

Torque & Rotational Dynamics

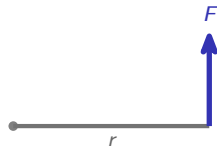
AP Physics 1 ■ Unit 5

AP Physics 1



What we will cover

- 1 Rotational kinematics
- 2 Linking linear and rotational motion
- 3 Torque
- 4 Rotational inertia
- 5 Rotational equilibrium
- 6 Newton's second law for rotation



Rotation mirrors straight-line motion

Spinning is described by angles:

- **angular velocity** 角速度 ω – rate the angle turns
- **angular acceleration** 角加速度 α – rate ω changes

For constant α , the equations have the **same form** as the linear ones. Rotation rate:

$$\omega = 2\pi f.$$



Rotation links angle, angular speed, and linear speed

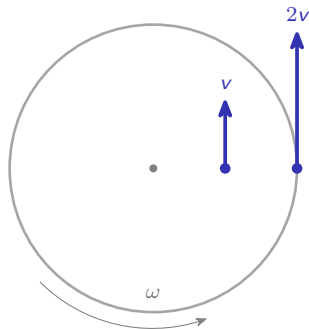
Linking linear and rotational motion

Every point on a spinning body moves along a curve, tied to the **radius** 半径 r :

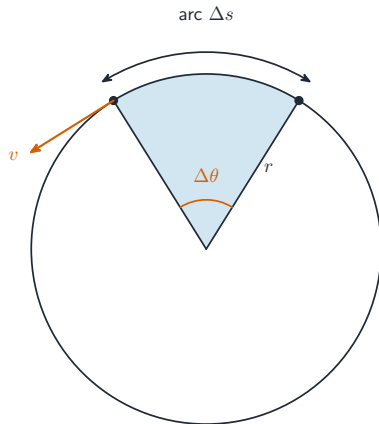
Same ω , different v

$$v = r\omega, \quad a_t = r\alpha, \quad s = r\theta$$

All points share one ω , but points farther out move **faster**.



Linking linear and rotational motion



Torque

Torque 力矩 is the rotational version of force
– how strongly a force twists:

Twisting strength

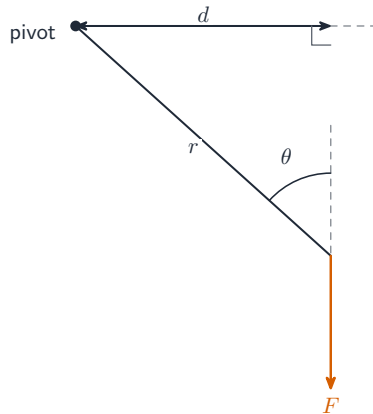
$$\tau = rF \sin \theta$$

The **lever arm** 力臂 is the perpendicular distance to the force. A bigger lever arm gives **more torque**.



Torque is the rotational analogue of force

Torque



Rotational inertia

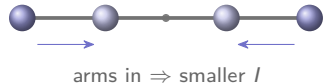
Rotational inertia 转动惯量 I resists a change in spin – and depends on **how the mass is spread**:

Distance is squared

$$I = \sum mr^2$$

Parallel-axis theorem 平行轴定理: $I = I_{cm} + md^2$.

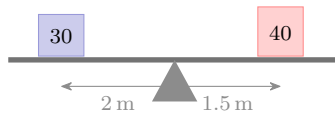
Move mass out to $2\times$ the radius $\Rightarrow I$ grows $4\times$.



Rotational equilibrium

In **rotational equilibrium** 转动平衡 the **net torque** 净力矩 is zero. Full **static equilibrium** 静平衡: both net force and net torque zero.

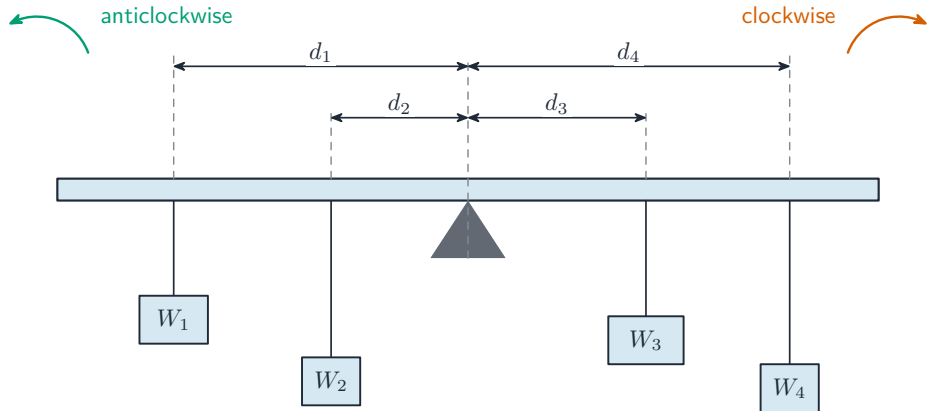
- pick the axis at an unknown force to drop it out
- set clockwise torque = counterclockwise torque



Heavier child

sits **closer** to balance.

Rotational equilibrium



Newton's second law for rotation

A nonzero net torque gives an angular acceleration – just like $F = ma$:

Rotational form

$$\tau_{net} = I\alpha$$

torque $\leftrightarrow$ force, $I \leftrightarrow$ mass, $\alpha \leftrightarrow$ acceleration.

Worked example

$\tau = 12 \text{ N m}$ on $I = 3 \text{ kg m}^2$:

$$\alpha = \frac{\tau}{I} = \frac{12}{3} = 4 \text{ rad/s}^2$$

Double $I \Rightarrow$ half the α – harder to spin up.

Unit 5 – key takeaways

1 Analogy

$$x, v, a, m, F \leftrightarrow \theta, \omega, \alpha, I, \tau; v = r\omega$$

2 Torque

$\tau = rF \sin \theta$; the lever arm sets the twist

3 Inertia

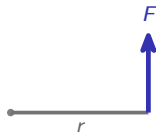
$I = \sum mr^2$; mass farther out counts far more

4 Dynamics

equilibrium: net $\tau = 0$; otherwise
 $\tau = I\alpha$

Check yourself

- 1 Push a door near the hinge vs at the handle – which gives more torque?
- 2 Move a mass to twice the radius – what happens to I ?
- 3 On a balanced seesaw, who sits closer to the pivot?
- 4 $\tau = 20 \text{ N m}$ on $I = 5 \text{ kg m}^2$ – find α .



Answers 1. the handle (bigger lever arm) 2. it quadruples 3. the heavier one
4. 4 rad/s^2