

## Question 1: Mathematical Routines (MR)

10 points

A (i) For sketching **one** of the following:

Point A1

- A constant  $p_x$  for  $t < t_1$
- A constant  $p_x$  for  $t > t_1$

For sketching a line that demonstrates momentum is constant

Point A2

Accept **one** of the following:

- A continuous, nonzero, horizontal line from  $t = 0$  to  $t = t_2$
- A continuous, nonzero, horizontal line from  $t = 0$  to  $t > t_2$

## Example Response



(ii) For including a conservation of momentum equation

Point A3

**Scoring Note:** Part A (ii) and part A (iii) may be scored together, if necessary.For setting  $m_c v_c$  equal to the momentum of the block-cart system after the collision

Point A4

**Scoring Note:** A correct, isolated, final expression of  $v_f = \frac{5}{6}v_c$  earns points A3 and A4.

## Example Response

$$p_i = p_f$$

$$m_c v_c = (m_c + m_b) v_f = \left( m_c + \frac{1}{5} m_c \right) v_f$$

$$v_f = \frac{5}{6} v_c$$

- (iii) For a multistep derivation that includes the correct relationship between kinetic energy, mass, and speed (i.e.,  $K = \frac{1}{2}mv^2$ ) **Point A5**

**Scoring Note:** The minimum requirement to earn this point is to show an expression of kinetic energy or change in energy that goes beyond the given equation on the reference information. For example, substituting quantities from the problem into  $K = \frac{1}{2}mv^2$  or indicating the change in kinetic energy is  $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$  earns the point.

For an indication that  $m_c$  and  $\frac{6}{5}m_c$  are the initial and final masses, respectively, of the objects moving horizontally **Point A6**

For including an expression for  $v_f$  consistent with part A (ii) **Point A7**

**Scoring Note:** A correct, isolated, final expression of  $\Delta K = -\frac{1}{12}m_c v_c^2$  earns points A3, A4, A6, and A7.

### Example Response

$$\Delta K = K_f - K_0$$

$$K_0 = \frac{1}{2}m_c v_c^2$$

$$K_f = \frac{1}{2}\left(\frac{6}{5}m_c\right)\left(\frac{5}{6}v_c\right)^2 = \frac{5}{12}m_c v_c^2$$

$$\Delta K = -\frac{1}{12}m_c v_c^2$$

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|          |                                   |                 |
|----------|-----------------------------------|-----------------|
| <b>B</b> | For indicating “Remains constant” | <b>Point B1</b> |
|----------|-----------------------------------|-----------------|

**Scoring Note:** This point is scored independently from the graph drawn in Figure 2 in part A (i).

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|  |                                             |                 |
|--|---------------------------------------------|-----------------|
|  | For indicating <b>one</b> of the following: | <b>Point B2</b> |
|--|---------------------------------------------|-----------------|

- The frictional force exerted on the new block by the cart is equal in magnitude and opposite in direction to the frictional force exerted on the cart by the new block.
- The frictional force is internal to the new block-cart system.
- The net external force is still zero.
- The frictional force does not cause a net external force.

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|  |                                             |                 |
|--|---------------------------------------------|-----------------|
|  | For indicating <b>one</b> of the following: | <b>Point B3</b> |
|--|---------------------------------------------|-----------------|

- The change in momentum of the new block is equal in magnitude and opposite in direction to the change in momentum of the cart.
- The impulse exerted on the cart and the impulse exerted on the new block are equal in magnitude and opposite in direction.
- The correct relationship between the change in the momentum of the block-cart system and the net external force.

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**Example Response**

*The momentum of the new block-cart system will remain constant because the frictional force exerted on the new block by the cart is internal to the system and only a net external force will cause a change in momentum of the system.*

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**Question 2: Experimental Design****12 points**

- 
- |            |                                                                                                           |                |
|------------|-----------------------------------------------------------------------------------------------------------|----------------|
| <b>(a)</b> | For measuring the mass of at least one cylinder with the digital scale                                    | <b>1 point</b> |
|            | For measuring the period of oscillation of the cylinder-spring system with the stopwatch                  | <b>1 point</b> |
|            | For a procedure that indicates that the cylinder hung on the spring should be set into oscillatory motion | <b>1 point</b> |
|            | For a procedure that indicates a method to reduce experimental uncertainty                                | <b>1 point</b> |
- Accept **one** of the following:
- For using multiple masses
  - For doing multiple trials with a single mass
  - For measuring multiple oscillations and dividing by the number of oscillations
- 

**Example Response**

*Place a cylinder on the digital scale and record the mass. Hang the cylinder from the spring and pull the cylinder down a small distance so that the spring is stretched. Release the cylinder. Use the stopwatch to measure the amount of time necessary for the cylinder to complete ten full cycles (from maximum stretch length back to maximum stretch length). Repeat the procedure for cylinders of different masses.*

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**Total for part (a) 4 points**

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- (b)(i)** For listing quantities that can be measured with a stopwatch and a digital scale and could be plotted to produce a linear graph whose slope can be used to determine  $k$  **1 point**

Accept **one** of the following:

- $m$  vs.  $T^2$
- $T^2$  vs.  $m$
- $4\pi^2 m$  vs.  $T^2$
- $T^2$  vs.  $4\pi^2 m$
- $\frac{T^2}{4\pi^2}$  vs.  $m$
- $m$  vs.  $\frac{T^2}{4\pi^2}$
- $T$  vs.  $\sqrt{m}$
- $\sqrt{m}$  vs.  $T$
- $\frac{T}{2\pi}$  vs.  $\sqrt{m}$
- $\sqrt{m}$  vs.  $\frac{T}{2\pi}$
- $T$  vs.  $2\pi\sqrt{m}$
- $2\pi\sqrt{m}$  vs.  $T$

**Scoring Note:** This point may be earned for any of the bullets above substituting  $\frac{1}{f}$  for  $T$ .

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**Example Response**

Vertical axis:  $m$       Horizontal axis:  $T^2$

(b)(ii) For correctly relating the slope of the best-fit line to the value of  $k$ 

1 point

**Example Response**

Plotting the mass as a function of the period-squared would result in a graph whose slope could be used to find  $k$  by using the equation for the period of an oscillating cylinder-spring system.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{slope} = \frac{k}{4\pi^2}$$

$$k = (\text{slope})4\pi^2$$

| Graph                           | Slope                                    | $k$                                 |
|---------------------------------|------------------------------------------|-------------------------------------|
| $m$ vs. $T^2$                   | $\text{slope} = \frac{k}{4\pi^2}$        | $k = (\text{slope})4\pi^2$          |
| $T^2$ vs. $m$                   | $\text{slope} = \frac{4\pi^2}{k}$        | $k = \frac{4\pi^2}{\text{slope}}$   |
| $4\pi^2 m$ vs. $T^2$            | $\text{slope} = k$                       | $k = \text{slope}$                  |
| $T^2$ vs. $4\pi^2 m$            | $\text{slope} = \frac{1}{k}$             | $k = \frac{1}{\text{slope}}$        |
| $\frac{T^2}{4\pi^2}$ vs. $m$    | $\text{slope} = \frac{1}{k}$             | $k = \frac{1}{\text{slope}}$        |
| $m$ vs. $\frac{T^2}{4\pi^2}$    | $\text{slope} = k$                       | $k = \text{slope}$                  |
| $T$ vs. $\sqrt{m}$              | $\text{slope} = \sqrt{\frac{4\pi^2}{k}}$ | $k = \frac{4\pi^2}{\text{slope}^2}$ |
| $\sqrt{m}$ vs. $T$              | $\text{slope} = \sqrt{\frac{k}{4\pi^2}}$ | $k = \text{slope}^2 \times 4\pi^2$  |
| $\frac{T}{2\pi}$ vs. $\sqrt{m}$ | $\text{slope} = \sqrt{\frac{1}{k}}$      | $k = \frac{1}{\text{slope}^2}$      |
| $\sqrt{m}$ vs. $\frac{T}{2\pi}$ | $\text{slope} = \sqrt{k}$                | $k = \text{slope}^2$                |
| $T$ vs. $2\pi\sqrt{m}$          | $\text{slope} = \sqrt{\frac{1}{k}}$      | $k = \frac{1}{\text{slope}^2}$      |
| $2\pi\sqrt{m}$ vs. $T$          | $\text{slope} = \sqrt{k}$                | $k = \text{slope}^2$                |

Total for part (b) 2 points

(c)(i) For using the kinetic energy equation  $K = \frac{1}{2}mv^2$  **1 point**

For substituting **one** of the following: **1 point**

- 0.25 kg as the mass
- 0.3 m/s as the initial velocity

For an answer that approximates the change in kinetic energy to be  $\Delta K \approx -0.0113 \text{ J}$  **1 point**

**Scoring Note:** A correct response with no supporting work earns this point only.

**Scoring Note:** The unit and the negative sign are not required to earn this point.

**Example Response**

$$\Delta K = K_f - K_i$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Delta K = \frac{1}{2}(0.25)(0)^2 - \frac{1}{2}(0.25)(0.3)^2$$

$$\Delta K = -0.0113 \text{ J}$$

(c)(ii) For indicating the magnitude of the change in momentum is zero **1 point**

For indicating the area under the force-time graph represents the value of the change in momentum **1 point**

**Example Response**

*The area under the curve for a force vs time graph represents the impulse or change in momentum. The area under the curve for 0.5 s to 2.5 s is zero.*

(c)(iii) For an explanation that compares the estimated value of the change in momentum from (c)(ii) to the data from the velocity-time graph **1 point**

**Example Response**

*The velocity-time graph shows that velocity is 0.3 m/s at both 0.5 s and 2.5 s, and momentum is mass times velocity, so the momentum is the same at both times. This agrees with my estimation from part (c)(ii) that the change in momentum is zero.*

**Total for part (c) 6 points**

**Total for question 2 12 points**

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# **AP Physics 1: Algebra-Based Scoring Guidelines**

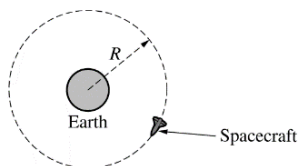
**2018 SCORING GUIDELINES****General Notes About 2018 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value  $g = 9.8 \text{ m/s}^2$ , but the use of  $10 \text{ m/s}^2$  is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

## 2018 SCORING GUIDELINES

## Question 1

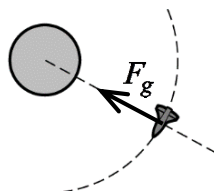
7 points total

Distribution  
of pointsNote: Figure not drawn to scale.

A spacecraft of mass  $m$  is in a clockwise circular orbit of radius  $R$  around Earth, as shown in the figure above. The mass of Earth is  $M_E$ .

- (a) LO / SP: 3.A.2.1 / 1.1; 3.B.2.1 / 1.1, 1.4  
2 points

In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.

Note: Figure not drawn to scale.

|                                                                                                                          |         |
|--------------------------------------------------------------------------------------------------------------------------|---------|
| For an arrow directed toward Earth's center                                                                              | 1 point |
| For a correct label on the arrow representing the gravitational force, where the arrow is pointing toward Earth's center | 1 point |
| <u>Note:</u> A maximum of 1 point may be earned if extraneous forces are present.                                        |         |

## 2018 SCORING GUIDELINES

## Question 1 (continued)

Distribution  
of points

(b) LO / SP: 2.B.2.1 / 2.2; 3.A.1.1 / 1.5, 2.2; 3.B.1.3 / 1.5, 2.2; 3.B.2.1 / 1.1, 1.4, 2.2, 3.C.1.2 / 2.2

4 points

i. 3 points

Derive an equation for the orbital period  $T$  of the spacecraft in terms of  $m$ ,  $M_E$ ,  $R$ , and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

|                                                                                                                                                                                     |  |         |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| For using (or implying) Newton's second law and equating the centripetal force to the gravitational force:<br>$F_g = ma = \frac{mv^2}{R} \qquad \frac{GmM_E}{R^2} = \frac{mv^2}{R}$ |  | 1 point |
| For explicitly or implicitly determining that the speed of the spacecraft is:<br>$v = \frac{2\pi R}{T}$                                                                             |  | 1 point |
| For a correct answer algebraically equivalent to:<br>$T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$                                                                                           |  | 1 point |
| <u>Note:</u> It is acceptable to leave answer in terms of $T^2$<br>$T^2 = \frac{4\pi^2 R^3}{GM_E}$                                                                                  |  |         |

ii. 1 point

A second spacecraft of mass  $2m$  is placed in a circular orbit with the same radius  $R$ . Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

\_\_\_\_ Greater than      \_\_\_\_ Less than      \_\_\_\_ Equal to

Briefly explain your reasoning.

|                                                                                                                                                                                                                                   |  |         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| Correct answer: "Equal to"<br><u>Note:</u> For an incorrect answer consistent with part (b)(i), the explanation is still graded for consistency with part (b)(i).                                                                 |  |         |
| For a correct explanation that the period of the spacecraft does not depend on the spacecraft mass (or only depends on the mass of Earth and the radius of the orbit)<br>OR an explanation consistent with the answer from (b)(i) |  | 1 point |
| <u>Note:</u> The explanation must be consistent with the checked answer.                                                                                                                                                          |  |         |

# 2018 SCORING GUIDELINES

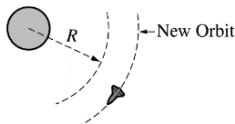
## Question 1 (continued)

### Distribution of points

(c) LO / SP: 2.B.1.1 / 2.2; 3.A.1.1 / 1.5, 2.2; 3.B.1.3 / 1.5, 2.2; 3.B.2.1 / 1.1, 1.4, 2.2; 3.C.1.2 / 2.2

1 point

The first spacecraft is moved into a new circular orbit that has a radius greater than  $R$ , as shown in the figure below.



Note: Figure not drawn to scale.

Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

\_\_\_\_ Greater than      \_\_\_\_ Less than      \_\_\_\_ Equal to

Briefly explain your reasoning.

|                                                                                        |  |         |
|----------------------------------------------------------------------------------------|--|---------|
| Correct answer: "Less than"                                                            |  |         |
| <u>Note:</u> If the wrong selection is made, the explanation is not graded.            |  |         |
| For a correct explanation of why speed decreases with increasing orbital radius        |  | 1 point |
| Example:<br>Derivation step in (b)(i) shows that speed decreases with increasing $R$ . |  |         |

**2018 SCORING GUIDELINES****Question 1 (continued)**

## Learning Objectives (LO)

- LO 2.B.1.1:** The student is able to apply  $F = mg$  to calculate the gravitational force on an object with mass  $m$  in a gravitational field of strength  $g$  in the context of the effects of a net force on objects and systems. [See Science Practices 2.2 and 7.2]
- LO 2.B.2.1:** The student is able to apply  $g = GM/r^2$  to calculate the gravitational field due to an object with mass  $M$ , where the field is a vector directed toward the center of the object of mass  $M$ . [See Science Practice 2.2]
- LO 3.A.1.1:** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [See Science Practices 1.5, 2.1, and 2.2]
- LO 3.A.2.1:** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. [See Science Practice 1.1]
- LO 3.B.1.3:** The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. [See Science Practices 1.5 and 2.2]
- LO 3.B.2.1:** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. [See Science Practices 1.1, 1.4, and 2.2]
- LO 3.C.1.2:** The student is able to use Newton's law of gravitation to calculate the gravitational force between two objects and use that force in contexts involving orbital motion (for circular orbital motion only in Physics 1). [See Science Practice 2.2]

# 2018 SCORING GUIDELINES

## Question 2

12 points total

Distribution  
of points

A group of students prepare a large batch of conductive dough (a soft substance that can conduct electricity) and then mold the dough into several cylinders with various cross-sectional areas  $A$  and lengths  $\ell$ . Each student applies a potential difference  $\Delta V$  across the ends of a dough cylinder and determines the resistance  $R$  of the cylinder. The results of their experiments are shown in the table below.

| Lab Station | $A$ ( $\text{m}^2$ ) | $\ell$ (m) | $\Delta V$ (V) | $R$ ( $\Omega$ ) | Example:<br>$RA$ ( $\Omega \cdot \text{m}^2$ ) | Example:<br>$\ell/A$ ( $\text{m}^{-1}$ ) |
|-------------|----------------------|------------|----------------|------------------|------------------------------------------------|------------------------------------------|
| 1           | 0.00049              | 0.030      | 1.02           | 23.6             | 0.012                                          | 61                                       |
| 2           | 0.00049              | 0.050      | 2.34           | 31.5             | 0.015                                          | 102                                      |
| 3           | 0.00053              | 0.080      | 3.58           | 61.2             | 0.032                                          | 151                                      |
| 4           | 0.00057              | 0.150      | 6.21           | 105.0            | 0.060                                          | 263                                      |

- (a) LO / SP: 1.E.2.1 / 4.1; 5.B.9.3 / 2.2; 5.C.3.2 / 4.1, 5.1  
7 points

The students want to determine the resistivity of the dough cylinders.

- i. 2 points

Indicate below which quantities could be graphed to determine a value for the resistivity of the dough cylinders. You may use the remaining columns in the table above, as needed, to record any quantities (including units) that are not already in the table.

Vertical Axis: \_\_\_\_\_ Horizontal Axis: \_\_\_\_\_

|                                                                                              |         |
|----------------------------------------------------------------------------------------------|---------|
| For quantities derived from $R$ , $\ell$ , and $A$ only                                      | 1 point |
| For choosing two quantities that, graphed together, can be used to determine the resistivity | 1 point |

## 2018 SCORING GUIDELINES

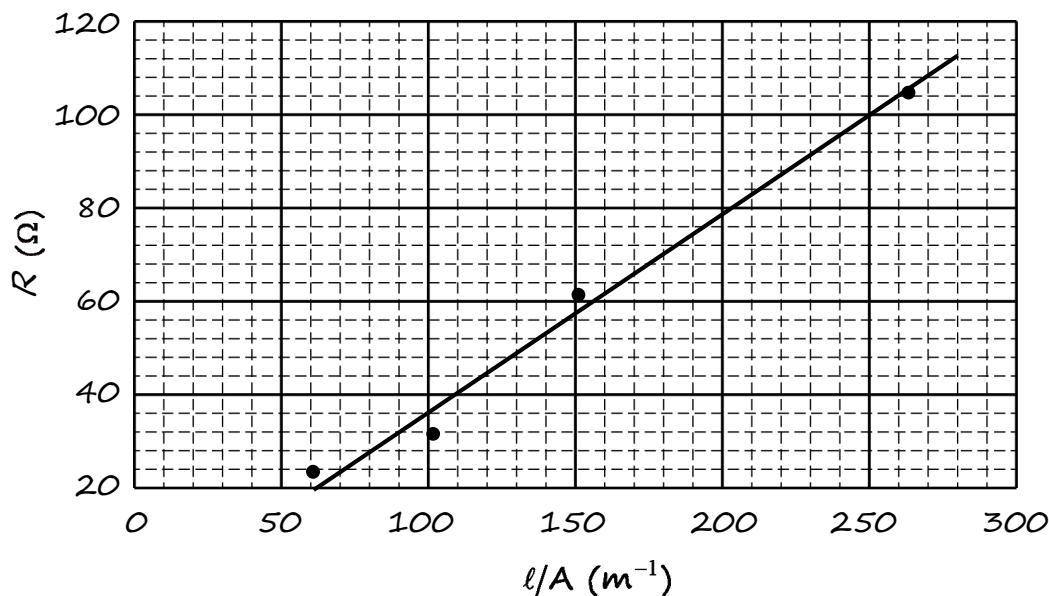
## Question 2 (continued)

Distribution  
of points

(a) (continued)

ii. 3 points

On the grid below, plot the appropriate quantities to determine the resistivity of the dough cylinders. Clearly scale and label all axes, including units as appropriate.



|                                                                                                                                      |  |         |
|--------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| <u>Note:</u> The following points can be earned only if the graphed points correspond to the quantities chosen in (a)(i).            |  |         |
| For a linear scale where the plotted data uses at least half the grid                                                                |  | 1 point |
| For labeling both axes, with units as appropriate                                                                                    |  | 1 point |
| For data points plotted that represent the appropriate trend based on chosen quantities                                              |  | 1 point |
| <u>Note:</u> The appropriate trend depends on the choice of graphed quantities and may be e.g., linear, inversely proportional, etc. |  |         |

## 2018 SCORING GUIDELINES

## Question 2 (continued)

Distribution  
of points

(a) (continued)

iii. 2 points

Use the above graph to estimate a value for the resistivity of the dough cylinders.

|                                                                                                                                                           |         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For correctly using the slope of the graph to find the resistivity or a statement that a calculator was used to find the appropriate value from the graph | 1 point |
| For a correct value of the resistivity $\rho = 0.42 \, \Omega \cdot \text{m}$ ( $\pm 0.03$ ) calculated from graph or data                                | 1 point |

(b) LO / SP: 1.E.2.1 / 4.1

1 point

Another group of students perform the experiment described in part (a) but shape the dough into long rectangular shapes instead of cylinders. Will this change affect the value of the resistivity determined by the second group of students?

☐ Yes ☐ No

Briefly justify your reasoning.

|                                                                                                     |         |
|-----------------------------------------------------------------------------------------------------|---------|
| Correct answer: “No”<br><u>Note:</u> If the wrong selection is made, the explanation is not graded. |         |
| For indicating that resistivity only depends on material, not shape                                 | 1 point |

(c) LO / SP: 1.E.2.1 / 4.1; 5.B.9.2 / 4.2; 5.B.9.3 / 2.2; 5.C.3.2 / 4.1, 4.2; 5.C.3.3 / 1.4, 2.2

4 points

Describe an experimental procedure to determine whether or not the resistivity of the dough cylinders depends on the temperature of the dough. Give enough detail so that another student could replicate the experiment. As needed, include a diagram of the experimental setup. Assume equipment usually found in a school physics laboratory is available.

|                                                                                                                                                                                                                                                                                                                                             |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For explicitly or implicitly controlling all variables (e.g., length and area) except for temperature                                                                                                                                                                                                                                       | 1 point |
| For a plausible way to change the temperature of the dough                                                                                                                                                                                                                                                                                  | 1 point |
| For indicating that the temperature will be varied                                                                                                                                                                                                                                                                                          | 1 point |
| For a valid and feasible experiment in which resistance, OR voltage and current, are appropriately measured                                                                                                                                                                                                                                 | 1 point |
| Example: Make two dough cylinders of the same length and cross-sectional area. Keep one cylinder at room temperature, and heat the other cylinder on a hot plate to 30°C (measured with a thermometer). Apply the same voltage to each cylinder (measured with a voltmeter), and measure the current through each cylinder with an ammeter. |         |

**2018 SCORING GUIDELINES****Question 2 (continued)**

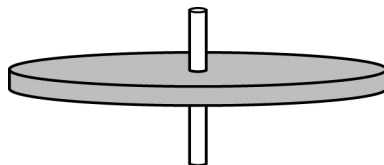
## Learning Objectives (LO)

- LO 1.E.2.1:** The student is able to choose and justify the selection of data needed to determine resistivity for a given material. [See Science Practice 4.1]
- LO 5.B.9.2:** The student is able to apply conservation of energy concepts to the design of an experiment that will demonstrate the validity of Kirchhoff's loop rule ( $\Delta V = 0$ ) in a circuit with only a battery and resistors either in series or in, at most, one pair of parallel branches. [See Science Practices 4.2, 6.4, and 7.2]
- LO 5.B.9.3:** The student is able to apply conservation of energy (Kirchhoff's loop rule) in calculations involving the total electric potential difference for complete circuit loops with only a single battery and resistors in series and/or in, at most, one parallel branch. [See Science Practices 2.2, 6.4, and 7.2]
- LO 5.C.3.2:** The student is able to design an investigation of an electrical circuit with one or more resistors in which evidence of conservation of electric charge can be collected and analyzed. [See Science Practices 4.1, 4.2, and 5.1]
- LO 5.C.3.3:** The student is able to use a description or schematic diagram of an electrical circuit to calculate unknown values of current in various segments or branches of the circuit. [See Science Practices 1.4 and 2.2]

## 2018 SCORING GUIDELINES

## Question 3

12 points total

Distribution  
of points

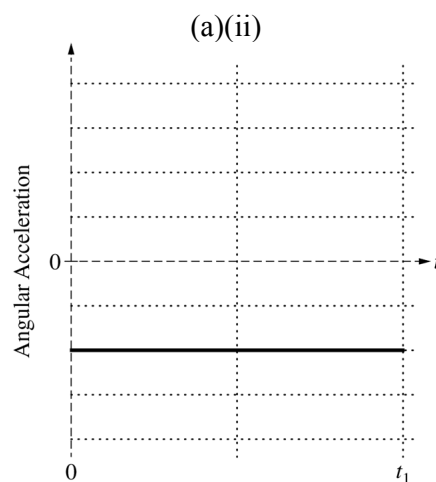
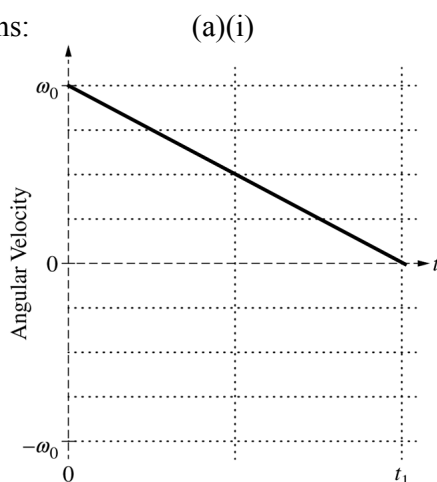
The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

- (a) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.F.1.1 / 1.4; 3.F.2.1 / 6.4; 4.D.2.1 / 1.2, 1.4  
4 points

At time  $t = 0$  the disk has an initial counterclockwise (positive) angular velocity  $\omega_0$ . The disk later comes to rest at time  $t = t_1$ .

- On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time  $t$  from  $t = 0$  until the disk comes to rest at time  $t = t_1$ .
- On the grid at right below, sketch the disk's angular acceleration as a function of time  $t$  from  $t = 0$  until the disk comes to rest at time  $t = t_1$ .

Example graphs:



- i. 2 points

|                                                                                                                                                                                             |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For a curve that has an angular velocity of $+\omega_0$ at time $t = 0$ and decreases to zero at time $t = t_1$                                                                             | 1 point |
| For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative) | 1 point |

## 2018 SCORING GUIDELINES

## Question 3 (continued)

Distribution  
of points

(a) (continued)

ii. 2 points

|                                                  |         |
|--------------------------------------------------|---------|
| For a curve that is negative for the entire time | 1 point |
| For a curve that is a constant nonzero function  | 1 point |

(b) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.F.2.1 / 6.4; 4.D.1.1 / 1.2, 1.4; 4.D.2.1 / 1.2, 1.4; 4.D.3.1 / 2.2  
3 points

The magnitude of the frictional torque exerted on the disk is  $\tau_0$ . Derive an equation for the rotational inertia  $I$  of the disk in terms of  $\tau_0$ ,  $\omega_0$ ,  $t_1$ , and physical constants, as appropriate.

|                                                                                                                                                                                                                                             |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For using an equation for the rotational version of Newton's second law                                                                                                                                                                     | 1 point |
| For using an appropriate rotational kinematics equation<br>$\alpha = \Delta\omega/\Delta t$                                                                                                                                                 | 1 point |
| For a correct answer in terms of the listed quantities, derived from first principles<br>$I = \frac{\tau_0 t_1}{\omega_0}$<br><u>Note:</u> This point is still earned if there is a minus sign, e.g., from using $-\tau_0$ or $-\omega_0$ . | 1 point |

|                                                                                                                                                                                                                                             |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| <i>Alternate solution using angular momentum and rotational impulse:</i>                                                                                                                                                                    |         |
| For defining and using angular momentum<br>$L = I\omega$                                                                                                                                                                                    | 1 point |
| For using rotational impulse<br>$\Delta L = \tau\Delta t$                                                                                                                                                                                   | 1 point |
| For a correct answer in terms of the listed quantities, derived from first principles<br>$I = \frac{\tau_0 t_1}{\omega_0}$<br><u>Note:</u> This point is still earned if there is a minus sign, e.g., from using $-\tau_0$ or $-\omega_0$ . | 1 point |

## 2018 SCORING GUIDELINES

## Question 3 (continued)

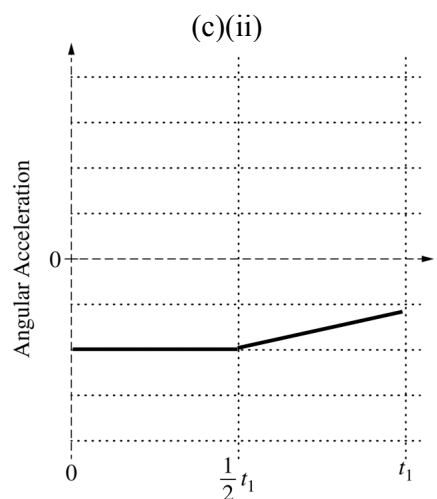
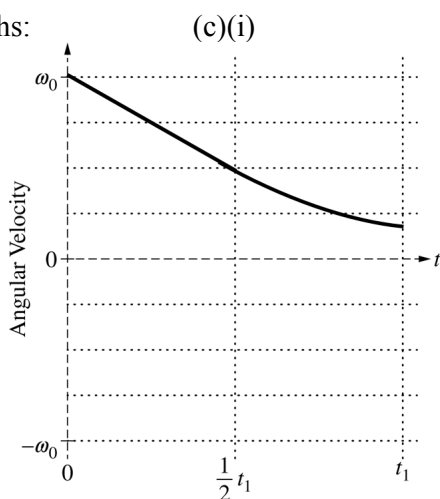
Distribution  
of points

- (c) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.A.3.1 / 6.4, 7.2; 3.F.1.1 / 1.4; 3.F.2.1 / 6.4; 4.D.1.1 / 1.2, 1.4; 4.D.2.1 / 1.2, 1.4  
4 points

In another experiment, the disk again has an initial positive angular velocity  $\omega_0$  at time  $t = 0$ . At time  $t = \frac{1}{2}t_1$ , the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

- On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from  $t = 0$  to  $t = t_1$ , which is the time at which the disk came to rest in part (a).
- On the grid at right below, sketch the disk's angular acceleration as a function of time from  $t = 0$  to  $t = t_1$ .

Example graphs:



- i. 3 points

|                                                                                                                    |         |
|--------------------------------------------------------------------------------------------------------------------|---------|
| For curve with a clear change of slope or curvature at $\frac{1}{2}t_1$ and showing a decrease in speed thereafter | 1 point |
| For a curve that indicates slowing at a decreasing rate between times $\frac{1}{2}t_1$ and $t_1$                   | 1 point |
| For a curve that does not reach zero at or before time $t_1$                                                       | 1 point |

## 2018 SCORING GUIDELINES

## Question 3 (continued)

Distribution  
of points

(c) (continued)

ii. 1 point

|                                                                                                                                                                                                             |         |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For a curve with decreasing magnitude between times $\frac{1}{2}t_1$ and $t_1$<br><u>Note:</u> The acceleration may reach zero at or before time $t_1$ . If so, it must remain zero for the remaining time. | 1 point |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|

(d) LO / SP: 3.F.2.1 / 6.4

1 point

The student is trying to mathematically model the magnitude  $\tau$  of the torque exerted by the axle on the disk when the oil is present at times  $t > \frac{1}{2}t_1$ . The student writes down the following two equations, each of which includes a positive constant ( $C_1$  or  $C_2$ ) with appropriate units.

$$(1) \tau = C_1 \left( t - \frac{1}{2}t_1 \right) \quad \left( \text{for } t > \frac{1}{2}t_1 \right)$$

$$(2) \tau = \frac{C_2}{\left( t + \frac{1}{2}t_1 \right)} \quad \left( \text{for } t > \frac{1}{2}t_1 \right)$$

Which equation better mathematically models this experiment?

\_\_\_\_ Equation (1)      \_\_\_\_ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

|                                                                                                                                                                                                                                                                                                                        |  |         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| Correct answer: “Equation (2)”<br><u>Note:</u> If the wrong selection is made, the explanation is not graded.                                                                                                                                                                                                          |  |         |
| For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time                                                                                                                                                 |  | 1 point |
| Examples:<br>Equation (2) because $\tau$ decreases. In Equation (1), it doesn't.<br>Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time. |  |         |

**2018 SCORING GUIDELINES****Question 3 (continued)**

## Learning Objectives (LO)

- LO 3.A.1.1:** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [See Science Practices 1.5, 2.1, and 2.2]
- LO 3.A.3.1:** The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. [See Science Practices 6.4 and 7.2]
- LO 3.F.1.1:** The student is able to use representations of the relationship between force and torque. [See Science Practice 1.4]
- LO 3.F.2.1:** The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. [See Science Practice 6.4]
- LO 4.D.1.1:** The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. [See Science Practices 1.2 and 1.4]
- LO 4.D.2.1:** The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. [See Science Practices 1.2 and 1.4]
- LO 4.D.3.1:** The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. [See Science Practice 2.2]

## 2018 SCORING GUIDELINES

## Question 4

7 points total

Distribution  
of points

A transverse wave travels to the right along a string.

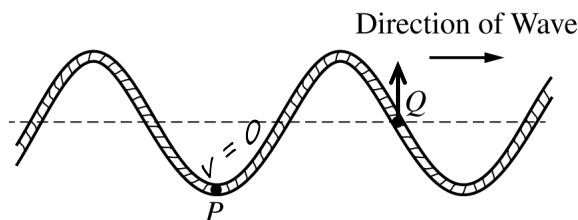
(a) LO / SP: 3.A.1.1 / 1.5; 6.A.1.2 / 1.2

4 points

Two dots have been painted on the string. In the diagrams below, those dots are labeled  $P$  and  $Q$ .

i. 2 points

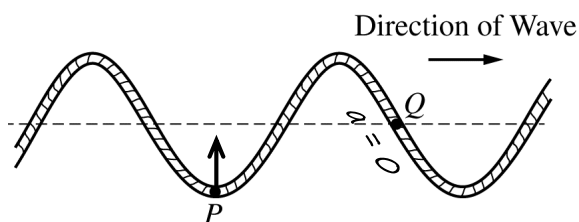
The figure below shows the string at an instant in time. At the instant shown, dot  $P$  has maximum displacement and dot  $Q$  has zero displacement from equilibrium. At each of the dots  $P$  and  $Q$ , draw an arrow indicating the direction of the instantaneous velocity of that dot. If either dot has zero velocity, write “ $v = 0$ ” next to the dot.



|                                  |         |
|----------------------------------|---------|
| For “ $v = 0$ ” at point $P$     | 1 point |
| For an upward arrow at point $Q$ | 1 point |

ii. 2 points

The figure below shows the string at the same instant as shown in part (a)(i). At each of the dots  $P$  and  $Q$ , draw an arrow indicating the direction of the instantaneous acceleration of that dot. If either dot has zero acceleration, write “ $a = 0$ ” next to the dot.



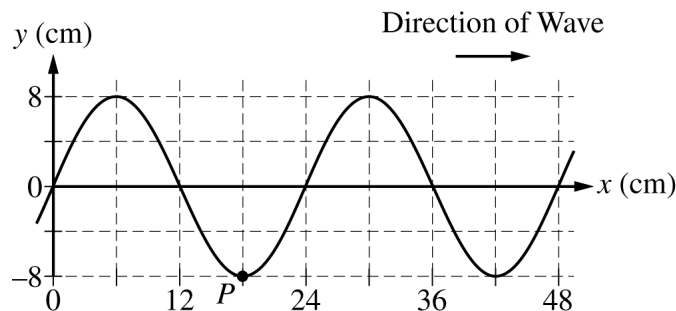
|                                  |         |
|----------------------------------|---------|
| For an upward arrow at point $P$ | 1 point |
| For “ $a = 0$ ” at point $Q$     | 1 point |

## 2018 SCORING GUIDELINES

## Question 4 (continued)

Distribution  
of points

The figure below represents the string at time  $t = 0$ , the same instant as shown in part (a) when dot  $P$  is at its maximum displacement from equilibrium. For simplicity, dot  $Q$  is not shown.



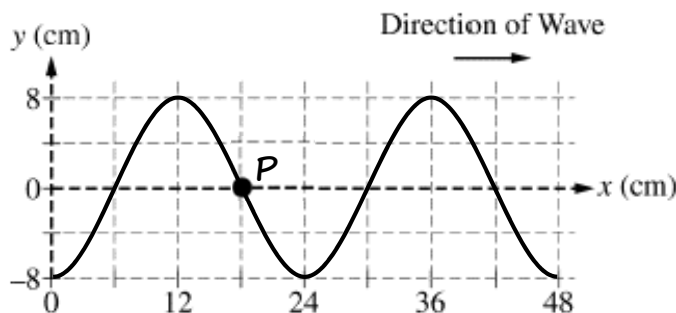
(b) LO / SP: 3.A.1.1 / 1.5; 6.A.1.2 / 1.2

2 points

i. 1 point

On the grid below, draw the string at a later time  $t = T/4$ , where  $T$  is the period of the wave.

Note: Do any scratch (practice) work on the grid at the bottom of the page. Only the sketch made on the grid immediately below will be graded.



|                                                                                                                                                                                  |         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For a curve shifted to the right by 1/4 of a wavelength (1 gridline) with the same wavelength as the original wave (for example, as indicated by a negative minimum at $x = 0$ ) | 1 point |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|

ii. 1 point

On your drawing above, draw a dot to indicate the position of dot  $P$  on the string at time  $t = T/4$  and clearly label the dot with the letter  $P$ .

|                                                                                                                           |         |
|---------------------------------------------------------------------------------------------------------------------------|---------|
| For point $P$ on the string and at the dot's original $x$ position, 3/4 of a wavelength (3 grid units) to right of origin | 1 point |
|---------------------------------------------------------------------------------------------------------------------------|---------|

**2018 SCORING GUIDELINES****Question 4 (continued)****Distribution  
of points**

(c) LO / SP: 6.A.3.1 / 1.4  
1 point

Now consider the wave at time  $t = T$ . Determine the distance traveled (not the displacement) by dot  $P$  between times  $t = 0$  and  $t = T$ .

|                                         |  |         |
|-----------------------------------------|--|---------|
| For the correct numerical answer: 32 cm |  | 1 point |
|-----------------------------------------|--|---------|

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**Learning Objectives (LO)**

**LO 3.A.1.1:** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [See Science Practice 1.5]

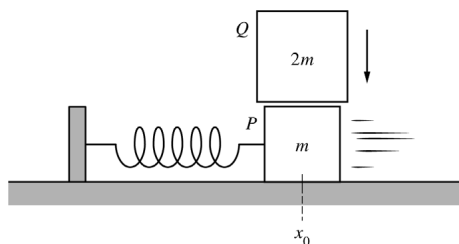
**LO 6.A.1.2:** The student is able to describe representations of transverse and longitudinal waves. [See Science Practice 1.2]

**LO 6.A.3.1:** The student is able to use graphical representation of a periodic mechanical wave to determine the amplitude of the wave. [See Science Practice 1.4]

## 2018 SCORING GUIDELINES

## Question 5

7 points total

Distribution  
of points

Block  $P$  of mass  $m$  is on a horizontal, frictionless surface and is attached to a spring with spring constant  $k$ . The block is oscillating with period  $T_P$  and amplitude  $A_P$  about the spring's equilibrium position  $x_0$ . A second block  $Q$  of mass  $2m$  is then dropped from rest and lands on block  $P$  at the instant it passes through the equilibrium position, as shown above. Block  $Q$  immediately sticks to the top of block  $P$ , and the two-block system oscillates with period  $T_{PQ}$  and amplitude  $A_{PQ}$ .

- (a) LO / SP: 3.B.3.1 / 6.4  
1 point

Determine the numerical value of the ratio  $T_{PQ}/T_P$ .

|                                  |         |
|----------------------------------|---------|
| For a correct answer: $\sqrt{3}$ | 1 point |
|----------------------------------|---------|

## 2018 SCORING GUIDELINES

## Question 5 (continued)

Distribution  
of points

- (b) LO / SP: 3.B.3.1 / 6.4, 7.2; 3.B.3.4 / 2.2, 6.2; 4.C.1.1 / 1.4, 2.2; 4.C.1.2 / 6.4;  
5.B.3.1 / 2.2, 6.4; 5.B.3.3 / 1.4, 2.2; 5.B.4.1 / 6.4, 7.2; 5.B.4.2 / 1.4, 2.1, 2.2; 5.D.2.1 / 6.4, 7.2  
6 points

How does the amplitude of oscillation  $A_{PQ}$  of the two-block system compare with the original amplitude  $A_P$  of block  $P$  alone?

\_\_\_\_\_  $A_{PQ} < A_P$       \_\_\_\_\_  $A_{PQ} = A_P$       \_\_\_\_\_  $A_{PQ} > A_P$

In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

|                                                                                                                                                                                                                                                                                                                         |  |          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|----------|
| Correct answer: $A_{PQ} < A_P$ .                                                                                                                                                                                                                                                                                        |  |          |
| <u>Note:</u> The response is graded even if an incorrect selection is made.                                                                                                                                                                                                                                             |  |          |
| For applying conservation of momentum to the collision                                                                                                                                                                                                                                                                  |  | 1 point  |
| For correctly finding that the post-collision speed has decreased (or, for determining that $v_f = \frac{1}{3}v_i$ )                                                                                                                                                                                                    |  | 1 point  |
| <u>Note:</u> The first 2 points can be earned for stating that the collision is inelastic                                                                                                                                                                                                                               |  |          |
| For stating or implying that the system's kinetic energy has decreased (or, for calculating a lower final kinetic energy)                                                                                                                                                                                               |  | 1 point  |
| For stating or implying kinetic energy of blocks right after collision equals maximum potential energy of spring<br>OR<br>For stating or implying that the maximum potential energy equals the total mechanical energy just after the collision (Simply stating that $E_{\text{tot}} = \frac{1}{2}kA^2$ is sufficient.) |  | 1 point  |
| For stating or implying that maximum potential energy is reached when the displacement from equilibrium equals the amplitude of oscillation                                                                                                                                                                             |  | 1 point  |
| <u>Note:</u> The previous 2 points can be earned in a single sentence in which one or both of the points is implicit.                                                                                                                                                                                                   |  |          |
| For a logical, relevant, and internally consistent argument that addresses the required argument or question asked and follows the guidelines described in the published requirements for the paragraph-length response                                                                                                 |  | 1 point  |
| <i>Alternate solution for first 2 points in part (b):</i>                                                                                                                                                                                                                                                               |  |          |
| <i>A frictional force is exerted on block P by Q, so it slows down.</i><br><i><u>Note:</u> The third point can be earned if energy loss is stated.</i>                                                                                                                                                                  |  | 2 points |

# 2018 SCORING GUIDELINES

## Question 5 (continued)

Example:

The second block ( $Q$ ) adds mass without changing the horizontal momentum of the two-block system. In effect, block  $P$  (mass  $m$ ) becomes block  $PQ$  (mass  $3m$ ). This reduces the speed at equilibrium from  $v_{\max}$  to  $v_{\max}/3$  according to conservation of momentum. To see how this affects amplitude, we must analyze what happens to the maximum kinetic energy ( $K$ ) of the oscillating mass:

$$K_P = \frac{1}{2}mv_{\max}^2 \qquad K_{PQ} = \frac{1}{2}(3m)\left(\frac{v_{\max}}{3}\right)^2 = \frac{1}{6}mv_{\max}^2 = \frac{1}{3}K_P$$

Because the maximum  $K$  is reduced, this means the maximum potential energy in the spring is also reduced (to  $1/3$  of its former value). Because amplitude is related

to maximum potential energy  $\left(U_{\max} = \frac{1}{2}kA^2\right)$ , the amplitude of block  $PQ$  is less than that of block  $P$ .

**2018 SCORING GUIDELINES****Question 5 (continued)**

## Learning Objectives (LO)

- LO 3.B.3.1:** The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. [See Science Practices 6.4, 7.2]
- LO 3.B.3.4:** The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. [See Science Practices 2.2, 6.2]
- LO 4.C.1.1:** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. [See Science Practices 1.4, 2.1, 2.2]
- LO 4.C.1.2:** The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. [See Science Practice 6.4]
- LO 5.B.3.1:** The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. [See Science Practices 2.2, 6.4, 7.2]
- LO 5.B.3.3:** The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. [See Science Practices 1.4, 2.2]
- LO 5.B.4.1:** The student is able to describe and make predictions about the internal energy of systems. [See Science Practices 6.4, 7.2]
- LO 5.B.4.2:** The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. [See Science Practices 1.4, 2.1, 2.2]
- LO 5.D.2.1:** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. [See Science Practices 6.4, 7.2]