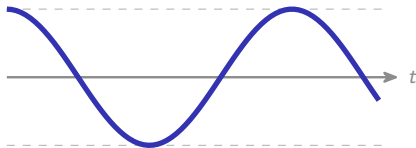


Oscillations

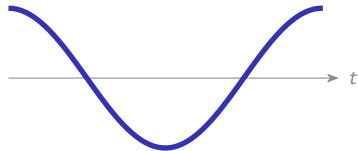
AP Physics 1 ■ Unit 7

AP Physics 1



What we will cover

- 1 Defining simple harmonic motion
- 2 Frequency and period
- 3 Representing and analyzing SHM
- 4 Energy of an oscillator



1

Defining SHM

Simple harmonic motion

Simple harmonic motion 简谐运动 happens when the **restoring force** 回复力 is proportional to the displacement:

The defining law

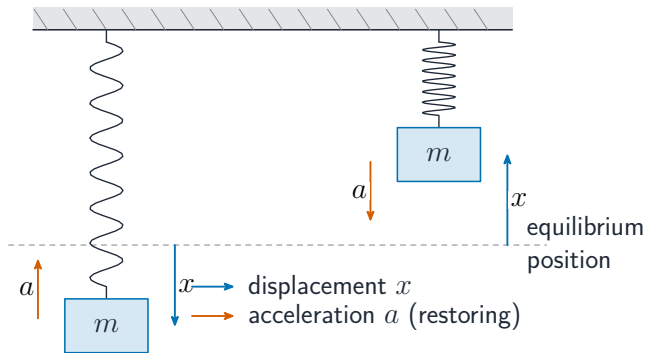
$$F = -kx$$

Pull it **twice** as far, and it pulls back **twice** as hard – always toward the middle.



SHM restores toward equilibrium

Defining Simple Harmonic Motion



In SHM the acceleration always points back towards equilibrium, opposite the displacement

2

Period

Frequency and period

An oscillation repeats every **period** 周期 T ;
the **frequency** 频率 $f = 1/T$.

Period depends on the system, not the swing

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

$$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}$$

The period does **not** depend on the **amplitude** 振幅 – a big swing and a small swing take the same time.

0.5 kg on $k = 50 \text{ N/m}$: $T \approx 0.63 \text{ s}$, whether you pull it 2 or 10 cm.

A pendulum's period ignores mass entirely.

Three ways to say how fast it oscillates

Period

T : seconds for one complete cycle.

Frequency

$f = 1/T$: cycles per second, in hertz.

Angular frequency

Angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$, in radians per second – the one that appears inside $x = A \cos(\omega t + \phi)$.

Why three

ω makes the algebra clean, f and T are what an experiment measures.

For a spring $\omega = \sqrt{k/m}$; for a simple pendulum $\omega = \sqrt{g/L}$. Neither depends on the amplitude.

Worked example – two periods

(a) A 0.25 kg mass on a spring with $k = 100 \text{ N/m}$. (b) A pendulum clock ticking with period exactly 2.0 s: how long is it?

- (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{100}} = 0.31 \text{ s}$

- (b) rearrange $T = 2\pi\sqrt{L/g}$:

$$L = g\left(\frac{T}{2\pi}\right)^2$$

- $= 9.8\left(\frac{2.0}{2\pi}\right)^2 = 0.99 \text{ m}$

A one-metre pendulum has a two-second period – one second per swing. That is not a coincidence: the metre was very nearly defined that way.

The period of SHM does not depend on amplitude

the claim

for simple harmonic motion, period and frequency depend **only on the system**, not on how far it swings

a spring

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}} - \text{mass and stiffness only}$$

a pendulum

$$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}} - \text{length and gravity only, with no mass in it}$$

why it matters

a pendulum clock keeps time as its swing decays, which is the whole reason it works

the small-angle limit

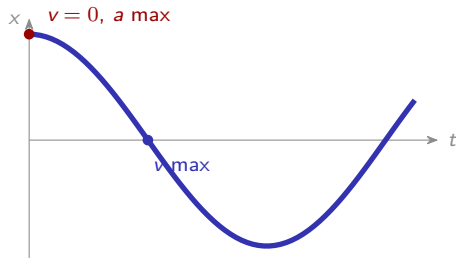
the pendulum formula holds only for small angles, where $\sin \theta \approx \theta$

“Independent of amplitude” is the defining property of SHM. If a question changes the amplitude and asks about the period, the answer is **no change**.

3

Analyzing SHM

Reading an oscillation



Displacement, velocity and acceleration trace smooth sine curves:

- **velocity** greatest at the **equilibrium** 平衡位置 (middle)
- **acceleration** greatest at the extremes

The **phase** 相位 sets where in the cycle it is: v peaks a quarter-cycle before x .

Reading the oscillation graph as a story

at the turning point

x is **largest**; v has just fallen to **zero**; a is at its **most negative**, hauling the mass back

passing through the centre

$x = 0$; $a = 0$; v is at its **maximum**

the phases

v leads x by a quarter cycle; a is a half cycle out of phase with x

a against x

a straight line through the origin with a **negative** gradient – and that graph *is* the definition of SHM

the energy

U is greatest at the ends, K greatest at the centre, and the total is constant

Never have maximum speed and maximum acceleration at the same point. If your answer does, a quarter cycle has gone missing.

4

Energy

Energy of an oscillator

Energy sloshes between two forms, but the total is constant:

Set by the amplitude

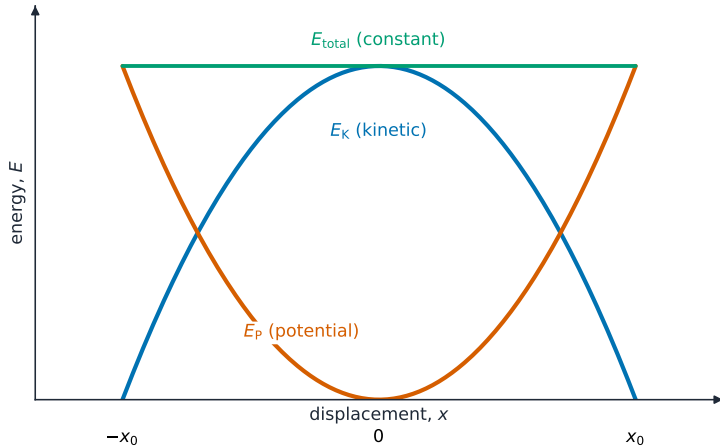
$$E = \frac{1}{2}kA^2$$

- **kinetic energy** 动能 greatest in the middle
- **potential energy** 势能 greatest at the extremes



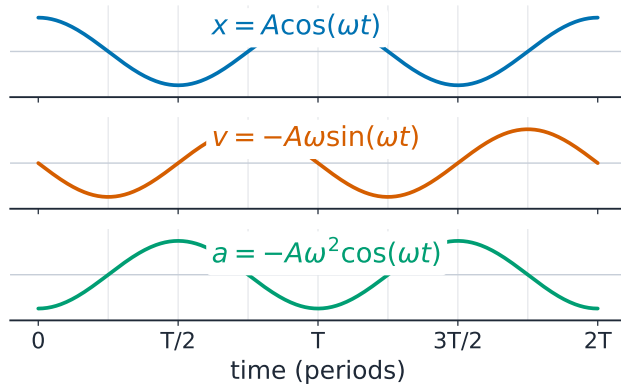
Oscillators store and exchange energy

Kinetic and potential energy over a cycle



Kinetic and potential energy swap over a cycle while the total energy stays constant

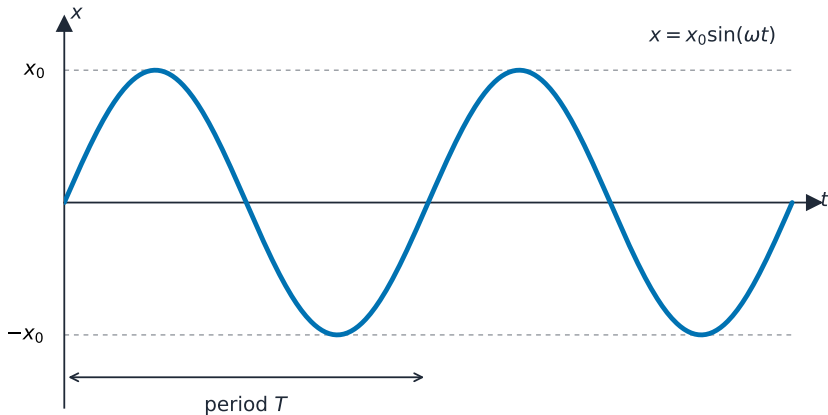
SHM: x , v and a a quarter-cycle apart



$a = -\omega^2 x$ · v leads x by $T/4$ · a is in antiphase with x

Displacement, velocity, and acceleration in SHM, each a quarter-cycle apart

SHM: displacement is sinusoidal



Displacement varies sinusoidally with time in simple harmonic motion

Exam tips

- Test for **SHM**: the acceleration must be proportional to the displacement and directed back toward the middle ($a = -\frac{k}{m}x$).
- The **period does not depend on amplitude** – use $T = 2\pi\sqrt{m/k}$ (spring) or $T = 2\pi\sqrt{L/g}$ (pendulum, small angles).
- Speed is **maximum at the middle** (all kinetic) and **zero at the extremes** (all potential); acceleration is largest at the extremes.
- Use energy ($\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$) to find the maximum speed quickly:
 $v_{\max} = A\sqrt{k/m}$.
- Total energy $\propto A^2$, so doubling the amplitude quadruples the energy.

5

Recap

Unit 7 – key takeaways

1

SHM

$F = -kx$: restoring force proportional to displacement

2

Period

$2\pi\sqrt{m/k}$ or $2\pi\sqrt{L/g}$; independent of amplitude

3

Motion

v max in the middle; a max at the extremes

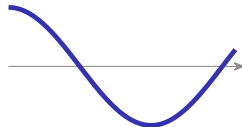
4

Energy

$E = \frac{1}{2}kA^2$; trades between KE and PE

Check yourself

- 1 Double the amplitude – does the period change?
- 2 Where in the swing is the velocity greatest?
- 3 Where is the acceleration greatest?
- 4 Does a pendulum's period depend on the mass?



Answers 1. no 2. in the middle (equilibrium) 3. at the extremes 4. no