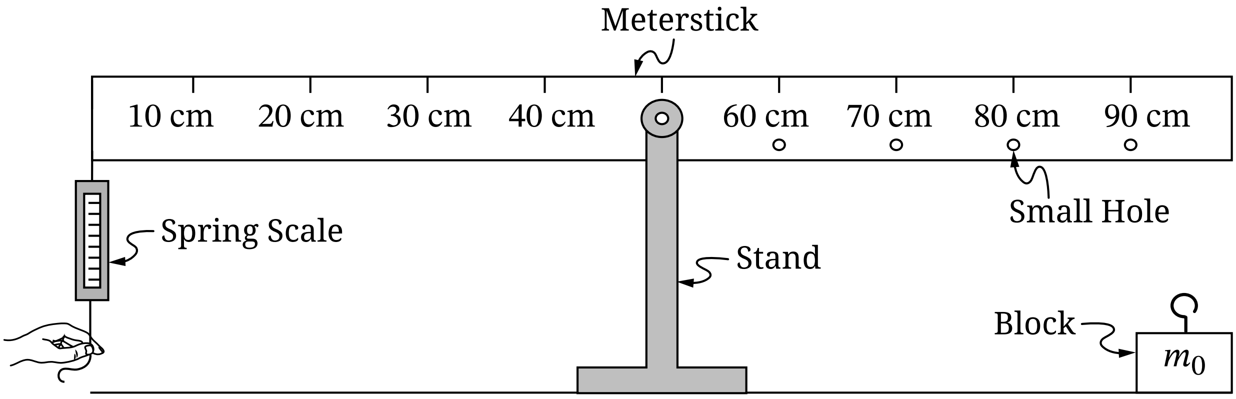
Question 3: Version J

3. Students are investigating balancing systems using the following setup. The students have a spring scale of negligible mass that is fixed to one end of a uniform meterstick. The center of the meterstick is attached to a stand on which the meterstick can pivot. There is a hook of negligible mass fixed to the top of a block of mass m_0. The hook can be attached to the meterstick through one of the small holes in the meterstick, as shown in Figure 1. The students do not have a direct way to measure the mass of the block. The block cannot be attached to the spring scale.

Figure 1



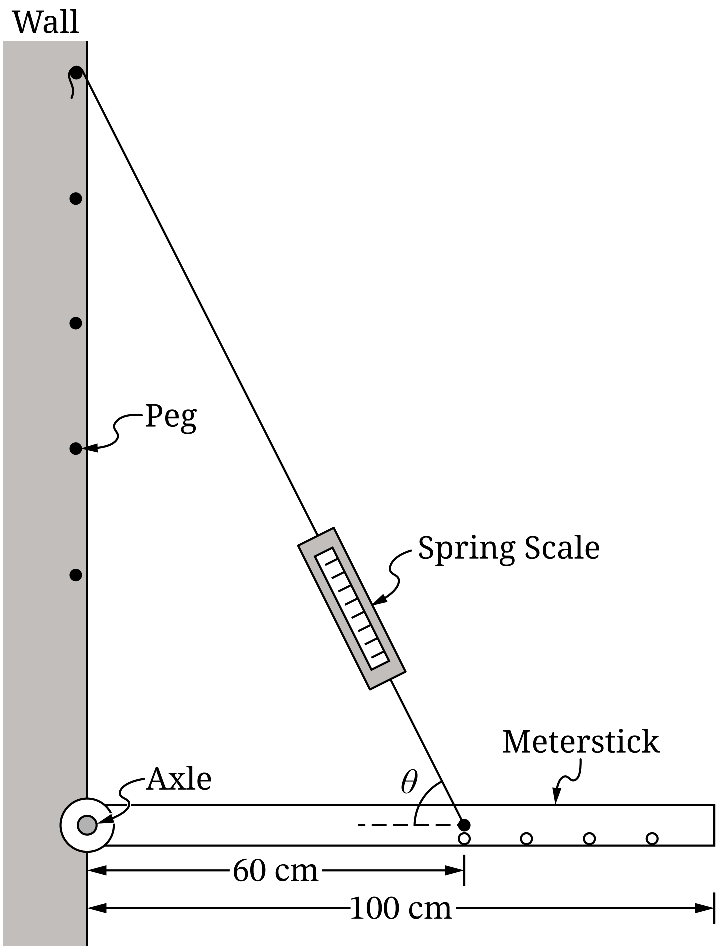
The students are asked to take measurements that will allow the students to create a linear graph whose slope could be used to determine the mass m_0 of the block.

A. Describe an experimental procedure to collect data that would allow the students to determine m_0. Include any steps necessary to reduce experimental uncertainty.

B. Describe how the data collected in part A could be graphed and how that graph would be analyzed to determine m_0.

The students have an identical meterstick of mass M that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

Figure 2



The angle θ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension F_T required to hold the meterstick horizontal. Table 1 shows the measured values of θ and F_T.

Table 1

θ (degrees)	F_T (N)
22	21
31	17
36	13
45	12
80	8

The students correctly determine that the relationship between F_T and θ is given by

$$F_T = \frac{5Mg}{6\sin\theta}.$$

The students create a graph with $\frac{1}{\sin\theta}$ plotted on the horizontal axis.

C.

i. **Indicate** what measured or calculated quantity could be plotted on the vertical axis to yield a linear graph whose slope can be used to calculate an experimental value for the mass M of the meterstick.

Vertical axis: ______ Horizontal axis: $\frac{1}{\sin\theta}$

ii. On the blank grid provided, create a graph of the quantities indicated in part C (i) that can be used to determine M.

- Use Table 2 to record the data points or calculated quantities that you will plot.
- Clearly **label** the vertical axis, including units as appropriate.
- **Plot** the points you recorded in Table 2.

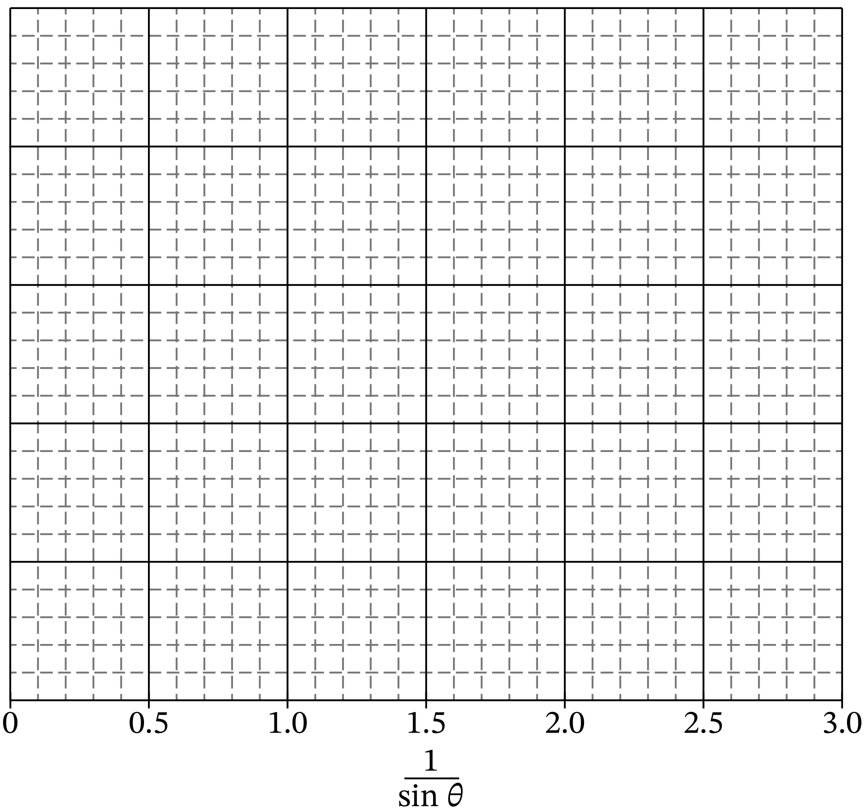


Figure 3

iii. **Draw** a straight best-fit line for the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for the mass M of the meterstick.

Begin your response to **QUESTION 3** on this page.

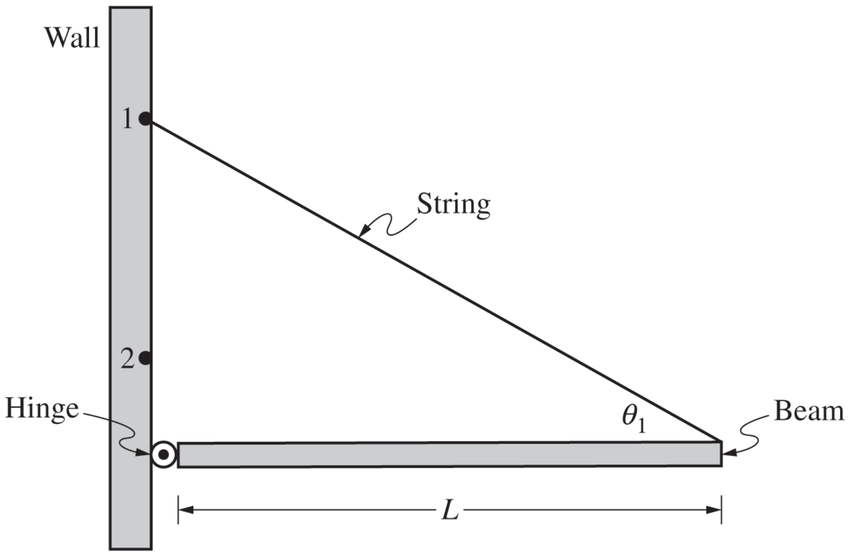


Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H, and the angle between the beam and the string is $\theta = \theta_1$.

(a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

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Continue your response to **QUESTION 3** on this page.

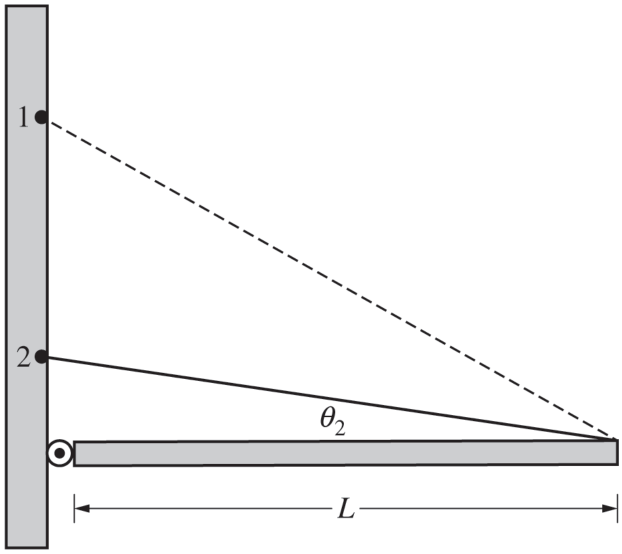

Figure 2

(b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is F_{T1}. The magnitude of the tension in the string shown in Figure 2 is F_{T2}. **Indicate** which of the following correctly compares F_{T2} with F_{T1}.

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

(c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of M, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

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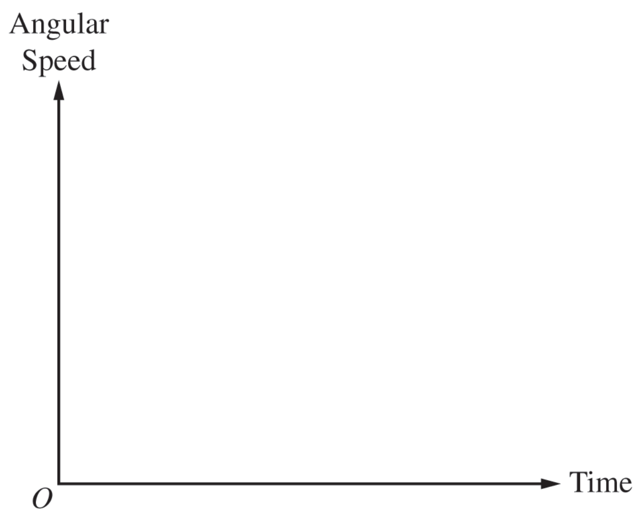
Continue your response to **QUESTION 3** on this page.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.



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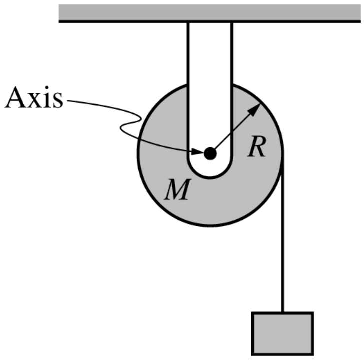

. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

(a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M, R, F_T, and physical constants as appropriate.

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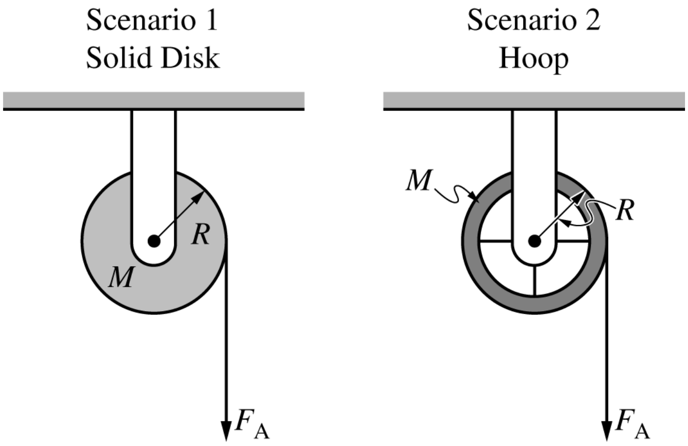
Continue your response to **QUESTION 4** on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

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