

On this exam the number is rarely the point. Each solution below is written the way a scored response is written: the principle named, the condition that lets it apply stated, then the algebra, then the answer with its unit and a sentence saying what it means. 本册按评分标准的行文顺序给出解答: 先说明原理与适用条件, 再计算。

Kinematics and dynamics

A 2.0 kg block is pulled along a horizontal surface by a 12 N force directed 30 degrees above the horizontal. The coefficient of kinetic friction is 0.25. Find the acceleration.

2.4 m s⁻², horizontally, in the direction of motion.

Resolve the applied force: horizontal component $12 \cos 30 = 10.4$ N, vertical component $12 \sin 30 = 6.0$ N upward. The normal force is NOT mg here, because the pull lifts part of the weight: $N = mg - 6.0 = (2.0)(9.8) - 6.0 = 13.6$ N. Friction opposes motion, so $f = 0.25 \times 13.6 = 3.4$ N. Net horizontal force $= 10.4 - 3.4 = 7.0$ N, and $a = F/m = 7.0/2.0 = 3.5$ m s⁻². Working to two significant figures with $g = 9.8$ gives 3.5 m s⁻². The step that carries the most credit is recognising that the vertical component changes the normal force; a solution using $N = mg$ loses it immediately.

A ball is thrown horizontally at 15 m/s from a 20 m cliff. How far from the base does it land, and what is its speed on impact?

30 m from the base; impact speed 25 m s⁻¹.

Treat the two directions independently. Vertically the initial velocity is zero, so $20 = \frac{1}{2}(9.8)t^2$, giving $t = 2.02$ s. Horizontally the velocity is constant, so $x = 15 \times 2.02 = 30$ m. For the impact speed, the vertical velocity is $v = 9.8 \times 2.02 = 19.8$ m s⁻¹ and the horizontal is still 15 m s⁻¹, so the resultant is $\sqrt{15^2 + 19.8^2} = 24.8$, i.e. 25 m s⁻¹. Note that the horizontal speed never appears in the time calculation – a common error is to use the resultant velocity in the vertical equation.

Energy and momentum

A 0.50 kg cart moving at 4.0 m/s collides with a stationary 1.5 kg cart and they stick together. They then slide up a frictionless ramp. How high do they rise?

0.051 m, about 5.1 cm.

This needs both principles, in order. Through the collision, momentum is conserved but kinetic energy is not, because the carts stick – say so. $(0.50)(4.0) = (2.0)v$, so $v = 1.0$ m s⁻¹. After the collision the ramp is frictionless, so mechanical energy IS conserved: $\frac{1}{2}(2.0)(1.0)^2 = (2.0)(9.8)h$, giving $h = 0.051$ m. Using energy conservation through the collision would give 0.82 m – sixteen times too large – and is the single most common error on this question type.

A 60 kg student runs up a 4.0 m staircase in 5.0 s. Calculate the average power developed against gravity.

470 W.

The work done against gravity is the gain in gravitational potential energy: $W = mgh = (60)(9.8)(4.0) = 2352$ J. Average power is work over time: $P = 2352/5.0 = 470$ W. The phrase “against gravity” matters: the student’s total power output is larger, because some energy goes to kinetic energy and to internal losses. Saying that explicitly is what distinguishes a full answer.

Circular motion and rotation

I A 0.30 kg ball on a 0.80 m string is swung in a vertical circle. What is the minimum speed at the top for the string to stay taut, and what is the tension at the bottom if the speed there is 6.0 m/s?

Minimum speed at the top 2.8 m s^{-1} ; tension at the bottom 16 N.

At the top, both gravity and tension point towards the centre, so $T + mg = mv^2/r$. The minimum condition is $T = 0$, which gives $v = \sqrt{gr} = \sqrt{(9.8 \times 0.80)} = 2.8 \text{ m s}^{-1}$. At the bottom, tension points towards the centre and gravity away, so $T - mg = mv^2/r$, giving $T = m(v^2/r + g) = 0.30(36/0.80 + 9.8) = 0.30(45 + 9.8) = 16 \text{ N}$. The reasoning to state is that the NET force must be centripetal at every point; “centripetal force” is not a separate force to be added to the diagram.

I A uniform 3.0 m beam of mass 20 kg is supported at both ends. A 50 kg person stands 1.0 m from the left end. Find the force from the left support.

About 260 N.

Take torques about the RIGHT support, which eliminates its unknown force from the equation – that choice is itself worth stating. The beam’s weight acts at its centre, 1.5 m from either end; the person is 2.0 m from the right support. So $F_{\text{left}}(3.0) = (20)(9.8)(1.5) + (50)(9.8)(2.0) = 294 + 980 = 1274$, giving $F_{\text{left}} = 425 \text{ N}$. Checking with vertical equilibrium: total weight is $(70)(9.8) = 686 \text{ N}$, so the right support carries 261 N. Always name the pivot you chose and why.

Waves and circuits

I A string fixed at both ends is 1.2 m long and vibrates in its third harmonic at 180 Hz. Find the wave speed.

144 m s^{-1} .

In the third harmonic a string fixed at both ends holds three half-wavelengths, so $1.2 = 3(\lambda/2)$ and $\lambda = 0.80 \text{ m}$. Then $v = f\lambda = 180 \times 0.80 = 144 \text{ m s}^{-1}$. Sketching the standing wave with its nodes and antinodes before writing anything is the reliable way to get the harmonic relationship right; from memory it is easy to use $\lambda = 2L/n$ with the wrong n .

I Two resistors, 4.0Ω and 12Ω , are connected in parallel across a 24 V battery of negligible internal resistance. Find the current from the battery.

8.0 A.

In parallel the voltage is common, so compute each branch separately: $24/4.0 = 6.0 \text{ A}$ and $24/12 = 2.0 \text{ A}$, giving 8.0 A total. Equivalently the combined resistance is $1/R = 1/4 + 1/12 = 1/3$, so $R = 3.0 \Omega$ and $I = 24/3.0 = 8.0 \text{ A}$. Doing it both ways is a free check, and the branch method makes it obvious why the combined resistance is smaller than either resistor.