Course Companion

AP Physics 1

Every topic handout in one volume

全部专题讲义合集

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Kinematics

AP Physics 1

Scalars and Vectors in One Dimension

A sprinter crosses 100 m in under 10 s, yet reaches top speed only after about 60 m. A long jumper's flight lasts less than a second, and the distance depends on a single number: the speed and angle at take-off. Kinematics turns such moments into numbers you can predict.



A car speedometer: speed is the magnitude of velocity – a scalar rate of motion

Kinematics 运动学 describes *how* objects move, without asking why. First, two kinds of quantity:

- A **scalar** 标量 has only size (**magnitude** 大小): **distance** 距离, speed, time, mass.
- A **vector** 矢量 has magnitude **and** direction: displacement, velocity, acceleration, force.

The difference matters. **Distance** is the total path length travelled – a scalar that only grows. **Displacement** is the straight-line change in position, with a direction. Walk 3 m east then 1 m back west: the distance is 4 m, but the displacement is only 2 m east.

In one dimension, direction is just a **sign** 符号 (+ or −) along a chosen axis. Choosing the **positive direction** 正方向 first is essential – every vector's sign depends on it. A velocity of −5 m/s does not mean "slow"; it means 5 m/s in the negative direction.

A vector is drawn as an **arrow**: its direction shows the vector's direction, and its length is proportional to the magnitude. A velocity of 10 m/s is drawn twice as long as one of 5 m/s. In equations a vector has an arrow over its symbol, $\vec{v}$. Along one axis

we drop the arrow and let the sign carry the direction, as in v_x.

In this unit an object is treated as a single point, the **object model**: its size, shape and what is inside it are ignored. A car, a ball and a planet can all be "points" when only their overall motion matters.

Displacement, Velocity, and Acceleration



A high-speed train: kinematics describes displacement, velocity and acceleration over time

Three linked vectors describe motion along a line:

- **Displacement** 位移 Δx is the change in position – a vector from start to end (not the total path length, which is distance).
- **Velocity** 速度 is the rate of change of position, $v = \dfrac{\Delta x}{\Delta t}$. Its sign gives direction; its magnitude is **speed** 速率.
- **Acceleration** 加速度 is the rate of change of velocity, $a = \dfrac{\Delta v}{\Delta t}$.

Be careful to separate **average velocity** 平均速度 (total displacement over total time) from **instantaneous velocity** 瞬时速度 (the velocity at one instant, the slope of the position–time graph at that point). When velocity is constant, they agree at every instant. With changing velocity, an instantaneous value can still equal the average at a particular moment. Taking the average over a shorter and shorter time interval brings it closer and closer to the instantaneous value.

An object **accelerates** whenever its velocity changes, in size **or** direction. A car going round a bend at a steady 20 m/s is accelerating, because its direction changes.

An object speeds up when v and a have the **same** sign, and slows down (**deceleration** 减速) when they have **opposite** signs. Note that a negative acceleration does not always mean slowing down – a ball falling faster and faster has negative velocity *and* negative acceleration.

For **constant acceleration** 恒定加速度, the four **kinematic equations** 运动学方程 (often called SUVAT) apply:

$$v = v_0 + at, \qquad \Delta x = v_0 t + \tfrac{1}{2}at^2, \qquad v^2 = v_0^2 + 2a\,\Delta x, \qquad \Delta x = \tfrac{1}{2}(v_0 + v)\,t.$$

Pick the equation that contains the three quantities you know plus the one you want, so only one unknown is left. They apply **only** while a is constant.

> **Worked example.** A car starts from rest and accelerates uniformly at 2.0 m/s^2 for 6.0 s. Find its **final velocity** 末速度 and the distance it travels.

List what you know: $v_0 = 0$, $a = 2.0$ m/s^2, $t = 6.0$ s.

$$v = v_0 + at = 0 + 2.0 \times 6.0 = 12 \text{ m/s},$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2} \times 2.0 \times 6.0^2 = 36 \text{ m}.$$

> **Worked example (free fall).** A ball is thrown straight up at 15 m/s. Taking $g = 9.8$ m/s^2 and up as positive, how high does it rise, and how long is it in the air before returning to the thrower's hand?

At the highest point the velocity is momentarily zero, and $a = -g = -9.8$ m/s^2 throughout (this is **free fall** 自由落体, ignoring **air resistance** 空气阻力):

$$v^2 = v_0^2 + 2a\,\Delta x \;\Rightarrow\; 0 = 15^2 + 2(-9.8)\Delta x \;\Rightarrow\; \Delta x = \frac{225}{19.6} = 11.5 \text{ m}.$$

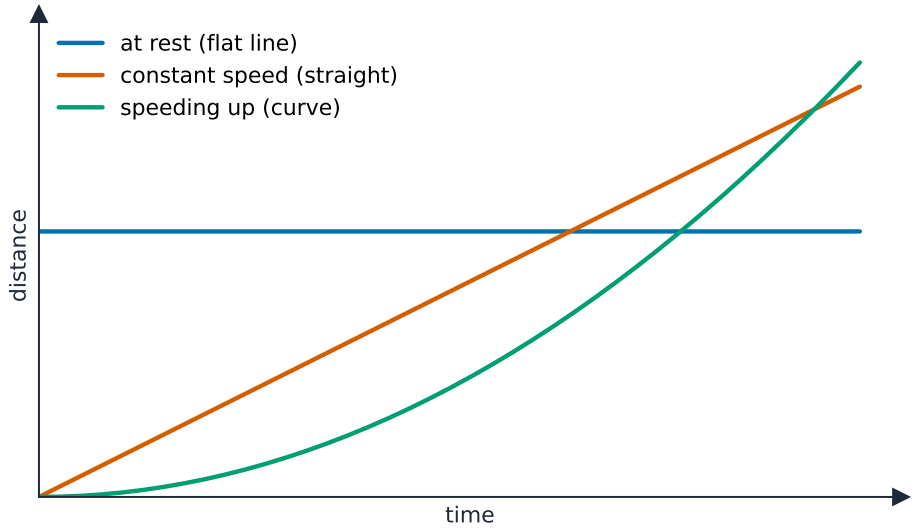
Time to the top: $0 = 15 - 9.8\,t \Rightarrow t = 1.53$ s. By **symmetry** 对称性 the fall takes the same time, so the total is 3.1 s.

On the AP exam you may use $g \approx 10$ m/s^2; with it this ball rises 11.25 m in 1.5 s. Either value is accepted, if you use it consistently.

Representing Motion

The same motion appears as a **description**, a **graph**, a **table**, or an **equation**, and you should move between them:

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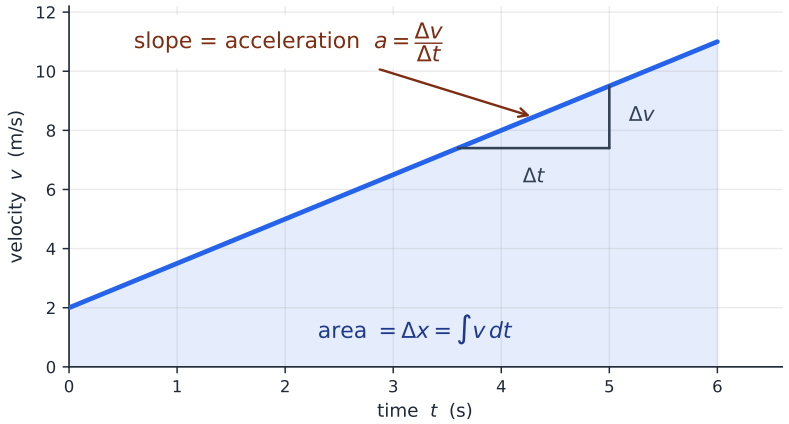


Reading a distance-time graph: flat means at rest, a straight slope means constant speed

- On a **position–time graph** 位置 – 时间图, the **slope** 斜率 is velocity (steeper = faster; a curve = changing velocity).
- On a **velocity–time graph** 速度 – 时间图, the slope is acceleration, and the **area** 面积 under the line is displacement.

Reading slopes and areas off graphs is a core exam skill. To get displacement from a velocity–time graph, split the area into triangles and rectangles and add them up; area **below** the time axis counts as **negative** displacement (motion the other way).

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$a =$ gradient of v-t · $s = \int v\,dt =$ area under the graph

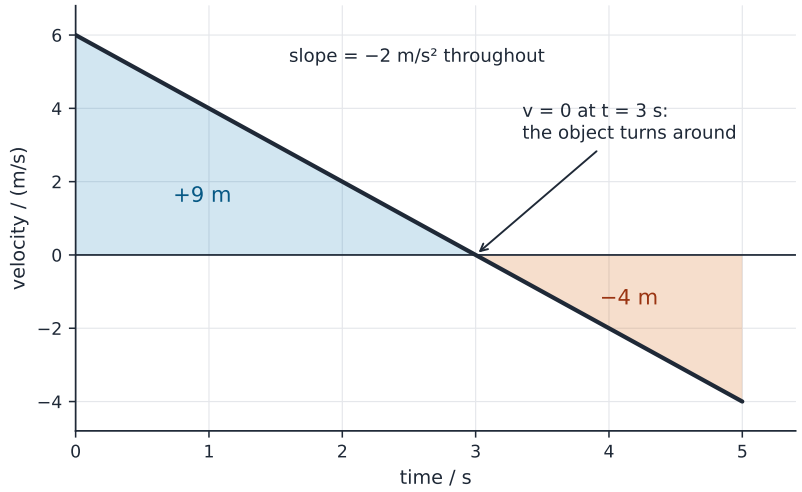
On a velocity-time graph the slope is the acceleration and the shaded area is the displacement

Worked example. A cyclist speeds up uniformly from rest to 8.0 m/s in 4.0 s, then holds 8.0 m/s for 6.0 s. Find the total distance from the velocity–time graph.

The area is a triangle followed by a rectangle:

$$\Delta x = \underbrace{\tfrac{1}{2} \times 4.0 \times 8.0}_{\text{triangle}} + \underbrace{6.0 \times 8.0}_{\text{rectangle}} = 16 + 48 = 64 \text{ m}.$$

Worked example (a graph that crosses the axis). A cart is pushed up a ramp and released. Its velocity is $v = 6.0 - 2.0t$ (up the ramp positive) for 5.0 s. Find (a) its acceleration, (b) when it turns around, (c) its displacement and (d) the distance it travels.



Area above the axis is motion up the ramp; area below it is motion back down

(a) The slope is $\dfrac{-4.0-6.0}{5.0} = -2.0$ m/s^2 – the same on both sides of $t = 3$ s. The acceleration does not change sign when the velocity does.

(b) The cart turns around when $v = 0$: $t = 3.0$ s.

(c) Above the axis: $\frac{1}{2} \times 3.0 \times 6.0 = +9.0$ m. Below: $\frac{1}{2} \times 2.0 \times (-4.0) = -4.0$ m. Displacement $= 9.0 - 4.0 = +5.0$ m (still up the ramp from the start).

(d) Distance adds the sizes: $9.0 + 4.0 = 13$ m.

Check (c) with $\Delta x = v_0 t + \frac{1}{2}at^2 = 6.0(5.0) + \frac{1}{2}(-2.0)(5.0)^2 = 30 - 25 = 5.0$ m. The kinematic equation gives the displacement, never the distance, when the motion reverses.

Average velocity on a graph. The average velocity over an interval is the constant velocity that gives the **same area** (same displacement) in the same time. Over the 5.0 s above it is $\dfrac{+5.0}{5.0} = +1.0$ m/s. On a velocity–time graph, draw it as a horizontal line at $v = 1.0$ m/s; the area under that line equals the net area under the real graph.

When the acceleration is not constant. AP Physics 1 does not ask you to **calculate** with changing acceleration, but you must sketch and explain it. A cart rolling over a series of bumps speeds up on each down slope and slows on each up slope: its velocity–time graph goes up and down. On a position–time graph, the slope of the tangent at a point is the instantaneous velocity; on a velocity–time graph, the slope of the tangent is the instantaneous acceleration. Where a v–t graph gets steeper, the magnitude of acceleration is growing; a steeper downward slope means a more negative acceleration.

Finding a from data: make a straight line. A cart is released from rest at the top of a ramp, and the time t to roll each distance x is measured. Since $x = \frac{1}{2}at^2$, a graph of x against t^2 is a straight line through the origin with slope $\frac{1}{2}a$.

1. Plot x on the vertical axis and t^2 on the horizontal axis, with labels and units.
2. Draw a **best-fit line** 最佳拟合线 that passes close to all points; do not join the dots.
3. Take the slope from two points on the line that are far apart, then $a = 2 \times$ slope. A slope of 0.60 m/s^2 gives $a = 1.2$ m/s^2.

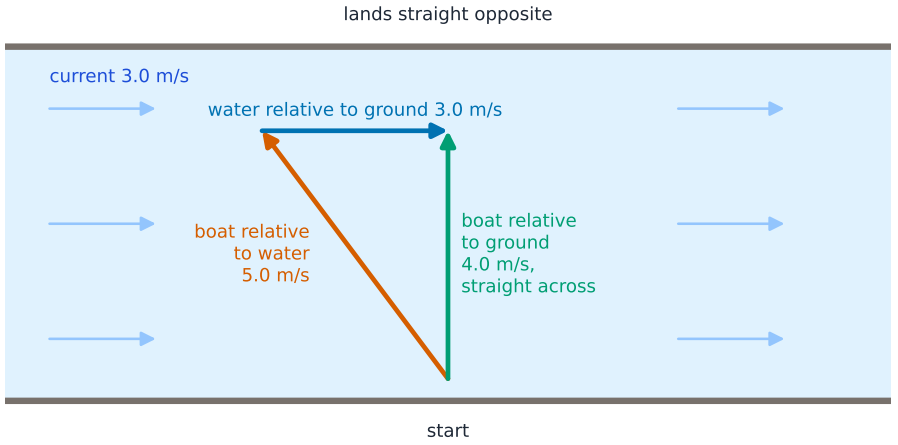
Choose the axes so that the expected equation becomes $y =$ (constant) $\times x$; then the constant, the slope, contains the quantity you want.

Reference Frames and Relative Motion

All motion is measured against a **reference frame** 参考系. Velocities measured in different frames differ, and you combine them by vector addition. The velocity of A relative to C is

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}.$$

A person walking on a moving train has one velocity relative to the train and another relative to the ground – this is **relative motion** 相对运动. A useful shortcut: the velocity of A relative to B is $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$ (subtract B's velocity).



To land straight across, the boat heads upstream: its velocity relative to the water plus the water's velocity relative to the ground points straight across

A frame that is not accelerating is an **inertial reference frame** 惯性参考系: one in which a free object (no net force) obeys Newton's first law, staying at rest or moving at constant velocity. A frame that accelerates – a braking car – is **non-inertial**, where

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objects seem to speed up with no force acting on them.

Observers in different inertial frames measure **different velocities** but the **same acceleration**. A ball dropped on a smoothly moving train falls straight down for a passenger and along a curve for someone on the platform, but both measure its acceleration as g downward.

> **Worked example (one dimension).** A train moves east at 20 m/s relative to the ground. A passenger walks towards the back of the train at 1.5 m/s relative to the train. Take east as positive.

1. $v_{P/T} = -1.5$ m/s and $v_{T/G} = +20$ m/s.
2. $v_{P/G} = v_{P/T} + v_{T/G} = -1.5 + 20 = +18.5$ m/s: east at 18.5 m/s.
3. A car on a road beside the track moving east at 25 m/s sees the passenger moving at $v_{P/C} = 18.5 - 25 = -6.5$ m/s, that is, backwards (west) at 6.5 m/s.

AP Physics 1 keeps relative-velocity questions to one dimension. The example below, with perpendicular velocities, is extra practice with vector addition.

> **Worked example.** A boat points straight across a river and moves at 3.0 m/s relative to the water. The current flows at 4.0 m/s along the river. Find the boat's speed and direction relative to the bank.

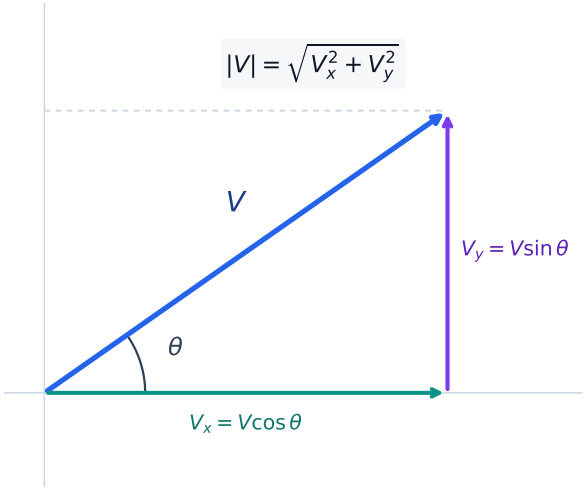
The two velocities are perpendicular, so add them as a right triangle:

$$v = \sqrt{3.0^2 + 4.0^2} = 5.0 \text{ m/s}, \qquad \theta = \tan^{-1}\frac{4.0}{3.0} = 53^\circ \text{ downstream from straight across.}$$

Vectors and Motion in Two Dimensions

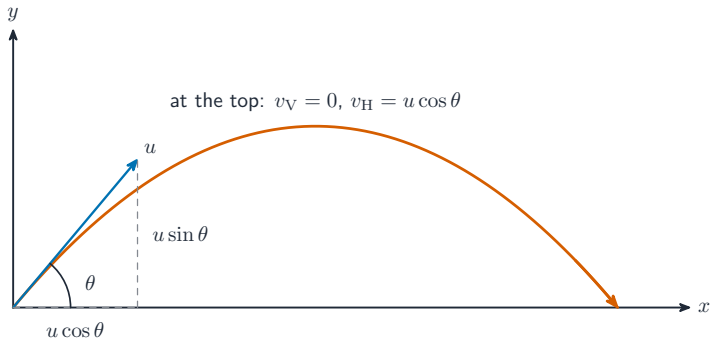
In two dimensions, resolve each vector into **components** 分量 along perpendicular axes (x and y), handle each axis separately, then recombine. A velocity v at angle θ to the horizontal has components $v_x = v\cos\theta$ and $v_y = v\sin\theta$.

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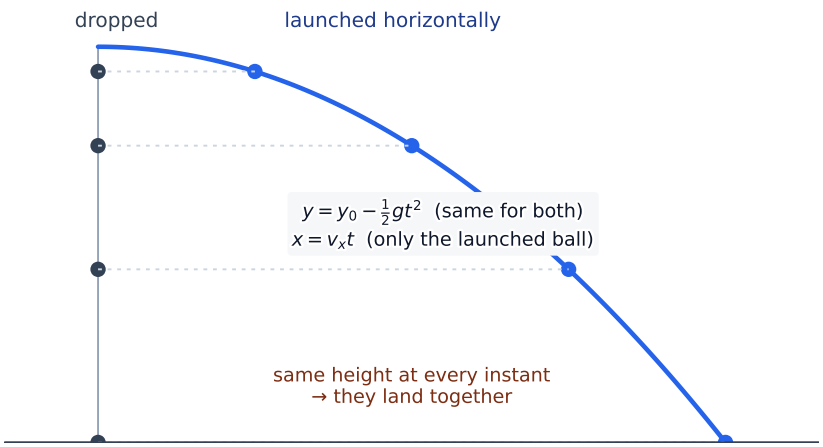


A velocity vector resolved into its horizontal and vertical components

For **projectile motion** 抛体运动 (an object moving under gravity alone): the horizontal and vertical motions are **independent** 相互独立. Horizontally, velocity is constant ($a_x = 0$); vertically, acceleration is $-g$ (down). The two motions share only the **time**. So a projectile's path (its **trajectory** 轨迹) is a **parabola** 抛物线, and you solve it as two one-dimensional problems joined by t.



A projectile launched at an angle: the horizontal and vertical motions are independent



A dropped ball and a horizontally launched ball fall together – the vertical motions are identical

Worked example. A ball is kicked at 20 m/s, 30° above the horizontal. Taking $g = 9.8$ m/s^2, find the **time of flight** 飞行时间, the **maximum height** 最大高度, and the horizontal **range** 射程 (assume it lands at launch height).

Split the launch velocity into components:

$$v_{0x} = 20\cos 30^\circ = 17.3 \text{ m/s}, \qquad v_{0y} = 20\sin 30^\circ = 10 \text{ m/s}.$$

Vertical motion sets the time. At the top $v_y = 0$, so $0 = 10 - 9.8\,t \Rightarrow t_{\text{up}} = 1.02$ s, and the total flight is $2t_{\text{up}} = 2.0$ s. The maximum height is

$$\Delta y = \frac{v_{0y}^2}{2g} = \frac{10^2}{19.6} = 5.1 \text{ m}.$$

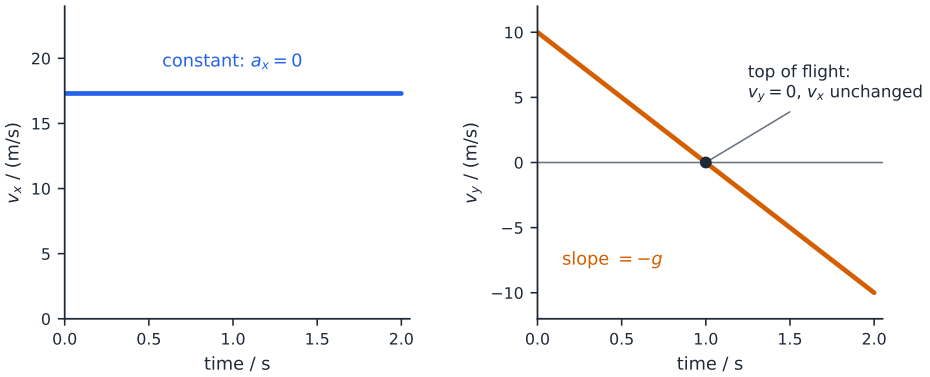
Horizontal motion runs at constant v_{0x} for the whole flight, so the range is

$$R = v_{0x} \times t_{\text{flight}} = 17.3 \times 2.0 = 35 \text{ m}.$$

A common trap: at the top of the flight the vertical velocity is zero, but the ball is **not** at rest – its horizontal velocity v_{0x} never changes. The speed at the top equals $v_{0x} = 17.3$ m/s.

Graphs of the components. FRQs often ask you to sketch v_x and v_y against time for a projectile. With $g = 10$ m/s^2 and the same launch:

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The horizontal component never changes; the vertical one falls steadily, passing through zero at the top

Explaining in a paragraph. Some questions ask for a "paragraph-length response". Example: two spheres leave a table at the same moment, one dropped and one launched horizontally. Explain why they land together although they travel different distances.

> Both spheres start with zero vertical velocity, and once they leave the table the only force on each is gravity, so both have the same downward acceleration, g. The vertical and horizontal motions are independent: the horizontal velocity of the launched sphere does not change how fast it falls. Starting at the same height with the same vertical velocity and acceleration, both cover the same vertical distance in the same time. The launched sphere also moves sideways at a constant speed, so it lands farther away, but at the same moment.

A good paragraph names the physics principle, applies it to **this** situation, and links the steps with "so" or "because", ending with the claim.

Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2 and ignore air resistance.

1. A car moving at 12 m/s brakes uniformly at 3.0 m/s^2 until it stops. Find the stopping time and the stopping distance.
2. A ball is dropped from rest 20 m above the ground. Find the time it takes to land and its speed just before landing.
3. A ball rolls off a table 1.25 m high at 8.0 m/s. How far from the table does it land?
4. A boat heads straight across a river at 3.0 m/s relative to the water. The current flows at 4.0 m/s. Find the boat's speed relative to the bank.

5. A student says: "At the top of its flight, a ball thrown straight up has zero acceleration." Correct the student.

Answers

1. $t = \dfrac{12}{3.0} = 4.0$ s. Distance $= \dfrac{v^2}{2a} = \dfrac{144}{6.0} = 24$ m.
2. $t = \sqrt{2(20)/9.8} = 2.0$ s; $v = gt = 9.8(2.02) = 20$ m/s.
3. Fall time $\sqrt{2(1.25)/9.8} = 0.51$ s, so it lands $8.0(0.51) = 4.0$ m away. The horizontal speed does not change during the fall.
4. The velocities are perpendicular: $\sqrt{3.0^2 + 4.0^2} = 5.0$ m/s, at $\tan^{-1}(3.0/4.0) = 37°$ to the bank.
5. At the top the **velocity** is zero for an instant, but gravity still acts, so the acceleration is still 9.8 m/s^2 downward. If the acceleration were zero, the ball would stay at the top.

Exam tips

- Choose the right **kinematic equation** by listing the three quantities you know plus the one you want, so only one unknown remains; the SUVAT equations apply **only** while acceleration is constant.
- Fix a **positive direction** first – every displacement, velocity, and acceleration then carries a sign; a negative velocity means "moving the other way", not "slow".
- Treat a projectile as **two independent 1-D problems** sharing only the time t: constant velocity horizontally, $a = -g$ vertically. At the top $v_y = 0$ but v_x is unchanged.
- On a **velocity–time graph** the gradient is the acceleration and the **area** is the displacement (area below the axis is negative).
- Distinguish **distance** (scalar, total path) from **displacement** (vector, start-to-end), and **speed** from **velocity**.

■ Common mistakes

- Saying the acceleration is zero at the top of a throw, or where a velocity–time graph crosses the axis. The velocity is zero there; the acceleration is the slope, which is not zero.
- Reading a position–time graph as if it were a velocity–time graph. Check the axis label before you use a slope or an area.
- Adding areas below the time axis as positive when the question asks for displacement. Below-axis area is negative displacement; add the sizes only for distance.
- Using $v_0 = 20$ m/s in the vertical equations of a projectile launched at 20 m/s at an angle. Resolve first: vertical uses $v_0 \sin\theta$, horizontal uses $v_0 \cos\theta$.

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- Using kinematic equations over a stretch where the acceleration changes (for example, braking that starts after a reaction time). Split the motion into parts with constant acceleration.
- Joining data points dot-to-dot, or taking a slope from two data points rather than from the best-fit line.
- Drawing v_y for a projectile as a curve, or as a V shape that bounces at the top. It is one straight line with slope $-g$ that passes through zero.
- Giving g a sign that does not match your positive direction. With up positive, $a = -9.8$ m/s^2 on the way up *and* on the way down.

Force and Translational Dynamics

AP Physics 1

Systems and Center of Mass

A lift accelerating upward makes you feel heavier; a car rounding a bend pushes you against the door; astronauts float although Earth's gravity on them is still about 90% of its value at the surface. Newton's laws explain all three, as long as you choose the right system and draw the right forces.



Internal forces spin the parts; the centre of mass goes straight.

A **system** 系统 is the object or group of objects you choose to analyze. A system can be treated as a single point at its **center of mass** 质心 – the average position of its mass. External forces change the motion of the center of mass; internal forces (between parts of the system) do not.

This is why a wrench spinning across a table still has its center of mass move in a straight line: the spinning is internal, and only the (near-zero) external force matters for the center of mass. For two masses m_1, m_2 on a line at positions x_1, x_2, the center of mass sits at $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ – always closer to the heavier mass.

> **Worked example (center of mass).** Three small masses lie on a line: 2.0 kg at $x = 0$, 1.0 kg at $x = 3.0$ m and 3.0 kg at $x = 4.0$ m.

$$x_{\text{cm}} = \frac{2.0(0) + 1.0(3.0) + 3.0(4.0)}{2.0 + 1.0 + 3.0} = \frac{15}{6.0} = 2.5 \text{ m}.$$

The center of mass is nearer the heavy end, and it need not be where any mass is. For a symmetric object, such as a uniform rod or ball, it lies on the lines of symmetry: at the middle. In two dimensions, use the same formula separately for x and for y.

One system or several objects? A system's center of mass moves as if all the external forces acted on one point there. Its parts can still move differently: when a firework bursts, the pieces fly apart, but their center of mass carries on along the same parabola, because the explosion forces are internal.

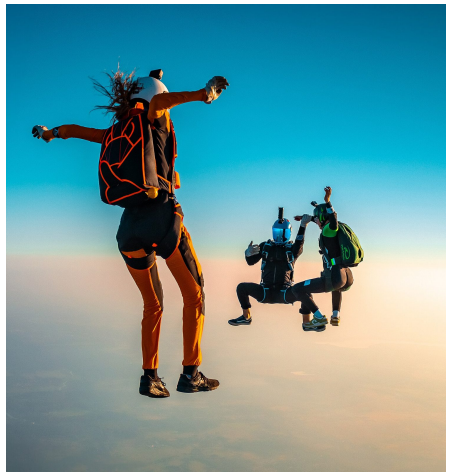
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Forces and Free-Body Diagrams

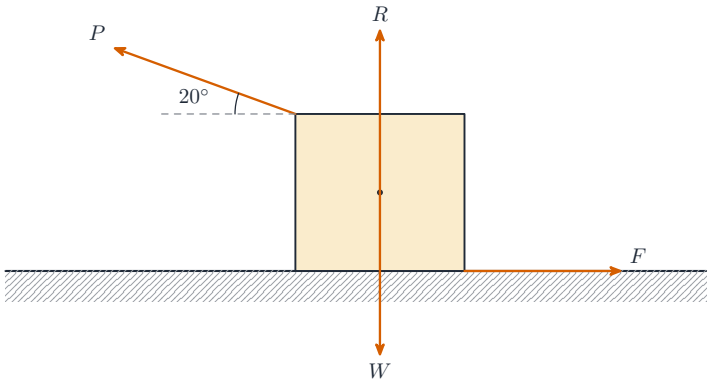
A **force** 力 is a push or pull – a vector, measured in newtons (N). A **free-body diagram** 受力图 shows one object as a dot with arrows for **every** force acting **on** it (weight 重力, normal, tension 张力, friction, applied), each labelled and pointing the right way. Draw it before any dynamics problem; it is where most marks are won or lost.



A rocket launch: net force changes momentum – F_net = ma for translational dynamics



A skydiver: free-body diagrams balance gravity and air resistance at terminal speed



A free-body diagram shows every force acting on one object

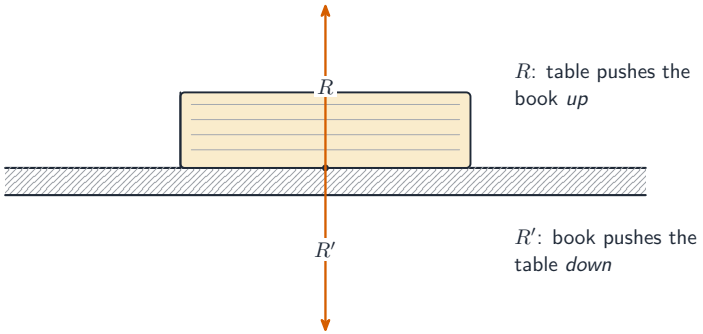
Two rules keep free-body diagrams honest: draw only forces acting **on** the chosen object (not forces it exerts on other things), and draw only **real, physical** forces (a rope, a surface, gravity, a hand) – never an "ma" arrow, which is the *result* of the forces, not a force itself.

Every force comes from an **interaction** with another object: no object can push itself forward. Contact forces (normal, friction, tension) are the large-scale effect of electric forces between the atoms of the two surfaces. On an AP free-body diagram, draw each force as **one arrow from the dot**, not as components; arrows drawn to scale should have lengths in proportion to the forces. To solve, choose one axis **along the acceleration** (for example, along a ramp), then split forces into components in your equations.

Newton's Third Law

Newton's third law 牛顿第三定律: if object A pushes on object B, then B pushes back on A with a force **equal in size and opposite in direction**. These two forces act on **different** objects, so they do not cancel in one object's free-body diagram. If both objects belong to the chosen system, the pair cancels when summing its internal forces. Identify third-law pairs by the "A on B / B on A" wording.

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A Newton's third-law pair: equal and opposite forces on two different objects

A classic trap: the weight of a book and the normal force from the table are **not** a third-law pair – they act on the **same** object (the book). The partner of the book's weight is the pull the book exerts on the Earth; the partner of the table's push is the push the book makes on the table.

Strings and pulleys. Tension is the pull that the parts of a string exert on each other. An **ideal string** has negligible mass and does not stretch, so the tension is the **same all along it**, and both ends move with the same speed. An **ideal pulley** has negligible mass and no friction, so it only changes the direction of the tension. A heavy rope is different: a heavy rope hanging at rest has its greatest tension at the top, where it must also hold up the rope below.

> **Worked example (checking with limiting cases).** Two blocks, m_A and m_B, hang on either side of an ideal pulley ($m_A > m_B$). Treating them as one system, the net force is $(m_A - m_B)g$ and the mass moved is $m_A + m_B$:

$$a = \frac{(m_A - m_B)g}{m_A + m_B}.$$

Test it at the extremes. If $m_A \gg m_B$, then $a \to g$: block A falls almost freely. If $m_A = m_B$, then $a = 0$: the blocks balance. An expression that gives sensible answers in the limits is probably right, and FRQs often ask "does your equation agree with your reasoning?": this is how you check.

Newton's First Law

Newton's first law 牛顿第一定律 (the law of **inertia** 惯性): an object's velocity stays constant unless a **net force** 合力 acts on it. So zero net force means constant velocity (including rest) – the object is in **translational equilibrium** 平衡. Inertia is the tendency to resist changes in motion, measured by mass.

Worked example. A 1200 kg car cruises at a steady 25 m/s on a level road. What is the net force on it? Because the velocity is constant, the acceleration is zero, so by the first law the **net** force is zero – the forward drive force exactly balances drag and friction. Constant velocity means balanced forces; steady speed around a bend still needs a net inward force.

Forces can be balanced along one axis and not along another. A puck sliding across ice has its weight balanced by the normal force, so it does not speed up vertically; with no horizontal force, it keeps a constant horizontal velocity too. A projectile has balanced (zero) horizontal force but unbalanced vertical force, so only its vertical velocity changes.

Newton's Second Law

Newton's second law 牛顿第二定律 relates net force to acceleration:

$$\vec{a} = \frac{\sum \vec{F}}{m}, \qquad \text{i.e.} \qquad \sum \vec{F} = m\vec{a}.$$

Apply it **one axis at a time**: add the force components along each axis and set the sum equal to ma for that axis. Acceleration points the same way as the net force.

Worked example. A 4.0 kg box is pulled along the floor by a horizontal force of 18 N. Friction on the box is 6.0 N. Find its acceleration. Along the direction of motion the net force is $18 - 6.0 = 12$ N, so

$$a = \frac{\sum F}{m} = \frac{12}{4.0} = 3.0 \text{ m/s}^2.$$

Worked example (incline 斜面). A block of mass m slides down a frictionless ramp tilted at angle θ. Find its acceleration. Resolve gravity into components along and perpendicular to the ramp; only the along-ramp part, $mg\sin\theta$, drives the motion, so

$$a = \frac{mg\sin\theta}{m} = g\sin\theta.$$

The steeper the ramp, the larger $\sin\theta$ and the faster it accelerates; at $\theta = 90°$ it is free fall.

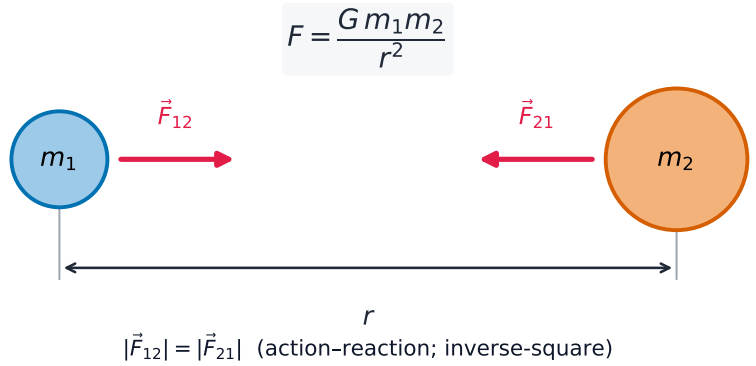
Gravitational Force

Near a planet's surface, the **gravitational force** (weight) is $F_g = mg$, directed down, where g is the **gravitational field strength** 重力场强度. More generally, **Newton's law of gravitation** 万有引力定律 gives the attraction between point masses, or non-overlapping spherically symmetric masses:

$$F_g = \frac{Gm_1m_2}{r^2},$$

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directed along the line joining them, weaker as the distance r grows (an inverse-square law). Doubling the separation quarters the force.



Two masses attract each other with equal, opposite, inverse-square forces along the line joining them

Worked example. A 2.0 kg object weighs 19.6 N on Earth ($g = 9.8$ m/s^2). On the Moon $g_{\text{Moon}} = 1.6$ m/s^2. Its mass is unchanged (2.0 kg), but its weight becomes $F_g = mg = 2.0 \times 1.6 = 3.2$ N. Mass measures inertia; weight is a force that depends on where you are.

Gravitational field. A mass creates a **gravitational field** 引力场 around it. The field strength at a point is the force per unit mass on a small test mass placed there: $g = \frac{F_g}{m} = \frac{GM}{r^2}$, measured in N/kg. Near Earth's surface $g \approx 10$ N/kg, and if gravity is the only force, the acceleration in m/s^2 equals the field strength in N/kg.

Apparent weight. A scale reads the normal force on you, your **apparent weight** 视重, not the gravitational force. In a lift accelerating **upward** at a, $N - mg = ma$, so $N = m(g + a)$: you feel heavier. Accelerating downward, $N = m(g - a)$: you feel lighter. In free fall, $a = g$ and $N = 0$: you feel **weightless**. Astronauts in orbit are weightless for this reason. They are not out of gravity's reach; they and their spacecraft are all falling around Earth together. An observer in a closed lift cannot tell whether the push of the floor comes from gravity or from the lift accelerating: this is the **equivalence principle**.

Mass actually plays two distinct roles. **Inertial mass** 惯性质量 sets how strongly an object resists acceleration ($F = ma$); **gravitational mass** 引力质量 sets how strongly it attracts other masses ($F = Gm_1m_2/r^2$). Experiments confirm the two are **equal** – which is exactly why every object, heavy or light, falls with the same g.

Kinetic and Static Friction

Friction 摩擦力 acts along a surface, opposing relative sliding (or the tendency to slide):



A climber relies on static friction: it grows to match the push, up to a maximum of $\mu s \cdot N$

- **Kinetic friction** 动摩擦 (while sliding): $f_k = \mu_k N$.
- **Static friction** 静摩擦 (while not yet sliding): $f_s \le \mu_s N$ – it adjusts up to a maximum to prevent motion.

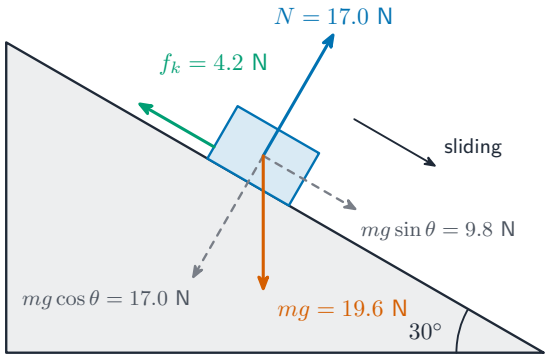
Here N is the **normal force** 法向力 (surface push, perpendicular to the surface) and μ is the **coefficient of friction** 摩擦系数.

Three facts are tested often. Friction does **not** depend on the area in contact: a brick slides the same on its side as on its end. The coefficient depends only on the two materials. And μ_s is usually **larger** than μ_k, so it takes more force to start an object sliding than to keep it sliding.

> **Worked example.** A 5.0 kg crate sits on a level floor with $\mu_s = 0.40$. Will a horizontal push of 15 N move it? On a level floor $N = mg = 5.0 \times 9.8 = 49$ N, so the largest static friction is $f_{s,\max} = \mu_s N = 0.40 \times 49 = 19.6$ N. The 15 N push is smaller than 19.6 N, so friction rises to match it and the crate stays still.

> **Worked example (incline with friction).** A 2.0 kg block slides down a ramp tilted at 30°, with $\mu_k = 0.25$. Find its acceleration.

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Tilt the axes along and perpendicular to the ramp; only the weight has to be split into components

1. Perpendicular to the ramp the block does not accelerate, so $N = mg\cos\theta = 2.0 \times 9.8 \times \cos 30° = 17.0$ N (not $mg = 19.6$ N).
2. Kinetic friction opposes the sliding, so it points **up** the ramp: $f_k = \mu_k N = 0.25 \times 17.0 = 4.2$ N.
3. Along the ramp (down the ramp positive): $\sum F = mg\sin\theta - f_k = 9.8 - 4.2 = 5.6$ N.

$$a = \frac{\sum F}{m} = \frac{5.6}{2.0} = 2.8 \text{ m/s}^2 \qquad \text{or, in symbols,} \qquad a = g(\sin\theta - \mu_k \cos\theta)$$

Check the limits: with $\mu_k = 0$ this gives $g\sin\theta = 4.9$ m/s^2, the frictionless result; friction opposes this downward sliding.

> **Worked example (pulling at an angle).** A 10 kg sled is pulled along level snow by a rope at 30° above the horizontal with a tension of 50 N; $\mu_k = 0.20$. Find the acceleration.

Vertical: the rope's upward component helps hold the sled up, so $N + 50\sin 30° = mg$, giving $N = 98 - 25 = 73$ N. Friction: $f_k = 0.20 \times 73 = 14.6$ N. Horizontal:

$$a = \frac{50\cos 30° - 14.6}{10} = \frac{43.3 - 14.6}{10} = 2.9 \text{ m/s}^2$$

Using $N = mg$ here would give friction 19.6 N and $a = 2.4$ m/s^2: wrong, because the normal force is whatever the perpendicular equation says it is.

> **Worked example (connected objects).** A 3.0 kg cart on a frictionless table is tied by a light string over a light, frictionless pulley to a hanging 1.0 kg mass. Find the acceleration and the tension.

Treat the cart and the hanging mass as one system moving along the string. The only external force along the string direction is the hanging mass's weight, $1.0 \times 9.8 = 9.8$ N

(the tension is internal):

$$a = \frac{9.8}{3.0 + 1.0} = 2.45 \text{ m/s}^2 \approx 2.5 \text{ m/s}^2$$

Now take the cart alone: the only horizontal force on it is the tension, so $T = 3.0 \times 2.45 = 7.4$ N. Check with the hanging mass alone: $mg - T = ma$ gives $9.8 - 7.4 = 2.4 \approx 1.0 \times 2.45$. The tension is **less** than the hanging weight; if it were equal, the hanging mass could not accelerate downward.

Measuring μ_s by tilting. Place a block on a board and slowly raise one end until the block **just** starts to slide, at angle θ_c. At that moment static friction is at its maximum and balances the weight component down the slope: $\mu_s mg\cos\theta_c = mg\sin\theta_c$, so

$$\mu_s = \tan\theta_c.$$

The mass cancels. To reduce uncertainty, raise the board slowly, repeat several times and average θ_c.

Measuring μ_k with a straight-line graph. Pull a block at constant velocity with a force sensor, so the pull equals kinetic friction. Repeat with extra masses on the block. Plot friction force f_k (vertical) against normal force $N = mg$ (horizontal): the slope is μ_k. Doubling the mass of the block doubles both N and f_k, so μ_k itself does not change.

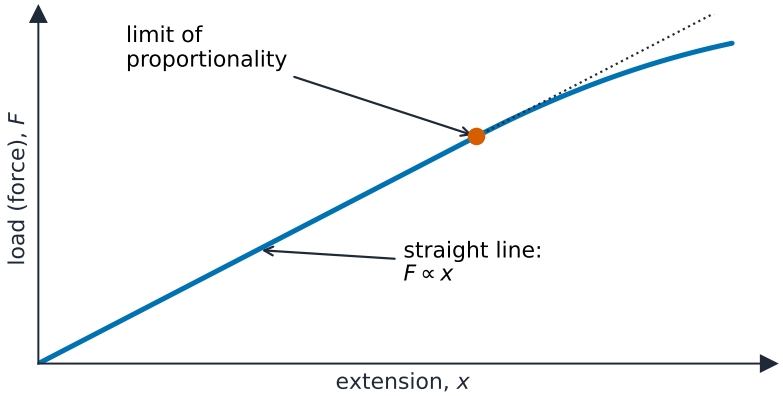
Spring Forces

An ideal spring exerts a **restoring force** 回复力 proportional to its stretch or compression – **Hooke's law** 胡克定律:

$$F_s = -kx,$$

where k is the **spring constant** 弹簧常数 (stiffness) and x is the displacement from the spring's natural length. The minus sign means the force points back toward equilibrium.

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$F = kx$ (straight section)
$k =$ gradient

Hooke's law: extension is proportional to load up to the limit of proportionality

Worked example. A spring with $k = 200$ N/m hangs vertically and a 0.50 kg mass is hung on it. How far does it stretch at rest? At rest the spring force balances the weight, $kx = mg$, so

$$x = \frac{mg}{k} = \frac{0.50 \times 9.8}{200} = 0.025 \text{ m} = 2.5 \text{ cm}.$$

Finding k in the lab. Hang different masses on the spring and measure each stretch x from the spring's relaxed length. Plot the spring force $F = mg$ (vertical) against x (horizontal). If the spring obeys Hooke's law, the points lie on a straight line through the origin, and the **slope is** k. A line that curves at large stretches shows the spring has gone past its limit of proportionality.

Circular Motion

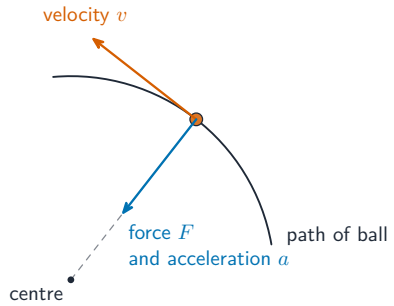


A hammer thrower: the wire's tension provides the centripetal force that keeps the ball moving in a circle

An object moving in a circle at constant speed still **accelerates**, because its velocity direction keeps changing. This **centripetal acceleration** 向心加速度 points toward the center:

$$a_c = \frac{v^2}{r}.$$

It is produced by a **net inward (centripetal) force** 向心力 $F_c = \dfrac{mv^2}{r}$ – supplied by whatever real force points inward (tension, gravity, friction, normal). The time for one full revolution is the **period** 周期 T, and the number of revolutions per second is the **frequency** 频率 f; they are reciprocals, $f = 1/T$, and the same definitions describe any rotation or oscillation.There is no separate outward force; "centrifugal" is only an apparent effect.



The velocity points along the tangent; the centripetal force and acceleration point to the centre

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Worked example. A 0.30 kg ball on a horizontal string slides on a frictionless horizontal table in a circle of radius 0.80 m at 4.0 m/s. Find the tension in the string. The tension supplies the whole centripetal force:

$$T = \frac{mv^2}{r} = \frac{0.30 \times 4.0^2}{0.80} = 6.0 \text{ N}.$$

If the string can take at most 6.0 N, this is the fastest the ball can go at that radius – spin any faster and the string breaks.

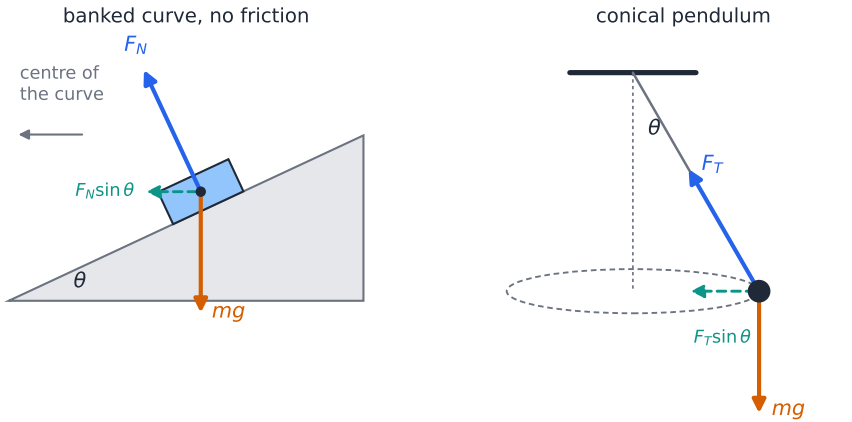
Worked example (top of a vertical loop). A roller-coaster car goes over the inside top of a loop of radius 8.0 m. What is the slowest speed at which it stays on the track? At the top, both the weight and the normal force from the track point **down**, toward the center: $mg + N = \dfrac{mv^2}{r}$. The car is just about to lose contact when $N = 0$, so $mg = \dfrac{mv^2}{r}$:

$$v_{\min} = \sqrt{gr} = \sqrt{9.8 \times 8.0} = 8.9 \text{ m/s}$$

The mass cancels. Faster than this, the track must push down as well ($N > 0$); slower, gravity alone is more than enough inward force and the car falls away from the track.

Speeding up on a circle. If an object's speed along the circle changes, it also has a **tangential acceleration** 切向加速度, along the tangent (forward when speeding up, backward when slowing down). Its total acceleration is the vector sum of the centripetal part (towards the center, v^2/r) and the tangential part. A car speeding up as it rounds a bend has an acceleration that points forward **and** inward.

When a tilted force does the job. The centripetal force is often a **component** of a force:



solid arrows: the real forces; dashed: the horizontal component that points to the centre

On a banked curve and in a conical pendulum, the horizontal component of a tilted force points to the center

- **Banked curve with no friction.** Vertically, $F_N \cos\theta = mg$. Horizontally, $F_N \sin\theta = \dfrac{mv^2}{r}$. Dividing: $\tan\theta = \dfrac{v^2}{rg}$. A curve of radius 100 m banked at θ with $\tan\theta = 0.25$ is safe at $v = \sqrt{0.25 \times 100 \times 10} \approx 16$ m/s, even on ice.
- **Conical pendulum.** Vertically, $F_T \cos\theta = mg$. Horizontally, $F_T \sin\theta = \dfrac{mv^2}{r}$. So again $\tan\theta = \dfrac{v^2}{rg}$: the faster it swings, the larger the angle.

Orbits. For a satellite in a circular orbit, gravity alone supplies the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \;\Rightarrow\; v = \sqrt{\frac{GM}{r}}.$$

With $v = \dfrac{2\pi r}{T}$, the period is $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$. The satellite's own mass m cancels, so a spacecraft of mass $2m$ in the same orbit has the **same** period. A larger orbit is slower and takes longer. The **period** T and **frequency** $f = \dfrac{1}{T}$ also describe any uniform circular motion: $v = \dfrac{2\pi r}{T}$.

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Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2.

1. A 5.0 kg box is pulled along a floor by a 30 N horizontal force against 10 N of friction. Find its acceleration.
2. A 60 kg person stands on a scale in an elevator accelerating **upward** at 2.0 m/s^2. What does the scale read?
3. A block slides down a frictionless ramp inclined at 30°. Find its acceleration.
4. Blocks of 2.0 kg and 3.0 kg sit side by side on a frictionless floor. A 20 N push acts on the 2.0 kg block. Find the acceleration and the force between the blocks.
5. A student says: "Newton's third-law forces are equal and opposite, so they cancel and nothing can ever accelerate." Correct the student.

Answers

1. $a = \dfrac{30 - 10}{5.0} = 4.0$ m/s^2.
2. $N - mg = ma$, so $N = 60(9.8 + 2.0) = 708$ N. The scale reads more than the weight of 588 N.
3. $a = g \sin 30° = 4.9$ m/s^2.
4. $a = \dfrac{20}{5.0} = 4.0$ m/s^2. The 3.0 kg block is pushed only by the other block: $F = 3.0(4.0) = 12$ N.
5. A third-law pair acts on **two different** objects, so the two forces never appear in the same free-body diagram and cannot cancel. To decide whether one object accelerates, add only the forces acting **on** it.

Exam tips

- Always draw a **free-body diagram** first: only real forces **on** the chosen object (weight, normal, tension, friction, applied) – never an "ma" arrow.
- Apply $\sum F = ma$ **one axis at a time**; on an incline resolve gravity into $mg\sin\theta$ (along) and $mg\cos\theta$ (perpendicular).
- A **Newton's third-law pair** acts on **two different objects** – a book's weight and the table's normal force are *not* a pair (both act on the book).
- **Static friction** adjusts up to $\mu_s N$ (use it to test whether motion starts); once sliding, use **kinetic** friction $f_k = \mu_k N$.
- Circular motion needs a **net inward (centripetal) force** mv^2/r supplied by a real force – there is no outward "centrifugal" force.

Common mistakes

- Writing $N = mg$ on a ramp, or when a force pulls up at an angle. Find N from the perpendicular force equation each time.
- Pointing friction against the applied force instead of against the (tendency of) sliding. On a ramp it points up the slope for a block sliding down.
- Using $\mu_s N$ as the static friction force. $\mu_s N$ is only the *maximum*; below it, static friction equals whatever is needed to prevent sliding.
- Drawing a "centripetal force" arrow as an extra force on a free-body diagram. The centripetal force is the net inward part of the real forces (tension, gravity, normal, friction).
- Setting the tension equal to the hanging weight in a connected system that is accelerating. Tension equals the hanging weight only when the acceleration is zero.
- Thinking an astronaut in orbit is weightless because there is no gravity there. Gravity there is almost as strong as on the ground; the astronaut is in free fall.
- Thinking a heavier satellite needs a faster speed for the same orbit. The mass cancels.
- Saying a larger contact area gives more friction. Friction depends on the normal force and the materials, not on the area.
- Adding a third-law partner to the same free-body diagram. The partner force acts on the *other* object.

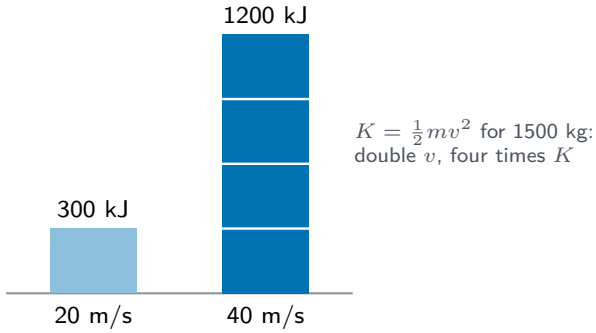
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Work, Energy, and Power

AP Physics 1

Translational Kinetic Energy

A pole vaulter sprints, plants the pole, and rises over a bar five metres up. No engine lifts her. The speed she brought to the pole becomes bend in the pole, then height. This unit tracks energy as it moves between stores like these, and shows how to account for every joule.



Double the speed, four times the kinetic energy.

Energy 能量 is the capacity to do work, measured in **joules** 焦耳 (J). A moving object has **kinetic energy** 动能:

$$K = \tfrac{1}{2}mv^2.$$

It depends on the **square** of the speed, so doubling the speed **quadruples** the kinetic energy. Kinetic energy is a **scalar** 标量 and is never negative.

> **Worked example.** A 1500 kg car travels at 20 m/s. Its kinetic energy is $K = \frac{1}{2} \times 1500 \times 20^2 = 3.0 \times 10^5$ J $= 300$ kJ. If it speeds up to 40 m/s (double), the kinetic energy becomes 4× larger, 1200 kJ – which is why **stopping distance** 制动距离 grows so fast with speed.

Kinetic energy depends on the frame. Speed is measured relative to an observer, so kinetic energy is too. A 70 kg passenger sitting on a train moving at 30 m/s has $K = 0$ for someone on the train, but $K = \frac{1}{2}(70)(30)^2 = 31.5$ kJ for someone on the platform. Both are correct; just use one frame for the whole problem.

A one-object particle model has only kinetic energy. Potential energy belongs to the **interaction** 相互作用 between two or more objects (a ball and the Earth, a block and a spring). When a single object is treated as a particle, its internal energy

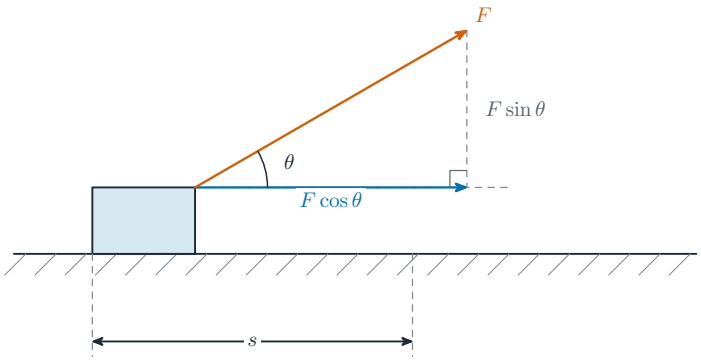
stores are ignored; changes in its kinetic energy come from net external work. A real object can also store internal energy.

Work

Work 功 is energy transferred by a force acting over a **displacement** 位移. For a constant force:

$$W = F\,d\cos\theta,$$

where θ is the angle between the force and the displacement. Work is **positive** when the force has a component along the motion (adds energy), **negative** when it opposes the motion (removes energy), and **zero** when the force is **perpendicular** 垂直. On a force–position graph, work is the **area** under the curve. The **work–energy theorem** 动能定理 states that the net work equals the change in kinetic energy: $W_{\text{net}} = \Delta K$.



Only the force component along the displacement does work

Worked example. A 2.0 kg block moving at 3.0 m/s on a **frictionless** 无摩擦的 floor is pushed by a 5.0 N force over 4.0 m in the direction of motion. Find its final speed. The net work is $W = Fd = 5.0 \times 4.0 = 20$ J, and by the work–energy theorem $W = \frac{1}{2}m(v^2 - v_0^2)$:

$$20 = \tfrac{1}{2} \times 2.0 \times (v^2 - 3.0^2) \;\Rightarrow\; v^2 = 29 \;\Rightarrow\; v = 5.4 \text{ m/s}.$$

A perpendicular force changes direction, not speed. If a force is always perpendicular to the velocity, it does no work, so $\Delta K = 0$ and the speed stays constant. It can still change the **direction** of motion: this is exactly what the tension does on a ball whirled in a circle.

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Worked example (twice the compression, how far does it slide?). A spring launches a block onto a rough floor, where the block slides a distance d before stopping. The spring is now compressed **twice** as much. How far does the block slide?

- Energy stored: $U_s = \frac{1}{2}kx^2$, so doubling x gives 4× the energy.
- On the rough floor, friction takes it all: $\frac{1}{2}kx^2 = \mu_k mg\,d$, so $d \propto x^2$.
- The block slides $4d$, not $2d$.

Exam questions often ask you to *justify* this kind of answer: say which quantity is squared, and write the equation that links it to the distance.

Potential Energy

Potential energy 势能 is stored energy that depends on position or configuration:



A conservative force's work depends only on the end points.

- **Gravitational potential energy** 重力势能 near the surface: $U_g = mgh$ (height h above a **reference level** 参考面).
- **Elastic potential energy** 弹性势能 in a spring: $U_s = \frac{1}{2}kx^2$.

Potential energy is defined only for **conservative forces** 保守力 (gravity, springs), for which the stored energy depends on position, not path. Only *changes* in potential energy matter, so you may put the zero level wherever is convenient.

Conservative forces and path. The work done by a conservative force between two points is the same for **every path**. Lift a book 1 m straight up, or carry it round the room first: gravity does the same $-mg(1\text{ m})$ of work. Around a **closed path** 闭合路径 (back to the start) a conservative force does **zero** total work. Kinetic friction against a stationary surface is different: it does negative work along the sliding path, so a longer path loses more energy for the same friction force.

Total potential energy is a sum over pairs. Each pair of interacting objects has its own potential energy. For a block hanging on a spring near the Earth, the system's potential energy is $U = U_g + U_s$: one term for the block–Earth pair and one for the block–spring pair.

The $U_g = mgh$ form only holds near a surface where g is roughly constant. The **general** form, for two non-overlapping spherically symmetric masses with centers a distance r apart, is

$$U_g = -\frac{Gm_1m_2}{r}.$$

It is **negative** and defined to be **zero at infinite separation**, so gravitational PE rises toward zero as the masses move apart. mgh is simply its near-surface approximation. (This is the form you need for **satellites** 卫星 and **escape speed** 逃逸速度.)

Conservation of Energy



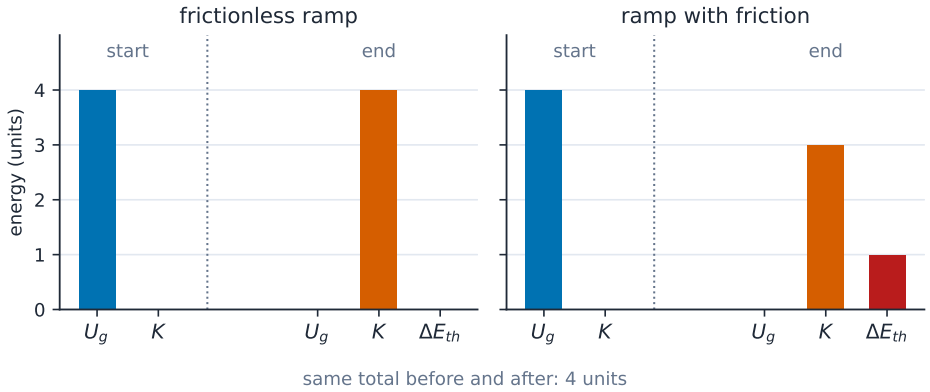
A roller coaster trades energy back and forth: it is highest (most potential energy) at the top and fastest (most kinetic energy) at the bottom

The **total mechanical energy** 机械能 is $E = K + U$. When only conservative forces do work, mechanical energy is **conserved** 守恒:

$$K_1 + U_1 = K_2 + U_2.$$

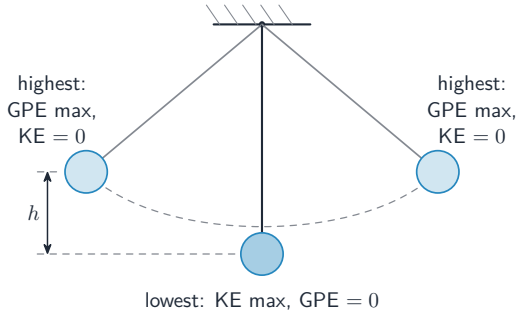
Friction 摩擦力 can transfer mechanical energy to **thermal energy** 热能. Other **non-conservative forces** 非保守力 can add mechanical energy as well. Total energy is conserved when all stores and transfers across the chosen system boundary are included. Energy bar charts are a good way to track where the energy goes.

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Energy bar charts: friction moves one unit into thermal energy, but the total stays four

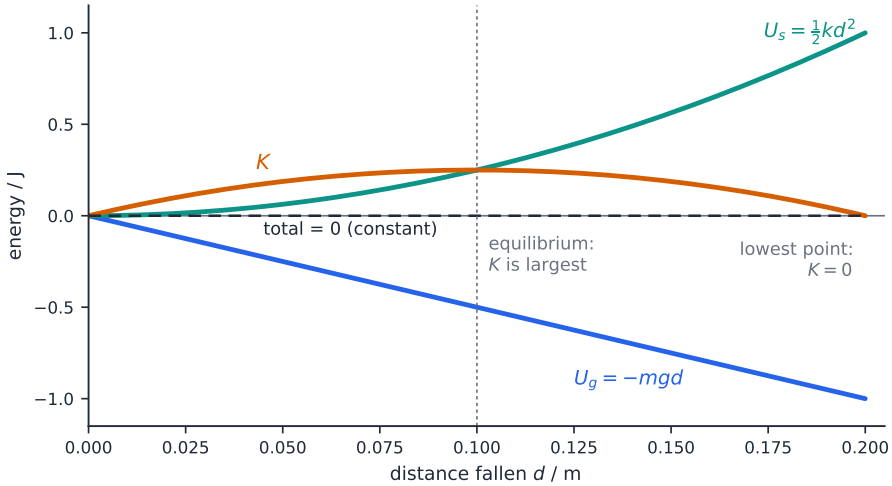
An **energy bar chart** 能量柱状图 shows each store at the start and the end. The bars must add to the same total, unless work, heat or another energy transfer crosses the system boundary. Draw the chart before writing an equation: the equation is then just the chart in symbols, $U_{g1} = K_2 + \Delta E_{th}$.



A swinging pendulum trades gravitational potential energy for kinetic energy and back

Energy against position. Free-response questions often ask you to sketch each energy store against position. Take a 0.50 kg block hung on an unstretched spring ($k = 50$ N/m) and released from rest. Choose the system *block + spring + Earth* and the start as zero for both potential energies. As the block falls a distance d:

- $U_g = -mgd$ is a straight line going down;
- $U_s = \frac{1}{2}kd^2$ is a parabola going up;
- K is whatever is left, because the total stays 0.



Energy against position for a block on a spring: the three stores always add to the same total

Kinetic energy is largest where the forces balance ($mg = kd$, at $d = 0.10$ m), and the block stops at $d = 0.20$ m, twice that distance. On a graph of K against U for a system with constant total energy E, the line is $K = E - U$: a straight line of slope -1.

> **Worked example.** A ball is released from rest at the top of a frictionless ramp 5.0 m high. Find its speed at the bottom. All the gravitational potential energy becomes kinetic energy:
>
> $$mgh = \tfrac{1}{2}mv^2 \;\Rightarrow\; v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5.0} = 9.9 \text{ m/s}.$$
>
> The mass cancels, so every object reaches the same speed – exactly the free-fall result, now got from energy. If instead 30 J were lost to friction, you would subtract it: $mgh - 30 = \tfrac{1}{2}mv^2$.

> **Worked example (a qualitative–quantitative question).** The fourth free-response question on the exam, *Qualitative Quantitative Translation*, asks for the same idea three ways: in words, as a derived equation, and then a check that the two agree. A block of mass m is released from rest at height H on a frictionless ramp. At the bottom it slides onto a rough floor, with coefficient of kinetic friction μ, and stops after a distance d.

(a) *A student says a heavier block of the same material would slide farther. Without using equations, explain whether the student is correct.* The student is not correct. A heavier block does start along the floor with more kinetic energy, because its gravitational potential energy at the top was larger. But the floor also pushes up on it harder, so the friction force on it is larger in the same proportion. Twice the mass means twice the energy to remove and twice the force removing it, so the block stops in the same distance.

(b) *Derive an expression for d in terms of H, μ and physical constants.* Take the block and the Earth as the system. All the gravitational potential energy ends up as thermal energy from friction, which equals the friction force times the distance:

$$mgH = \mu mg\, d \quad\Longrightarrow\quad d = \frac{H}{\mu}.$$

(c) *Explain how your expression supports your answer to (a).* The mass m cancels, so d does not depend on the mass, which is what (a) argued. The expression also shows what does change d: a higher ramp gives a longer slide, and a rougher floor (larger μ) a shorter one.

In part (c), point to a feature of the equation (here, that m cancels) and connect it to the words in part (a). Repeating the derivation does not earn the mark.

Choosing the system for energy

The most common error in this unit is counting gravity **twice**. Decide first whether the Earth is inside the **system** 系统.

- **Earth inside the system:** gravity is an **internal force** 内力, so you use gravitational potential energy $U_g = mgh$. Gravity does no "outside" work.
- **Earth outside the system:** there is no U_g; instead gravity does work on the object, $W_g = -mg\,\Delta h$.

Both give the same answer. Using U_g **and** W_g together counts the same energy twice. For a 10 kg box lifted 5.0 m at constant speed, the person does 490 J of work, not 980 J.

Choose the system to make energy constant. If every force that does work is internal (gravity with the Earth inside, a spring with the spring inside) and there is no friction, the system's total energy is constant. When a question says "use conservation of energy", pick that system and write $E_{\text{start}} = E_{\text{end}}$. When an outside force does work, the change in the system's energy equals that work: $\Delta E = W_{\text{ext}}$.

Work from a force–displacement graph

When the force changes, $W = Fd$ no longer works directly. Instead, work is the **area** under a force–displacement graph. A spring's force grows steadily, $F = kx$, so the graph is a triangle with area $\frac{1}{2}(kx)(x) = \frac{1}{2}kx^2$. That is where the elastic potential energy formula comes from.

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Worked example (spring launcher). A spring with $k = 200$ N/m is compressed by 0.10 m and launches a 0.050 kg ball straight up. Ignore friction and the small height change while the spring extends.

- Energy stored: $U_s = \frac{1}{2}(200)(0.10)^2 = 1.0$ J.
- Launch speed: $\frac{1}{2}(0.050)v^2 = 1.0$, so $v = \sqrt{40} = 6.3$ m/s.
- Highest point: $(0.050)(9.8)h = 1.0$, so $h = 2.0$ m.

Measuring a spring constant from launch height (a lab). A student compresses a spring launcher by different amounts x, fires a ball of mass m straight up, and measures the height h it reaches. Energy gives $\frac{1}{2}kx^2 = mgh$, so

$$h = \frac{k}{2mg}\,x^2.$$

Plot h against x^2: the points should lie on a straight line through the origin, with **slope** $\dfrac{k}{2mg}$. Then $k = 2mg \times$ slope. For $m = 0.050$ kg and a slope of 200 m^{-1}, $k = 2(0.050)(9.8)(200) = 196$ N/m. Plotting h against x would give a curve, from which the slope tells you nothing directly.

Worked example (friction on an incline). A 2.0 kg block starts from rest at the top of a ramp 3.0 m long and 1.5 m high. A friction force of 4.0 N acts along the ramp. Find its speed at the bottom.

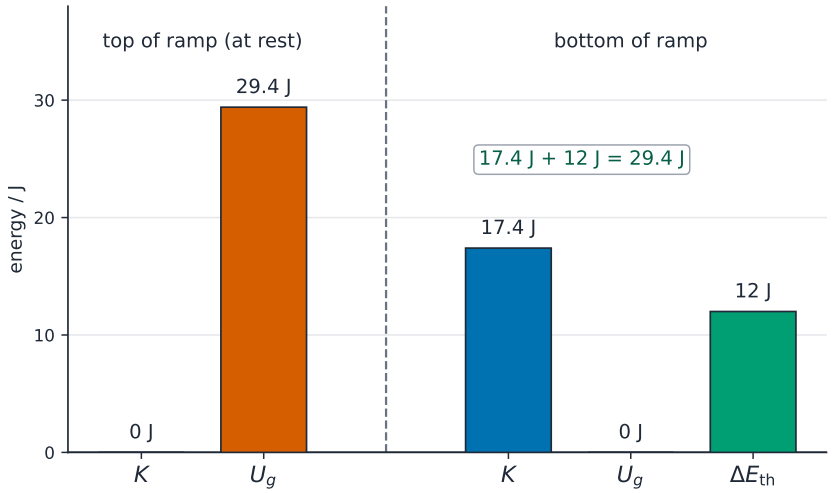
$$mgh = \tfrac{1}{2}mv^2 + f\,d: \qquad (2.0)(9.8)(1.5) = \tfrac{1}{2}(2.0)v^2 + (4.0)(3.0)$$

$$29.4 = v^2 + 12 \;\Rightarrow\; v = \sqrt{17.4} = 4.2 \text{ m/s}.$$

Friction turned 12 J into thermal energy. Without it the block would reach $\sqrt{29.4} = 5.4$ m/s.

Drawing it as an energy bar chart. Free-response questions often ask for this as bars. Choose the system first: here the block, the Earth and the ramp, so friction's work stays inside as thermal energy ΔE_{th}. Draw one bar for each energy store at the start and at the end. The totals must match, because no outside force does work on this system.

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An energy bar chart: the bars before and after have the same total, and the thermal bar shows where friction put the missing kinetic energy

If the system were the block alone, there would be no U_g bar: gravity would be an outside force doing +29.4 J of work, and friction an outside force doing −12 J. Either choice gives the same speed; what changes is which bars you draw.

Worked example (energy and forces together: a loop). A small cart starts from rest at height h on a frictionless track and goes round a vertical loop of radius $R = 0.40$ m. What is the smallest h for which it stays on the track at the top of the loop?

- *Forces at the top* (from the circular-motion unit): the cart just keeps contact when the **normal force** 法向力 is zero, so $mg = \dfrac{mv^2}{R}$, giving $v_{\text{top}}^2 = gR$.
- *Energy from the start to the top* (height $2R$): $mgh = mg(2R) + \frac{1}{2}mv_{\text{top}}^2 = 2mgR + \frac{1}{2}mgR$.

$$h = 2.5R = 2.5 \times 0.40 = 1.0 \text{ m}$$

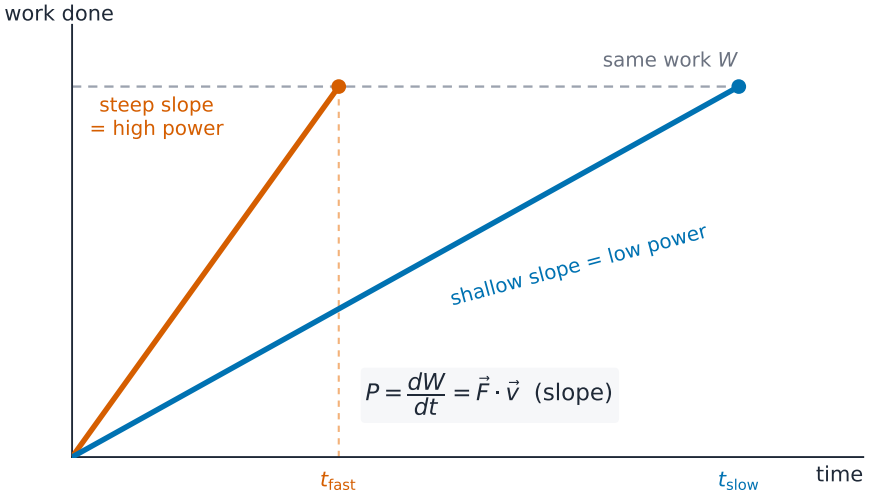
Neither idea alone is enough: energy gives the speed at the top, and Newton's second law says how fast it must be. Starting at only $h = 2R$ fails, because the cart would need zero speed at the top, and then it falls off before it gets there.

Power

Power 功率 is the **rate** of doing work or transferring energy, measured in **watts** 瓦特 (W):

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}, \qquad \text{and instantaneously} \qquad P = \vec{F}\cdot\vec{v}.$$

So the same job done faster requires more power. On an energy–time graph, power is the **slope** 斜率.



Power is the slope of the work-time graph: the same work in less time means more power

Worked example. A motor lifts a 50 kg load at a steady 2.0 m/s. Because it moves at constant speed, the lifting force equals the weight, so

$$P = Fv = mgv = 50 \times 9.8 \times 2.0 = 980 \text{ W}.$$

Writing power in symbols. Many questions give only letters. A crane lifts a load of mass M through a height H at constant speed in a time Δt. The energy transferred is the gain in gravitational potential energy, MgH, so the average power is

$$P = \frac{MgH}{\Delta t}.$$

Check that it makes sense: a heavier load, a greater height or a shorter time each need more power.

Real machines waste some energy, so we quote **efficiency** 效率 – useful output power divided by total input power. If this motor draws 1400 W of electrical power to deliver 980 W of useful lifting, its efficiency is $980/1400 = 0.70$, or 70%; the other 30% becomes heat and sound.

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A waterfall: power is how quickly energy is transferred – the same drop in less time is more power

Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2.

1. A 0.20 kg ball is thrown straight up at 12 m/s. Ignoring **air resistance** 空气阻力, how high does it rise?
2. A spring with $k = 400$ N/m is compressed 0.050 m and launches a 0.10 kg block along a frictionless floor. Find the block's speed.
3. A 10 kg box is pushed 5.0 m across a floor at constant speed against a 30 N friction force. Find the work done by the person, the net work, and the thermal energy produced.
4. A 60 kg student climbs stairs 4.0 m high in 8.0 s. Find the student's average power output for the lifting.
5. A student says: "Lifting the box 5.0 m needs 980 J: gravity does −490 J of work, and the potential energy also rises by 490 J." Correct the student.

Answers

1. $\frac{1}{2}mv^2 = mgh$, so $h = \dfrac{v^2}{2g} = \dfrac{144}{19.6} = 7.3$ m. The mass cancels.
2. $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = x\sqrt{k/m} = 0.050\sqrt{4000} = 3.2$ m/s.

3. Person: $(30)(5.0) = 150$ J. Net work is zero, because the speed is constant. Thermal energy produced: 150 J.
4. $P = \dfrac{mgh}{t} = \dfrac{(60)(9.8)(4.0)}{8.0} = 290$ W.
5. The student has counted gravity twice. With the Earth in the system, use only $\Delta U_g = 490$ J. With the Earth outside, use only gravity's work, -490 J. Either way the person must supply 490 J.

Exam tips

- Use $W = Fd\cos\theta$: work is zero when the force is perpendicular to the motion, and negative when it opposes it.
- Reach for the **work–energy theorem** ($W_{\text{net}} = \Delta K$) or **energy conservation** ($K_1 + U_1 = K_2 + U_2$) instead of forces whenever the path is complicated.
- When friction acts, mechanical energy is **not** conserved – subtract the energy lost to heat.
- Remember $K \propto v^2$: doubling the speed **quadruples** the kinetic energy (and, for a constant braking force, the stopping distance).
- Use $P = Fv$ for a force parallel to velocity: a motor can supply power at constant velocity even though the net force and net power are zero.
- In a qualitative–quantitative question, argue in words first, derive the equation, then name the feature of the equation (a cancelled mass, a square, a quantity on the bottom) that matches your words.

Common mistakes

- Counting gravity twice: using U_g *and* the work done by gravity in the same equation. Choose one, depending on whether the Earth is in the system.
- Using $W = Fd$ when the force is at an angle to the motion. Only the component along the displacement does work: $W = Fd\cos\theta$.
- Saying the normal force or a **centripetal force** 向心力 does work on an object moving in a circle. A force perpendicular to the velocity does no work.
- Writing $\frac{1}{2}kx$ or kx^2 for spring energy. The stored energy is $\frac{1}{2}kx^2$ (the triangle under $F = kx$).
- Treating energy as a vector. Kinetic and potential energy have no direction, so a ball's speed at a given height is the same whether it moves up or down (no friction).
- Leaving out thermal energy when friction acts. The lost mechanical energy equals $f\,d$ and appears as thermal energy; it does not disappear.
- Saying that doubling the spring compression doubles the sliding distance. The energy is $\frac{1}{2}kx^2$, so it **quadruples**, and so does the distance.
- Giving a single particle gravitational potential energy without including the Earth. That potential energy belongs to the interaction between the particle and the Earth.

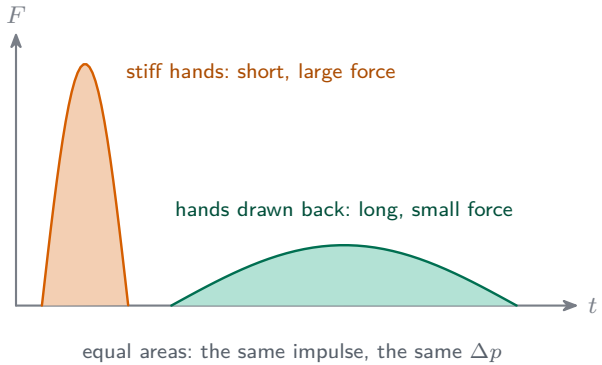
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- Plotting h against x for a spring launcher and calling the curve "the slope". Plot h against x^2 to get a straight line.
- Thinking a longer path changes the work done by gravity. Only the start and end heights matter for a conservative force.

Linear Momentum

AP Physics 1

Catch a raw egg with stiff hands and it breaks; draw your hands back as it lands and it survives. The egg stops either way, so its change in momentum is the same. What changes is how long the stopping takes. This unit is about that trade, and about the one quantity a collision can never create or destroy.



Same impulse, longer time, smaller force.

Linear momentum 动量 is mass times velocity – a vector pointing the same way as the velocity:

$$\vec{p} = m\vec{v}.$$

It measures "how hard it is to stop" a moving object. A heavy slow truck and a light fast ball can have the same momentum. Its unit is kg m/s, the same as N s.

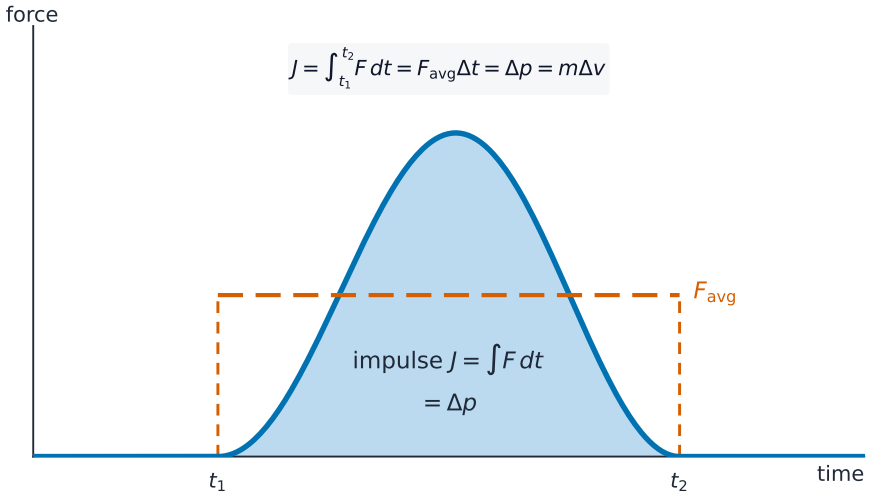
Two models: collisions and explosions. A **collision** is a model for a short interaction in which the forces between the objects are much **larger** than any outside force during it. Because we look only at the moment just before and just after, each object can be treated as a single point (the object model), whatever its shape. An **explosion** is the reverse: forces **inside** the system push its parts apart, as when a spring between two carts is released or a firework bursts.

Change in Momentum and Impulse

A net force acting over time changes momentum. Using a constant force, or its average over the interval, the **impulse** 冲量 delivered is

$$\vec{J} = \vec{F}\,\Delta t = \Delta\vec{p}.$$

This is the **impulse–momentum theorem** 冲量 – 动量定理: impulse equals the change in momentum. On a **force–time graph** 力 – 时间图, impulse is the **area** 面积 under the curve. It explains why **airbags** 安全气囊 and follow-through help – spreading the same momentum change over a longer time reduces the force.



Impulse is the area under the force-time curve, equal to the average force times the contact time

Worked example. A 0.15 kg ball hits a wall at 20 m/s and bounces straight back at 15 m/s. The contact lasts 0.020 s. Find the average force on the ball. Take the rebound direction as positive, so $u = -20$ m/s and $v = +15$ m/s:

$$\Delta p = m(v-u) = 0.15\big(15-(-20)\big) = 5.25 \text{ kg m/s}, \qquad F = \frac{\Delta p}{\Delta t} = \frac{5.25}{0.020} = 260 \text{ N}.$$

The sign work matters: forgetting that the velocity **reverses** is the most common mistake here.

Worked example (reading a force–time graph). A 0.050 kg golf ball at rest is struck by a club. The force rises steadily from zero to 400 N and falls back to zero, a triangle lasting 0.010 s. The impulse is the triangle's area:

$$J = \tfrac{1}{2}(400)(0.010) = 2.0 \text{ N s}, \qquad v = \frac{J}{m} = \frac{2.0}{0.050} = 40 \text{ m/s}.$$

The **average force** 平均力 is $J/\Delta t = 200$ N, half the **peak** 峰值, as it must be for a triangle.

Force is the slope of a momentum–time graph. Rearranged, the impulse–momentum theorem says

$$\vec{F}_{\text{net}} = \frac{\Delta\vec{p}}{\Delta t}.$$

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So on a graph of momentum against time, the **slope** is the net force: a straight line means a constant force, and a flat line means zero net force. For an object whose mass does not change, $\Delta\vec{p} = m\Delta\vec{v}$, and this becomes $\vec{F}_{\text{net}} = m\Delta\vec{v}/\Delta t = m\vec{a}$: Newton's second law is a special case of the impulse–momentum theorem.

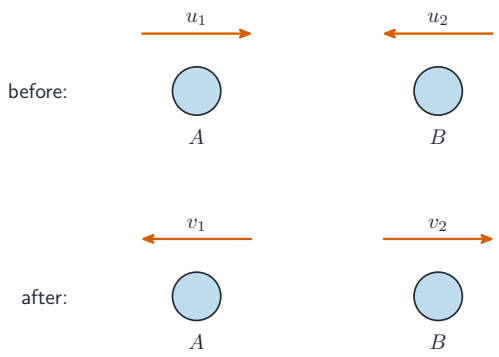
Checking one graph against another (a lab). A cart of mass 0.50 kg is pushed by a force sensor. The force–time graph is a rectangle of 2.0 N lasting from $t = 0.5$ s to $t = 2.5$ s, so the area gives $\Delta p \approx (2.0)(2.0) = 4.0$ kg m/s. The cart's velocity–time graph should then show a change of $\Delta v = \Delta p/m = 8.0$ m/s over the same interval. If the measured Δv is smaller, some other force (often friction) must also be acting, since the sensor reads only the push.

Conservation of Linear Momentum

If the **net external force** 合外力 on a system is zero, its total momentum is **conserved** 守恒:

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}.$$

Internal forces (like the push between two colliding carts) come in **third-law pairs** 第三定律力对 and cancel, so they cannot change the total momentum. This is the key tool for **collisions** 碰撞 and **explosions** 爆炸 – apply it separately in the x and y directions.



A head-on collision: total momentum before equals total momentum after

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Worked example (recoil 反冲). A 60 kg skater, initially at rest on frictionless ice, throws a 2.0 kg ball at 8.0 m/s relative to the ice. Find her recoil speed. The total momentum starts at zero and stays zero:

$$0 = (60)v + (2.0)(8.0) \;\Rightarrow\; v = -\frac{16}{60} = -0.27 \text{ m/s},$$

so she moves at 0.27 m/s in the opposite direction to the ball – the principle behind **rockets** 火箭 and guns.

Equal and opposite impulses. During any interaction, the force of A on B and the force of B on A are a third-law pair: equal in size, opposite in direction, and acting for the **same time**. So the impulses are equal and opposite, and

$$|\Delta\vec{p}_A| = |\Delta\vec{p}_B|.$$

This holds even when the masses are very different: a truck and a fly gain momentum changes of the same size in a collision. Their **velocity** changes differ, because the fly's mass is so much smaller.

Worked example (push-off, in symbols). A student of mass M_S on frictionless ice pushes a disk of mass M_D with a constant force F_H for a time t_f. Both start at rest.

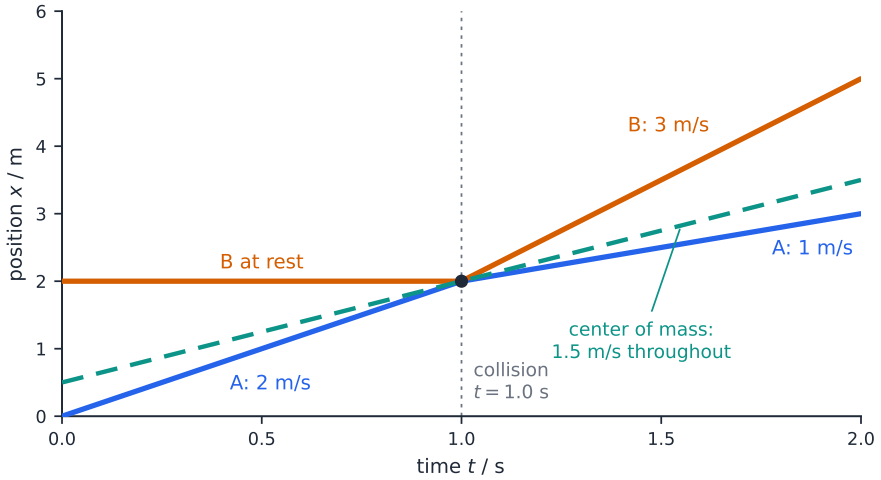
- The disk: $F_H t_f = M_D v_D$, so $v_D = \dfrac{F_H t_f}{M_D}$.
- Writing v_S and v_D as speeds, the student gets an equal and opposite impulse: $M_S v_S = M_D v_D$, so $\dfrac{v_D}{v_S} = \dfrac{M_S}{M_D}$.

The lighter object moves faster. On a speed–time graph, both speeds rise in straight lines until t_f (constant force), the disk's more steeply, and then both stay constant.

A whole collection of objects can be described by a single **center-of-mass velocity** 质心速度:

$$\vec{v}_{\text{cm}} = \frac{\sum m_i\vec{v}_i}{\sum m_i} = \frac{\vec{p}_{\text{total}}}{M_{\text{total}}}.$$

Since total momentum is conserved when no net external force acts, $\vec{v}_{\text{cm}}$ then stays **constant** – a v_{cm}-versus-time graph is flat unless an outside force acts. An internal collision or explosion never changes it, however violently the parts fly apart.



The center of mass keeps the same velocity through the collision, because the collision forces are internal

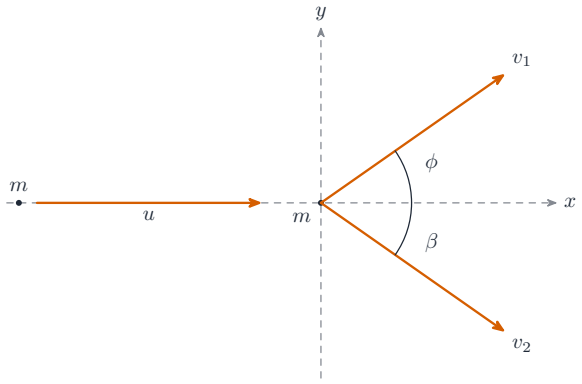
Reading the graph. The center-of-mass velocity is $v_{\text{cm}} = \dfrac{(6)(2)+(2)(0)}{8} = 1.5$ m/s, before **and** after: one straight line with no bend at $t = 1.0$ s. If the blocks had stuck together instead, both would follow that same dashed line after the collision. The center-of-mass line only bends when an **external** force acts: a plunger pushing the first block, or a rough patch of floor slowing it down.

Limiting cases. A disk of speed v_1 hits a block at rest and they stick: $v_{\text{cm}} = \dfrac{M_D v_1}{M_D + M_B}$. If $M_D \gg M_B$, the block barely matters and $v_{\text{cm}} \to v_1$. If $M_D \ll M_B$, the disk barely moves the block and $v_{\text{cm}} \to 0$. Checking that your formula gives these limits is a quick way to test it, and exams ask for it.

Elastic and Inelastic Collisions

Momentum is conserved in **every** collision (with no external force). **Kinetic energy** 动能 is not:

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A glancing collision, resolved along two perpendicular axes

- In an **elastic collision** 弹性碰撞, kinetic energy is **also** conserved (objects bounce apart cleanly).
- In an **inelastic collision** 非弹性碰撞, some kinetic energy becomes heat or **deformation** 形变. In a **perfectly inelastic** 完全非弹性 collision the objects **stick together** 粘在一起 and move with one common velocity afterward.

Strategy: always write momentum conservation; add energy conservation only if the collision is stated to be elastic. One elastic fact worth memorising: in a 1D elastic collision between **equal masses**, the two objects simply *swap velocities* (a moving ball striking an identical **stationary** 静止的 one stops dead, and the target flies off at the incoming speed).

> **Worked example.** A 1000 kg car moving at 20 m/s runs into a stationary 1500 kg car and they lock together. Find their common speed, and the kinetic energy lost. Momentum conservation gives
>
> $$1000 \times 20 = (1000 + 1500)\, v \;\Rightarrow\; v = \frac{20000}{2500} = 8.0 \text{ m/s}.$$
>
> Kinetic energy before is $\frac{1}{2}(1000)(20^2) = 2.0 \times 10^5$ J; after is $\frac{1}{2}(2500)(8.0^2) = 8.0 \times 10^4$ J. So 1.2×10^5 J (about 60%) is lost to crumpling and heat – momentum is still conserved, but kinetic energy is not.

> **Worked example (is it elastic?).** A 0.50 kg cart at 2.0 m/s hits a 1.5 kg cart at rest. Afterward the first cart moves back at 1.0 m/s and the second moves forward at 1.0 m/s. Momentum: $0.50(2.0) = 1.0$ before and $0.50(-1.0) + 1.5(1.0) = 1.0$ kg m/s after. Kinetic energy: $\frac{1}{2}(0.50)(2.0^2) = 1.0$ J before and $\frac{1}{2}(0.50)(1.0^2) + \frac{1}{2}(1.5)(1.0^2) = 0.25 + 0.75 = 1.0$ J after. Both are conserved, so this collision is elastic. Test kinetic energy only after momentum checks out.

Notice that in an elastic collision the **system's** kinetic energy is unchanged, but each object's kinetic energy can change a lot: here the first cart lost 0.75 J and the second gained exactly that.

> **Worked example (momentum arrows).** Disk R (m_0) moves at v_0 toward disk S ($3m_0$) at rest. After the collision R moves **back** at $\frac{1}{2}v_0$. Find S's velocity and kinetic energy.

$$m_0 v_0 = m_0\left(-\tfrac{1}{2}v_0\right) + 3m_0 v_S \;\Rightarrow\; v_S = \tfrac{1}{2}v_0, \qquad K_S = \tfrac{1}{2}(3m_0)\left(\tfrac{1}{2}v_0\right)^2 = \tfrac{3}{8}m_0 v_0^2.$$

On a momentum-vector diagram, if R's arrow before has length 2 units, then after R's arrow points **backward** with length 1 unit, and S's points forward with length 3 units. The arrows after must add to the arrow before: $-1 + 3 = 2$. The two changes in momentum have the same size, 3 units each, as the third law requires. Total kinetic energy after is $\frac{1}{8}m_0v_0^2 + \frac{3}{8}m_0v_0^2 = \frac{1}{2}m_0v_0^2$, so this collision happens to be elastic.

Bouncing gives a bigger impulse than sticking. Throw a ball of clay and a rubber ball of the same mass and speed at identical blocks. The clay sticks to its block and moves forward with it; the rubber ball bounces **back**. The rubber ball's change in momentum is larger (it goes from $+mv$ to a negative value), so by the third law it gives the block a **larger** impulse. The block hit by the rubber ball moves faster, and if both blocks then slide off a table, it lands farther away. The sphere–block **system** has the same total momentum in both cases, because the external impulse is zero: the extra forward momentum of the block is balanced by the sphere's backward momentum.



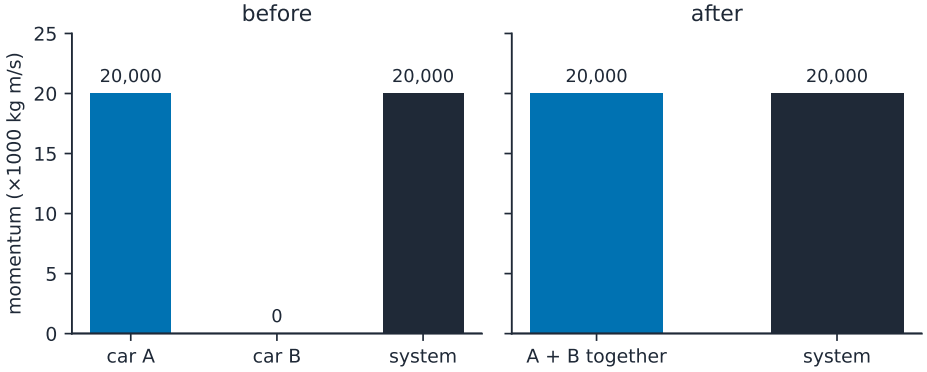
A Newton's cradle shows momentum and kinetic energy passing through a line of balls in a near-elastic collision

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Choosing the system

Every momentum answer starts with one decision: which objects are in the **system** 系统. Momentum is conserved only for a system on which the **net external impulse** 外部冲量 is zero, or small enough to ignore.

- **Internal forces** 内力 act between objects inside the system. They come in Newton's-third-law pairs, give equal and opposite impulses, and cannot change the total.
- **External forces** 外力 come from outside it. A ball falling alone is not a conserved system, because gravity is external. Ball **and** Earth together is: the Earth gains exactly the momentum the ball does, in the opposite direction.
- A short collision usually lets you ignore **friction** 摩擦力 and **gravity** 重力. The collision forces are huge, but they act for so little time that the outside forces add almost no impulse. So "conserved during the collision" is safe; "conserved for the whole trip" usually is not.



the system bars are equal: total momentum is conserved

Momentum bar charts: the system total is the same before and after the collision

Momentum bar charts 动量柱状图 are the AP way to show this: one bar per object before and after, plus the system total. The figure shows the car collision above. The system bars match, so the claim "momentum is conserved" is visible, not just asserted.

> **Worked example (two stages, two laws).** A 0.010 kg bullet at 300 m/s embeds in a 2.0 kg block hanging on a string, a **ballistic pendulum** 冲击摆. How high does the block swing?

- *Stage 1, the collision.* It is over in a fraction of a second, so the string and gravity add **negligible** 可忽略的 impulse: momentum is conserved. $0.010(300) = (2.010)\,v$, so $v = 1.49$ m/s. Kinetic energy is **not** conserved here: it falls from 450 J to about 2.2 J.

- *Stage 2, the swing.* Now gravity acts for a long time, so momentum is no longer conserved, but no energy is lost: $\frac{1}{2}v^2 = gh$, so $h = \dfrac{1.49^2}{2(9.8)} = 0.11$ m.

Using energy in stage 1, or momentum in stage 2, gives a wrong answer. Name the stage before you choose the law.

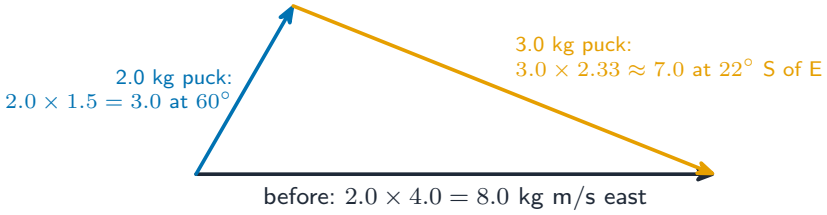
> **Worked example (dropping a block onto a moving cart).** A cart of mass m_c rolls at v_c on a frictionless track. A block of mass $\frac{1}{5}m_c$ is dropped from rest onto it and sticks. Choose the cart and block as the system.

- *Horizontal momentum is conserved*: there is no horizontal external force, and the falling block had no horizontal momentum. So a graph of p_x against time is a flat line, even during the collision.
- $m_c v_c = \frac{6}{5}m_c v_f$, so $v_f = \frac{5}{6}v_c$.
- $\Delta K_x = \frac{1}{2}\left(\frac{6}{5}m_c\right)\left(\frac{5}{6}v_c\right)^2 - \frac{1}{2}m_c v_c^2 = -\frac{1}{12}m_c v_c^2$: this is the loss of kinetic energy from horizontal motion only. The block also loses its vertical kinetic energy when it lands; the total kinetic-energy loss includes both.

Vertical momentum is **not** conserved: the track pushes up on the cart to stop the block's fall. If the block slides on the cart for a while before moving with it, the friction between them is internal, so p_x of the system still stays constant.

A collision in two dimensions

Momentum is a **vector** 矢量, so it is conserved separately along each **axis** 轴.



Momentum is conserved as a vector.

> **Worked example.** A 2.0 kg puck moving east at 4.0 m/s strikes a 3.0 kg puck at rest. Afterward the 2.0 kg puck moves at 1.5 m/s at 60° north of east. Find the velocity of the 3.0 kg puck.

- x (east): $2.0(4.0) = 2.0(1.5\cos 60°) + 3.0\,v_x$, so $8.0 = 1.5 + 3.0\,v_x$ and $v_x = 2.17$ m/s.
- y (north): $0 = 2.0(1.5\sin 60°) + 3.0\,v_y$, so $v_y = -0.87$ m/s, south.
- Speed $\sqrt{2.17^2 + 0.87^2} = 2.33$ m/s, at $\tan^{-1}(0.87/2.17) = 22°$ south of east.

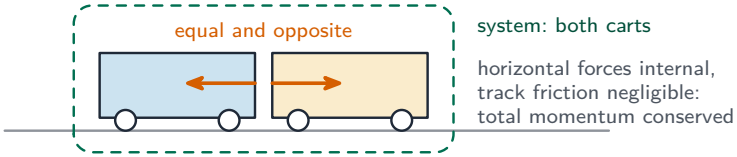
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Check the energy: 16 J before and $2.25 + 8.2 = 10.4$ J after. Kinetic energy fell, so this collision is inelastic. A rise would require stored internal energy to be released; without such a source, check the arithmetic.

On the AP Physics 1 exam you will not have to solve two-dimensional simultaneous equations. You may be asked to **set up** the x and y equations correctly, or to reason about what happens if a mass, speed or angle changes. For example, if the first puck had bounced off at a **larger** angle with the same speed, its x momentum after would be smaller, so the second puck must carry **more** x momentum.

Justify it in words

Free-response questions often ask you to **justify** 论证 a conservation claim without numbers. A full-credit answer names the system, the forces and the law. For two **carts** 小车 colliding on a level, low-friction **track** 轨道:



Name the system, the forces and the law.

> *Yes, the total momentum of the two-cart system is conserved. During the collision, the only horizontal forces on each cart are the forces the carts exert on each other. These are internal to the system and, by Newton's third law, equal and opposite, so their impulses cancel. Friction from the track is negligible, so the net external impulse on the system is zero.*

"Momentum is always conserved" earns nothing: it is false for the wrong system, and it names no reason.

Designing a test of "perfectly elastic". A toy ball is said to bounce perfectly elastically. To test this at different speeds:

- Drop the ball from a measured height h_1 onto a hard floor and measure the height h_2 it bounces back to, filming against a metre rule. Repeat for a range of h_1, several trials each.
- The speed just before impact is $\sqrt{2gh_1}$. For a perfectly elastic bounce, kinetic energy is unchanged, so $h_2 = h_1$.
- Plot h_2 against h_1 and add the line $h_2 = h_1$. Points on the line are elastic; points falling further below it at large h_1 support the claim that higher-speed collisions lose more energy.

- A point **above** the line would mean the ball left the floor with more kinetic energy than it arrived with, which breaks energy conservation for a simple ball. It shows a measurement error, such as a ball that was thrown rather than dropped.

Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2.

1. A 0.40 kg ball moving at 5.0 m/s is caught and stopped in 0.10 s. Find the average force on it.
2. The same ball is stopped in 0.020 s instead. Find the force, and explain why a padded glove is used.
3. A 3.0 kg cart at 4.0 m/s hits a 1.0 kg cart at rest and they stick. Find their common speed and the kinetic energy lost.
4. A 50 kg astronaut floating at rest pushes off a 150 kg satellite, also at rest, and moves away at 0.60 m/s. Find the satellite's velocity. What happens to the center of mass of the pair?
5. A student says: "In question 3 kinetic energy was lost, so some momentum must have been lost too." Correct the student.

Answers

1. Taking the initial motion as positive, $\Delta p = 0.40(0 - 5.0) = -2.0$ kg m/s, so $F = -2.0/0.10 = -20$ N: 20 N opposite to the initial motion.
2. The force magnitude is $|F| = 2.0/0.020 = 100$ N, opposite to the initial motion. The momentum change is fixed; padding makes the stopping last longer, which lowers the force on the hand.
3. $3.0(4.0) = (4.0)v$, so $v = 3.0$ m/s. Kinetic energy falls from $\frac{1}{2}(3.0)(4.0^2) = 24$ J to $\frac{1}{2}(4.0)(3.0^2) = 18$ J: 6.0 J is lost.
4. $0 = 50(0.60) + 150\,v$, so $v = -0.20$ m/s: 0.20 m/s in the opposite direction. The total momentum stays zero, so the center of mass stays at rest.
5. Momentum and kinetic energy obey separate laws. With no net external impulse, momentum is conserved in every collision. Kinetic energy can turn into internal energy (heat, sound, deformation), so it may fall while momentum stays exactly the same.

Exam tips

- Momentum is a **vector** – assign $+/-$ signs before adding; a ball that **rebounds** reverses its velocity, giving a large Δp.
- Momentum is conserved in **every** collision (no external force); kinetic energy is conserved **only** if the collision is stated to be **elastic**.

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- In a **perfectly inelastic** collision the objects stick and move with one common velocity.
- Impulse $= F\,\Delta t = \Delta p =$ the **area** under a force–time graph; spreading a collision over a longer time reduces the force (airbags, bending knees).
- For recoil/explosions, set the total momentum equal before and after (often zero before).

■ Common mistakes

- Subtracting speeds for a rebound. A ball that hits a wall at 20 m/s and returns at 15 m/s changes velocity by 35 m/s, not 5 m/s.
- Claiming momentum is conserved for a single object acted on by gravity or friction. Conservation needs a system with zero net external impulse.
- Using kinetic energy conservation for a collision that is not stated (or shown) to be elastic. Write momentum first; test kinetic energy only if the question asks.
- Saying a larger force means a larger impulse. Impulse is force times time: a soft catch has a smaller force but the same impulse.
- Adding the magnitudes of momenta in a 2D collision. Conserve the x and y components separately, then combine.
- Using momentum conservation for the swing of a ballistic pendulum, or energy conservation for the impact. Each stage has its own law.
- Saying the heavier object gets the larger change in momentum in a collision. The impulses are equal and opposite, so $|\Delta p|$ is the same for both; only the velocity changes differ.
- Drawing a bend in the center-of-mass line at a collision. Internal forces never change v_{cm}; only external forces do.
- Thinking a ball that sticks gives a target a bigger push than one that bounces. A bounce reverses the ball's momentum, so it delivers the larger impulse.
- Conserving vertical momentum when a block is dropped onto a cart. The track gives an upward impulse; only the horizontal momentum is conserved.

Torque and Rotational Dynamics

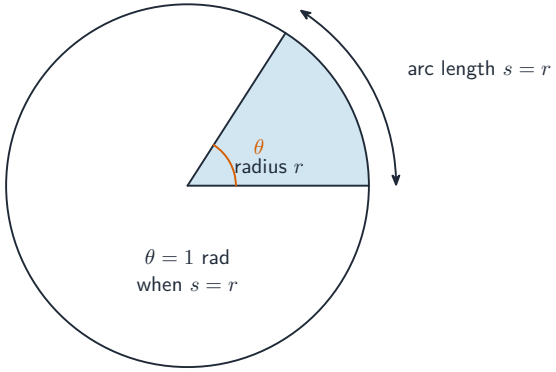
AP Physics 1

Rotational Kinematics

Why is a door handle on the side away from the hinges, and why does a tightrope walker carry a long pole? Both are about turning: how hard a force turns something, and how hard that something resists being turned. This unit gives each idea a number.

Until now we treated each object as a single point. A spinning wheel cannot be treated that way: its top moves forward while its bottom moves backward. We call it a **rigid system** 刚体系统, an object that keeps its shape while different points on it move in different directions. If the turning does not matter to the question (the Earth's daily spin when you study its orbit), you may still treat the system as a single object.

Rotation is described by angular quantities that mirror the linear ones:



One radian is the angle whose arc length equals the radius

- **angular displacement** 角位移 θ (in **radians** 弧度),
- **angular velocity** 角速度 $\omega = \dfrac{\Delta\theta}{\Delta t}$,
- **angular acceleration** 角加速度 $\alpha = \dfrac{\Delta\omega}{\Delta t}$.

For constant α, the **rotational kinematic equations** 转动运动学方程 have the same form as the linear ones, with θ, ω, α replacing x, v, a: $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, and $\omega^2 = \omega_0^2 + 2\alpha\theta$.

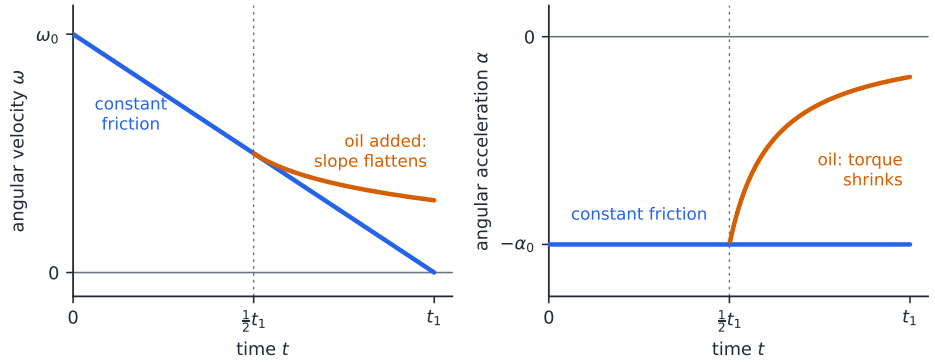
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Worked example. A wheel starts from rest and speeds up uniformly to 30 rad/s in 6.0 s. Find its angular acceleration and the total angle turned:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30}{6.0} = 5.0 \text{ rad/s}^2, \qquad \theta = \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2} \times 5.0 \times 6.0^2 = 90 \text{ rad}.$$

Signs. Choose one turning direction as positive, usually counterclockwise, and the other is negative. When ω and α have the **same** sign the rotation speeds up; when they have **opposite** signs it slows down.

Angular graphs work like linear ones. On a graph of ω against t, the slope is α and the area under the line is the angular displacement $\Delta\theta$. On a graph of θ against t, the slope is ω.



A constant friction torque gives a straight ω–t line; oil that keeps reducing the friction makes the line bend flatter

Worked example (a disk slowed by friction). A disk spins at ω_0 on an axle. Friction at the axle exerts a constant torque τ_0 that stops it at time t_1.

- The torque is constant, so α is constant and negative: the ω–t graph is a straight line down to zero, and the α–t graph is a flat line below the axis.
- $\tau_0 = I|\alpha| = I\dfrac{\omega_0}{t_1}$, so $I = \dfrac{\tau_0 t_1}{\omega_0}$.
- If oil reaches the axle from $\frac{1}{2}t_1$ on, and keeps reducing the friction, $|\alpha|$ falls toward zero. The ω line becomes flatter, and the disk is still turning at t_1.

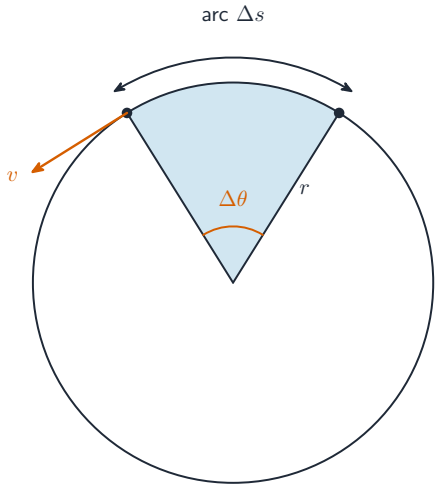
A model for the oiled torque must **decrease** with time. $\tau = \dfrac{C_2}{t + \frac{1}{2}t_1}$ does; $\tau = C_1\left(t - \frac{1}{2}t_1\right)$ grows with time, so it cannot describe friction that is getting weaker.

Connecting Linear and Rotational Motion

A point at radius r from the **axis of rotation** 转轴 has linear quantities tied to the angular ones:

$$s = r\theta, \qquad v = r\omega, \qquad a_t = r\alpha.$$

Points farther from the axis move faster. This link lets you switch between "how fast the wheel spins" and "how fast a point on its rim moves."



As the radius turns through an angle, a point moves along an arc at speed v

> **Worked example.** A bicycle wheel of radius 0.35 m spins at 12 rad/s. Relative to the axle, a point on the rim moves at $v = r\omega = 0.35 \times 12 = 4.2$ m/s, and a point halfway to the axis moves at half that speed. If the wheel rolls without slipping, the bike also moves at 4.2 m/s relative to the ground; the rim point's ground velocity combines this translation with its rotation.

Every point on a rigid system turns through the same angle in the same time, so **all points share the same ω and α**. Only the linear quantities v and a_t depend on r.

Torque

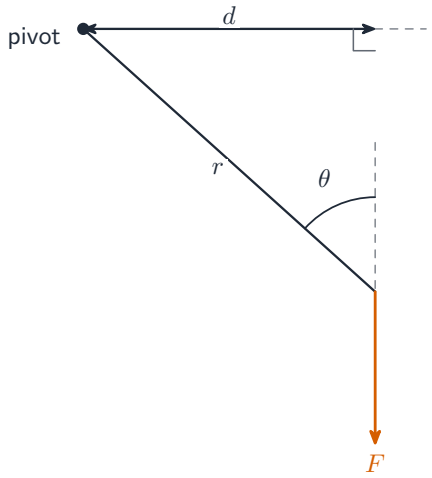
Torque 力矩 is the rotational effect of a force – how effectively it turns an object about an axis:

$$\tau = rF\sin\theta = F \cdot r_\perp,$$

where $r_\perp$ is the **moment arm** 力臂 (the perpendicular distance from the axis to the force's **line of action** 作用线). A force applied farther out, or more perpendicular,

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produces more torque. Torque has a sign (**clockwise** 顺时针 vs **counterclockwise** 逆时针).



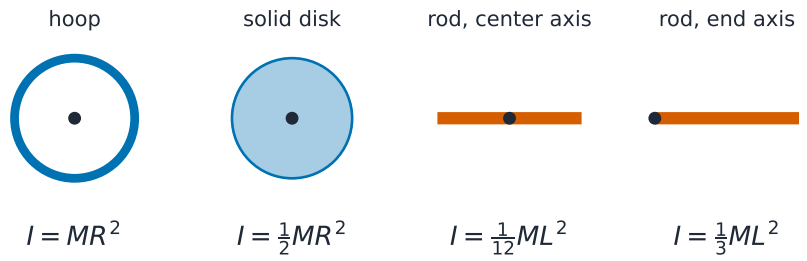
The moment of a force depends on the perpendicular distance from the pivot

> **Worked example.** You push with 20 N at the end of a 0.30 m wrench. Perpendicular to the wrench the torque is $\tau = rF = 0.30 \times 20 = 6.0$ N m. If you push at 60° to the wrench instead, only the perpendicular part counts: $\tau = rF\sin 60° = 0.30 \times 20 \times 0.87 = 5.2$ N m – which is why you push at a right angle for the most turning effect.

Lever arm, drawn. The **lever arm** (moment arm) $r_\perp$ is the perpendicular distance from the axis to the line drawn through the force. A force whose line passes **through** the axis has zero lever arm and no torque, however large it is. This is why, for a wheel rolling down a ramp, only **static friction** 静摩擦 changes its spin about its center: gravity acts at the center, and the normal force points straight at it, so neither has a torque about the center.

Rotational Inertia

Rotational inertia 转动惯量 (**moment of inertia** 惯性矩) I measures how hard it is to change an object's rotation – the rotational version of mass. It depends on both the mass **and how far** that mass sits from the axis: mass spread farther out gives a larger I. For a **point mass** 质点 $I = mr^2$; for extended bodies, standard formulas are provided (a hoop is mR^2, a solid disk $\frac{1}{2}mR^2$). This is why a figure skater spins faster when she pulls her arms in – she reduces I.



Equal masses, different distributions: the further the mass sits from the axis, the larger I

> **Worked example (hoop or disk?).** A **hoop** 圆环 and a **solid disk** 实心圆盘 each have mass 2.0 kg and radius 0.50 m. The same net torque of 1.0 N m acts on each. The hoop has $I = MR^2 = 0.50$ kg m^2, so $\alpha = 1.0/0.50 = 2.0$ rad/s^2. The disk has $I = \frac{1}{2}MR^2 = 0.25$ kg m^2, so $\alpha = 4.0$ rad/s^2. Same mass and same size, but the disk speeds up twice as fast, because its mass sits closer to the axis.

Adding up rotational inertia. For a collection of point masses about the same axis, $I_{\text{total}} = \sum m_i r_i^2$. For extended objects, add their rotational inertias about that axis, including each object's own size. Two 0.20 kg balls on a light rod, each 0.30 m from the axis, give $I = 2(0.20)(0.30)^2 = 0.036$ kg m^2. Slide them to 0.15 m and I falls to a **quarter**, because r is squared.

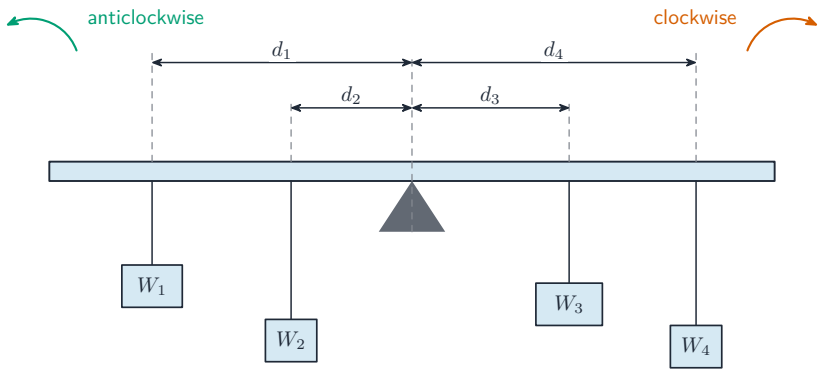
The parallel-axis theorem. For a given object, I is **smallest** about an axis through the center of mass. About any parallel axis a distance d away,

$$I = I_{\text{cm}} + Md^2.$$

Check it on a rod of length L: about its center $I_{\text{cm}} = \frac{1}{12}ML^2$, and the end is $d = \frac{1}{2}L$ away, so $I_{\text{end}} = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$, the value in the figure above.

Rotational Equilibrium

An object is in **rotational equilibrium** 转动平衡 when the **net torque** 合力矩 is zero, so its angular velocity stays constant. For a balanced (static) object, both the net force **and** the net torque are zero. Choosing the axis at an unknown force's location removes it from the torque equation – a useful trick for beam and ladder problems.



At balance the clockwise and anticlockwise moments about the pivot are equal

> **Worked example.** A 30 kg child sits 2.0 m from the **pivot** 支点 of a seesaw. Where must a 40 kg child sit on the other side to balance it? Set the clockwise torque equal to the anticlockwise torque (the g's cancel):
>
> $$30 \times 2.0 = 40 \times d \;\Rightarrow\; d = \frac{60}{40} = 1.5 \text{ m}.$$
>
> The heavier child sits closer to the pivot – less distance, same torque.

Rotational and translational equilibrium are **separate** conditions. A wheel spinning at a steady rate on a fixed axle is in rotational equilibrium but not at rest. A box sliding at constant velocity while two equal forces push on opposite corners is in translational equilibrium, but it starts to spin. To describe a rotating object fully, you may need **both** analyses, done independently.

> **Worked example (a beam held by a string).** A uniform beam of mass M and length L is hinged to a wall at its left end and held horizontal by a string from its right end to the wall above, at angle θ to the beam. Take torques about the hinge, so the unknown hinge force drops out:
>
> $$F_T \sin\theta\, L = Mg\,\frac{L}{2} \;\Rightarrow\; F_T = \frac{Mg}{2\sin\theta}.$$
>
> If the string is fixed **lower** on the wall, θ is smaller and the tension is **larger**. In words: a flatter string pulls mostly sideways, so it must pull harder to give the same upward torque. When the string is cut, gravity's torque about the hinge is largest at first and shrinks as the beam swings down (the lever arm shrinks), so the beam's angular speed rises with a slope that gets **flatter**. For the same reason, a rod falling from the top is speeding up fastest (largest α) when it passes horizontal.

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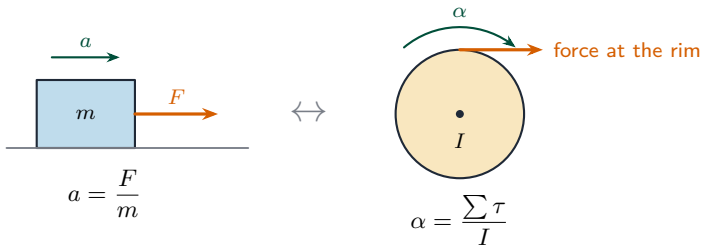
Newton's Second Law in Rotational Form

Net torque produces angular acceleration, in direct analogy with $F = ma$:

$$\sum \tau = I\alpha.$$

So a larger net torque, or a smaller rotational inertia, gives a larger angular acceleration. Solve rotation problems just like **translation** 平动 problems, with $\tau \leftrightarrow F$, $I \leftrightarrow m$, and $\alpha \leftrightarrow a$.

> **Worked example.** A net torque of 12 N m acts on a wheel with rotational inertia $I = 3.0$ kg m^2. Its angular acceleration is $\alpha = \tau/I = 12/3.0 = 4.0$ rad/s^2 – the exact rotational twin of $a = F/m$.



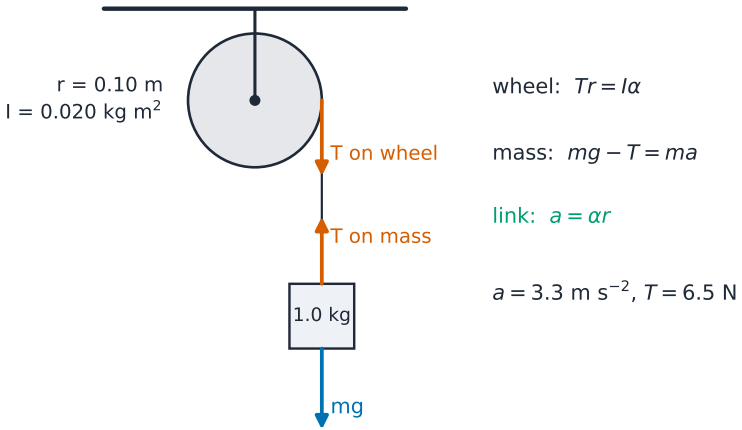
Torque, rotational inertia and angular acceleration play the parts of F, m and a

If the net torque is not zero, the angular velocity **must** change. The rotational form of Newton's first law says the reverse: ω is constant only when the net torque is zero.

Rotation and translation together

Many AP problems join a falling object to a turning one. Write Newton's second law for **each** object, then link them with $a = r\alpha$.

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The tension gives the wheel its torque and slows the mass: solve the two bodies together

> **Worked example (a block on a pulley).** A 2.0 kg block hangs from a light string wrapped around a pulley, a uniform disk of mass 4.0 kg and radius 0.10 m. The block is released from rest. Find its acceleration and the **tension** 张力 in the string.

- Pulley: $I = \frac{1}{2}(4.0)(0.10)^2 = 0.020$ kg m^2.
- Block (down positive): $mg - T = ma$.
- Pulley: $TR = I\alpha = I\frac{a}{R}$, so $T = \frac{I}{R^2}a = \frac{0.020}{0.010}a = 2.0a$.
- Add the two equations: $mg = (m + 2.0)a$, so $19.6 = 4.0a$ and $a = 4.9$ m/s^2. Then $T = 2.0(4.9) = 9.8$ N.

The block falls at half of g. The tension is less than the block's weight (19.6 N), as it must be for the block to accelerate down. A heavier pulley would mean a slower fall.

Compare the tensions. A block on a frictionless table is pulled by a string over a pulley to a hanging block. With a **massless** pulley, the tension is T_1. Replace it with a massive one: the system now accelerates more slowly, and for the hanging block $T = m_B(g - a)$. A smaller a means a **larger** tension in the vertical string, $T_2 > T_1$. The horizontal string carries less tension than the vertical one, because their difference is what turns the pulley.

> **Worked example (two pulleys on one axle).** A pulley of radius $2r_0$ and a pulley of radius r_0 are fixed together on a frictionless axle. Object 1 (m_0) hangs from the large one on one side; object 2 ($1.5m_0$) hangs from the small one on the other side. Released from rest, which way does it turn?

- Torque from object 1's side: $m_0 g(2r_0) = 2m_0 g r_0$. From object 2's side: $1.5m_0 g(r_0) = 1.5m_0 g r_0$.
- Net torque on the objects–pulleys system: $0.5m_0 g r_0$, in the direction that lowers object 1. Object 1 goes down even though it is **lighter**: its lever arm is twice as long.
- If object 1's string is cut while it is moving, only object 2's torque is left, and it acts **against** the rotation. Now ω and α point in opposite directions: on an ω–t graph, the line that was rising turns to fall in a straight line, passes through zero, and the pulleys then turn the other way.

A beam on two supports

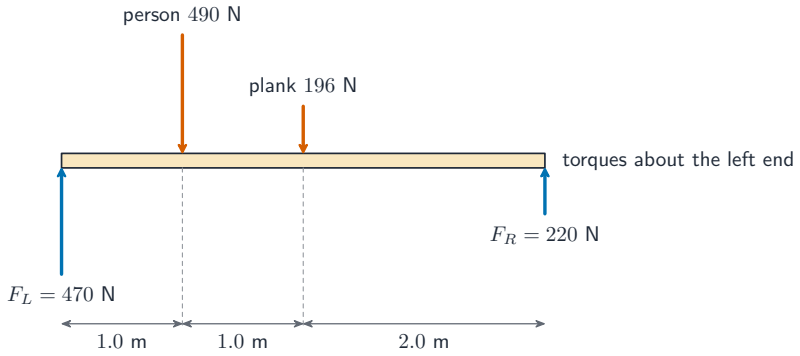
Static problems need **both** conditions: zero net force and zero net torque. Take torques about one support to remove its unknown force.

> **Worked example.** A **uniform** 均匀的 plank 4.0 m long with mass 20 kg rests on supports at its two ends. A 50 kg person stands 1.0 m from the left end. Find the force from each support.

- Torques about the left support: $F_R(4.0) = (20)(9.8)(2.0) + (50)(9.8)(1.0) = 392 + 490 = 882$, so $F_R = 220$ N.
- Forces: $F_L + F_R = (70)(9.8) = 686$ N, so $F_L = 470$ N.

The person stands nearer the left support, so that support carries more. The plank's weight acts at its center, 2.0 m from either end.

Start such a problem with a **force diagram** 力图. It is like a free-body diagram, but each force is drawn at the point where it acts on the rigid object, and the arrow lengths show the relative sizes. Every lever arm can then be read straight off the drawing.



A force diagram draws each force where it acts, so the lever arms can be read straight off it

Is the equation plausible? Exam questions sometimes give an equation and ask whether it can be right, without deriving it. Check two things.

- **Units.** Both sides must have the same units.
- **Behaviour.** Ask what should happen when one quantity grows, or goes to zero. A student writes, for a rod of rotational inertia I hit by a sliding disk, $\omega = \dfrac{Ixv_0}{m_{\text{disk}} d^4}$. Its units work out (both sides are s^{-1}), but it says a rod with **more** rotational inertia spins **faster** after the same hit. A heavier rod is harder to turn, so I should be on the bottom. The equation is not plausible.

Measuring a mass from a linear graph (a lab). A metre stick of mass M is hinged to a wall at one end and held horizontal by a spring scale fixed 0.60 m from the hinge, pulling at angle θ to the stick. Torques about the hinge:

$$F_T \sin\theta\,(0.60) = Mg(0.50) \;\Rightarrow\; F_T = \frac{5Mg}{6} \cdot \frac{1}{\sin\theta}.$$

Plot F_T on the vertical axis against $\dfrac{1}{\sin\theta}$: a straight line through the origin with slope $\dfrac{5Mg}{6}$. If the best-fit slope is 4.1 N, then $M = \dfrac{6(4.1)}{5(9.8)} = 0.50$ kg. Take several angles over a wide range, and repeat each reading to reduce uncertainty.

Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2.

1. A wheel speeds up uniformly from rest to 20 rad/s in 4.0 s. Find its angular acceleration and how many **revolutions** 转数 it makes.
2. A 15 N force pushes a door 0.40 m from the hinge. Find the torque when the force is perpendicular to the door, and when it is at 30° to the door.
3. A 25 kg child sits 1.6 m from a seesaw pivot. Where must a 40 kg child sit to balance?
4. A disk with $I = 0.50$ kg m^2 starts from rest. A net torque of 2.0 N m acts for 3.0 s. Find its final angular velocity.
5. A student says: "Two wheels with the same mass and radius will speed up equally under the same torque." Correct the student.

Answers

1. $\alpha = 20/4.0 = 5.0$ rad/s^2. $\theta = \frac{1}{2}(5.0)(4.0)^2 = 40$ rad $= 40/2\pi = 6.4$ revolutions.
2. Perpendicular: $(0.40)(15) = 6.0$ N m. At 30°: $(0.40)(15)\sin 30° = 3.0$ N m.
3. $(25)(1.6) = (40)d$, so $d = 1.0$ m.
4. $\alpha = 2.0/0.50 = 4.0$ rad/s^2, so $\omega = (4.0)(3.0) = 12$ rad/s.

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5. Rotational inertia depends on where the mass is, not only how much. A hoop has twice the I of a solid disk with the same mass and radius, so under the same torque it speeds up only half as fast.

Exam tips

- Torque $\tau = Fr_\perp$ uses the **perpendicular** distance from the pivot; a force through the pivot gives zero torque.
- For balance, set clockwise torque = anticlockwise torque; choosing the pivot at an unknown force removes it from the equation.
- Use the rotational **analogues** 类比量: $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$, so $\sum\tau = I\alpha$ mirrors $\sum F = ma$.
- **Rotational inertia** depends on **how far** the mass sits from the axis, not just its amount – a hoop resists spinning more than a disc of equal mass.
- Convert angles to **radians** and link linear to angular with $v = r\omega$, $a_t = r\alpha$.

Common mistakes

- Using the distance along the force's line of action, or the length of the rod, when the force is at an angle. Torque uses the perpendicular distance $r_\perp$ (or $rF\sin\theta$).
- Leaving angles in degrees or revolutions in $v = r\omega$ and the rotational kinematic equations. Use radians; one revolution is 2π rad.
- Setting the string tension equal to the hanging weight when a massive pulley turns. The block accelerates, so $T < mg$, and the tension on the two sides of a massive pulley can differ.
- Forgetting the weight of a uniform beam. It acts at the beam's center and must appear in the torque equation.
- Checking only $\sum\tau = 0$ for a static object. A static object also needs $\sum F = 0$; use it to find the second support force.
- Thinking a larger mass always means a larger rotational inertia. The distribution matters: I grows with r^2.
- Saying gravity changes a rolling wheel's spin about its center. Gravity acts at the center, so it has no torque about it; static friction provides the torque.
- Thinking the tension is the same with a massive pulley. The vertical string's tension rises (the system accelerates less), and the two sides of the pulley carry different tensions.
- Assuming a heavier hanging object always wins on a stepped pulley. Compare torques, mgr, not weights.
- Using $I = \frac{1}{12}ML^2$ for a rod turning about its end. Use the parallel-axis theorem: $I = \frac{1}{3}ML^2$.

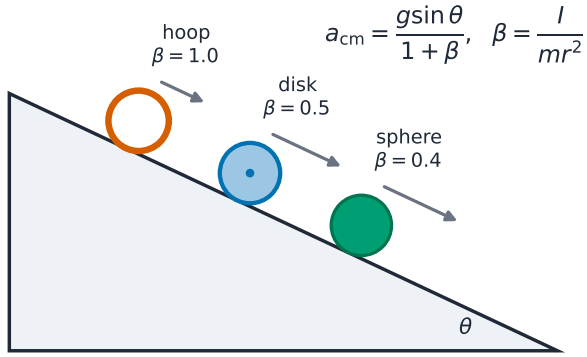
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Energy and Momentum of Rotating Systems

AP Physics 1

Rotational Kinetic Energy

Roll a full can of soup and an empty can down the same slope, side by side. Model the full can as a solid cylinder whose contents rotate rigidly with it, and the empty can as a hoop; the full can reaches the bottom first. Which one, and why, is the kind of question this unit answers with energy and angular momentum, not with forces.



Less rotational inertia per mass, more of the energy goes into speed.

A spinning object has **rotational kinetic energy** 转动动能, the rotational twin of $\frac{1}{2}mv^2$:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2.$$

An object that both moves and spins (like a rolling ball) has **both translational kinetic energy** 平动动能 and rotational kinetic energy, and its total is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with v the **centre-of-mass** 质心 speed and I measured about the centre of mass.

Three facts follow from this formula.

- **A spinning object can have kinetic energy while its center is at rest.** A flywheel on a fixed axle does not go anywhere, but every point on it moves, so it stores energy.

- $\frac{1}{2}I\omega^2$ is not a new kind of energy. Add up $\frac{1}{2}mv^2$ for every small piece of the object, with $v = r\omega$, and you get $\frac{1}{2}\left(\sum mr^2\right)\omega^2 = \frac{1}{2}I\omega^2$.
- Like all energy, rotational kinetic energy is a **scalar**: it has no direction.

Worked example (a rod swinging down). A rod with a sphere on its end has mass M and length L, and its center of mass is $\frac{3}{4}L$ from the axle. It is balanced pointing straight up, tapped, and swings down to point straight down. The center of mass falls from $\frac{3}{4}L$ above the axle to $\frac{3}{4}L$ below it, a drop of $\frac{3}{2}L$, so the rod–sphere–Earth system gains

$$\Delta K = Mg\left(\tfrac{3}{2}L\right) = \tfrac{3}{2}MgL.$$

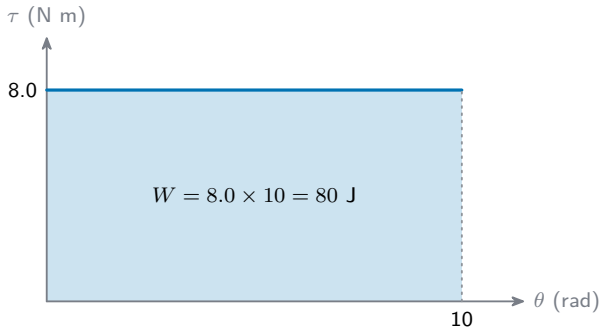
The rotational kinetic energy is greatest at the **bottom**, where the center of mass is lowest. If the Earth is **not** in the system, the answer is the same, but the reason changes: gravity is then an outside force that does positive work on the rod as its center of mass moves down.

Torque and Work

A torque acting through an angular displacement does **work** 功, changing rotational kinetic energy:

$$W = \tau\,\Delta\theta, \qquad P = \tau\omega.$$

This is the rotational form of $W = Fd$ and $P = Fv$, and it extends the **work–energy theorem** 动能定理 to rotation.



Work is the area under torque against angle.

Worked example. A motor applies a steady torque of 8.0 N m to a **flywheel** 飞轮 while it turns through 10 rad. The work done is $W = \tau\,\Delta\theta = 8.0 \times 10 = 80$ J, and if the flywheel started from rest this all becomes rotational kinetic energy.

If the torque changes as the object turns, the work is the **area** under a graph of torque against angular position, just as linear work is the area under a force–position graph.

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Worked example (pulling a string off a toy). A string of length ℓ_0 is wrapped round the fixed, frictionless axle of a toy initially at rest, with rotational inertia I_X. A constant force F_0 pulls the string until it comes off. The force acts through a distance ℓ_0, so it does work $F_0\ell_0$, and all of it becomes rotational kinetic energy:

$$F_0\ell_0 = \tfrac{1}{2}I_X\omega_X^2 \;\Rightarrow\; \omega_X = \sqrt{\frac{2F_0\ell_0}{I_X}}.$$

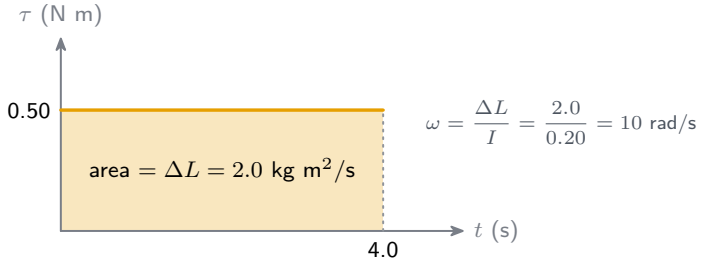
A second toy with half the rotational inertia receives the **same** work, so it gets the same kinetic energy. With a smaller I, it needs a **larger** ω to hold that energy: $\omega_Y = \sqrt{2}\,\omega_X$.

Angular Momentum and Angular Impulse

Angular momentum 角动量 describes rotational motion. For a **rigid body** 刚体 rotating about a fixed principal axis, its component along that axis is:

$$L = I\omega.$$

A net torque acting over time delivers an **angular impulse** 角冲量 that changes it: $\tau_{\text{avg}}\,\Delta t = \Delta L$ – the rotational **impulse–momentum theorem** 冲量 – 动量定理.



Angular impulse is the area under torque against time.

Worked example (angular impulse). A motor applies a torque of 0.50 N m for 4.0 s to a wheel with $I = 0.20$ kg m^2, starting from rest. The angular impulse is $\tau\,\Delta t = 2.0$ kg m^2/s, so $\omega = \Delta L/I = 2.0/0.20 = 10$ rad/s.

The graph rules match the linear ones from the momentum unit. The angular impulse is the **area** under a graph of torque against time, and the net torque is the **slope** of a graph of angular momentum against time. For a fixed I, $\tau_{\text{net}} = \dfrac{\Delta L}{\Delta t} = I\dfrac{\Delta\omega}{\Delta t} = I\alpha$: the rotational form of Newton's second law.

Same angular momentum, different energy. A solid disk and a hoop of the same mass M and radius R are each turned by a string pulled with the same force F_A for

the same time Δt. Each receives the same angular impulse $F_A R\,\Delta t$, so each gains the **same** angular momentum L. But $K = \dfrac{L^2}{2I}$, and the disk has the smaller rotational inertia, so it gains **more** kinetic energy. In terms of work: the disk spins up faster, so its string is pulled through a **longer** distance in the same time, and the same force does more work on it.

A single small object moving in a straight line also has angular momentum about a chosen point: $L = mvr_\perp$, where $r_\perp$ is the perpendicular distance from the point to its line of motion. This is what lets a moving object set something spinning.

More generally, $L = rmv\sin\theta$, where θ is the angle between $\vec{r}$ (from the point to the object) and $\vec{v}$. The value depends on which point you choose. An object moving in a straight line at constant speed has **constant** angular momentum about any fixed point, because $r\sin\theta = r_\perp$ stays the same as it passes.

Conservation of Angular Momentum

If the **net external torque** 合外力矩 on a system is zero, its total angular momentum is **conserved** 守恒:

$$I_1\omega_1 = I_2\omega_2.$$

So if I decreases, ω increases to keep L constant – this is why a spinning skater speeds up when pulling their arms in. It applies to **collisions** 碰撞 and **explosions** 爆炸 of rotating systems too.



A spinning skater pulls their arms in: rotational inertia falls, so angular speed rises to keep angular momentum constant

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> **Worked example.** A skater spins at 2.0 rev/s with rotational inertia $I_1 = 4.0$ kg m^2. She pulls their arms in, dropping her rotational inertia to $I_2 = 1.6$ kg m^2. With no external torque, angular momentum is conserved:
>
> $$\omega_2 = \frac{I_1}{I_2}\,\omega_1 = \frac{4.0}{1.6} \times 2.0 = 5.0 \text{ rev/s}.$$
>
> Her kinetic energy actually *rises* – the extra energy comes from the work her muscles do pulling her arms in against the outward pull.

The skater is a **nonrigid** system: she changes shape. Her angular **speed** changes although her angular **momentum** does not, because she moves mass closer to the axis. Only an outside torque can change the angular momentum of a system. Inside it, the angular impulses are equal and opposite (Newton's third law), so they cancel.

Check it with numbers. Writing $K = \dfrac{L^2}{2I}$, kinetic energy is **inversely proportional** 成反比 to I when L is fixed. Before: $\frac{1}{2}(4.0)(4.0\pi)^2 = 316$ J. After: $\frac{1}{2}(1.6)(10\pi)^2 = 790$ J, which is 2.5 times as much, the same factor by which I fell.

> **Worked example (a collision that starts a spin).** A uniform rod of mass 1.2 kg and length 0.80 m hangs from a pivot at its top end, at rest. A 0.10 kg lump of clay moving horizontally at 5.0 m/s hits the bottom end and sticks. Find the angular velocity just after.

- About the pivot, gravity and the pivot force exert no torque during the brief collision, so angular momentum is conserved.
- Before: only the clay, $L = mvr = (0.10)(5.0)(0.80) = 0.40$ kg m^2/s.
- After: $I = \frac{1}{3}(1.2)(0.80)^2 + (0.10)(0.80)^2 = 0.256 + 0.064 = 0.320$ kg m^2.
- $\omega = L/I = 0.40/0.320 = 1.25$ rad/s.

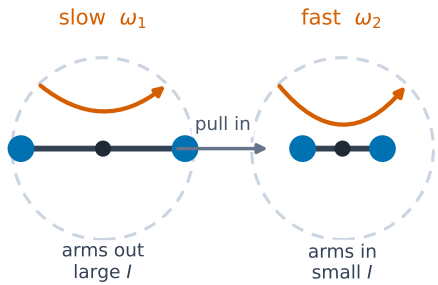
Linear momentum is **not** conserved here: the pivot pushes on the rod during the collision. Angular momentum about the pivot is, because that push passes through the pivot and has no torque.

Where should it hit, and does bouncing help? A disk of mass m slides at v_0 and sticks to a rod pivoted at one end, a distance x from the pivot. Its angular momentum about the pivot is mv_0x, so hitting **farther** from the pivot gives the combined rod–disk system more angular momentum, but also adds more rotational inertia. It does not always give a larger angular speed. Conservation gives

$$mv_0x = \left(I + mx^2\right)\omega \;\Rightarrow\; \omega = \frac{mv_0x}{I + mx^2}.$$

For fixed m, v_0 and rod inertia I, this angular speed is greatest at $x = \sqrt{I/m}$ if that point lies on the rod; beyond it, moving the hit farther out reduces ω.

If the disk **bounces back** instead of sticking, its angular momentum changes from $+mv_0x$ to a negative value. That is a larger change, so by the third law the rod receives a larger angular impulse and spins **faster** than when the disk sticks.



smaller I ⇒ larger ω (angular momentum conserved)

$L = I\omega$ conserved (no external torque): $I_1\omega_1 = I_2\omega_2$

Pulling mass inward lowers I, so ω rises to conserve $L = I\omega$

Rolling

Rolling without slipping 纯滚动 links the translational and rotational motions: the **contact point** 接触点 is momentarily at rest, so

$$v = r\omega \qquad \text{and} \qquad a = r\alpha.$$

A rolling object's energy splits between translation and rotation, so on an **incline** 斜面 it accelerates **more slowly** than a frictionless sliding object – some energy goes into spin.

> **Worked example.** For a solid disk ($I = \frac{1}{2}mR^2$) that rolls without slipping, what fraction of its kinetic energy is rotational? Using $v = R\omega$, the rotational part is $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mR^2)\omega^2 = \frac{1}{4}mv^2$, while the translational part is $\frac{1}{2}mv^2$. So the total is $\frac{3}{4}mv^2$ and the rotational share is $\frac{1/4}{3/4} = \frac{1}{3}$. A hoop, with its mass farther out, stores half its energy in spin and rolls down a ramp even more slowly.

Friction in rolling. When an object rolls **without slipping**, the contact point is at rest for an instant, so static friction acts at a point that does not move: ideally it does **no work** and takes no energy from the system. Its job is to provide the torque that makes the object spin. When an object **slips** (skids), $v \neq r\omega$, the contact point slides over the surface, and kinetic friction turns some kinetic energy into thermal energy.

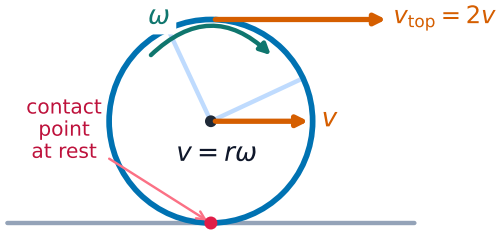
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> **Worked example (a sphere pushed through its center).** A plunger pushes a sphere with a force aimed at its center, from A to B, on a frictionless floor. The sphere then crosses a frictionless stretch B to C, a rough patch C to D, and a frictionless stretch D to E. Where does its angular velocity change?

- A to B: the push passes through the center, so it has no torque. The sphere slides without spinning.
- B to C: no horizontal force at all, so nothing changes.
- C to D: kinetic friction at the bottom acts backward, and it has a torque about the center. The sphere slows down **and** starts to spin. This is the only place ω changes.
- D to E: no friction again, so v and ω both stay constant.

Wheel against block on a ramp. A wheel rolls down a ramp, with static friction equal to 40% of the gravity component along the slope. Newton's second law along the slope gives $Ma = Mg\sin\theta - 0.4\,Mg\sin\theta$, so $a = 0.6\,g\sin\theta$. A block sliding down a **frictionless** ramp of the same angle reaches the bottom faster. In terms of forces: the net force down the slope is larger on the block. In terms of energy: the wheel's Mgh is shared between translation and rotation, while all of the block's goes into translation.

A rolling cylinder and a block of the same mass on a **rough** ramp can reach the bottom with the **same** speed. The cylinder puts part of its energy into spinning; the block loses part of its energy as thermal energy through kinetic friction. If those two parts are equal, the translational kinetic energy at the bottom is the same.



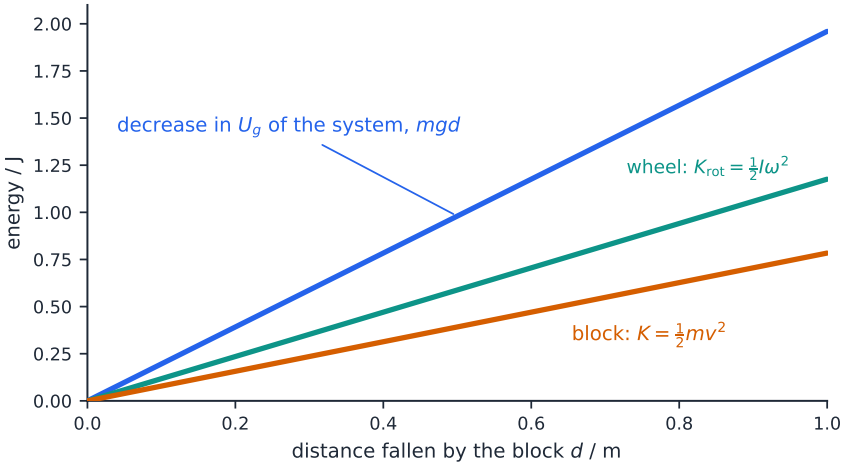
$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2\left(1 + \tfrac{I}{mr^2}\right)$$

rolling no slip: $v = r\omega$ · $a = r\alpha$

In rolling without slipping the contact point is at rest, so $v = r\omega$

Energy graphs for a block turning a wheel. A block hangs from a string wound round a wheel on a fixed axle. As the block falls a distance d, the system loses mgd of gravitational potential energy. The block gets part of it as translational kinetic

energy; the rest is the wheel's rotational kinetic energy, the gap between the two lines below.



The energy the block loses is shared: some goes into the block's motion, the rest into spinning the wheel

To **measure** the wheel's rotational inertia, record ω and K_{rot} at several points and plot K_{rot} against ω^2. The points lie on a straight line through the origin with slope $\frac{1}{2}I$, so $I = 2 \times$ slope. A slope of 0.015 J s^2 gives $I = 0.030$ kg m^2.

Similarly, for a cylinder rolling down a ramp and then along a level floor, the **total** kinetic energy grows in a straight line with the distance travelled down the ramp ($K = mg\,s\sin\theta$), then stays constant along the floor.

The rolling race

Release several round objects from rest at the top of the same ramp, height h. Energy conservation with $v = R\omega$ gives

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mR^2}\right) \;\Rightarrow\; v = \sqrt{\frac{2gh}{1 + I/(mR^2)}}.$$

Neither mass nor radius appears on its own: only the **shape factor** 形状因子 $I/(mR^2)$ matters. For $h = 1.2$ m and $g = 9.8$ m/s^2:

Object	$I/(mR^2)$	Speed at the bottom
block sliding without friction	0	4.8 m/s
solid sphere	2/5	4.1 m/s
solid disk (full can)	1/2	4.0 m/s
hoop (empty can, roughly)	1	3.4 m/s

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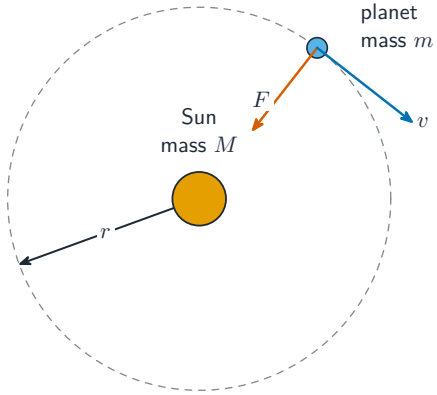
The full can wins against the empty one: less of its energy goes into spinning, so more goes into speed.

Motion of Orbiting Satellites

A **satellite** 卫星 in **orbit** 轨道 is in **free fall** 自由落体: gravity curves its path. For a circular orbit, gravity provides the **centripetal force** 向心力, so

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \;\Rightarrow\; v = \sqrt{\frac{GM}{r}}.$$

So a larger orbit means a slower speed. For a circular orbit, angular momentum and **mechanical energy** 机械能 are both constant; for an **elliptical** 椭圆的 orbit, angular momentum is conserved (no torque about the planet) while speed varies – fastest when closest.



Gravity provides the centripetal force that keeps a satellite in orbit

Worked example. Find the speed of a satellite in a low orbit just above the Earth, radius $r = 6.4 \times 10^6$ m, with $GM = 4.0 \times 10^{14}$ m^3/s^2:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{6.4 \times 10^6}} = \sqrt{6.25 \times 10^7} \approx 7.9 \times 10^3 \text{ m/s},$$

about 7.9 km/s – and notice that the satellite's mass cancels, so all low orbits share this speed.



The International Space Station: a real satellite held in orbit by gravity alone

Escape velocity 逃逸速度 is the launch speed that just lets an object leave for good: its total mechanical energy is exactly zero, so it slows to zero speed only at infinite distance. Setting $\frac{1}{2}mv^2 - \dfrac{GMm}{r} = 0$ (using the general gravitational PE $U_g = -GMm/r$) and solving for v,

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}.$$

It is $\sqrt{2}$ times the circular-orbit speed at the same radius, and (like orbital speed) does not depend on the escaping object's mass.

What stays constant in an orbit. When a satellite's mass is tiny compared with the planet's, the planet's own motion can be ignored. Gravitational potential energy is defined to be **zero** when the two are infinitely far apart, so for any orbit it is negative.

	Circular orbit	**Elliptical orbit**
total mechanical energy of the system	constant	constant
satellite's angular momentum	constant	constant
gravitational potential energy	constant	changes
satellite's kinetic energy	constant	changes

In an ellipse, as the satellite moves closer, U_g becomes more negative and K grows by the same amount. Angular momentum $L = mvr_\perp$ explains the same thing: a smaller $r_\perp$ needs a larger v.

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Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2.

1. A flywheel with $I = 2.0$ kg m^2 spins at 30 rad/s. Find its rotational kinetic energy.
2. A hoop and a solid disk of equal mass and radius roll from rest down the same ramp. Which reaches the bottom first? Explain without calculating.
3. A skater spinning at 1.5 rev/s reduces her rotational inertia from 3.0 to 1.2 kg m^2. Find her new rate of spin.
4. One satellite orbits at four times the radius of another. Compare their orbital speeds.
5. A student says: "Angular momentum is conserved when the skater pulls their arms in, so her kinetic energy is conserved too." Correct the student.

Answers

1. $K = \frac{1}{2}(2.0)(30)^2 = 900$ J.
2. The disk. A hoop has all its mass at the rim, so a larger share of its energy goes into rotation and less into speed. The disk's shape factor is $\frac{1}{2}$, the hoop's is 1.
3. $\omega_2 = \dfrac{3.0}{1.2}(1.5) = 3.75$ rev/s.
4. $v \propto 1/\sqrt{r}$, so the outer satellite moves at half the speed.
5. With L fixed, $K = L^2/2I$ rises as I falls. Her muscles do work pulling her arms inward, and that work becomes the extra kinetic energy. Angular momentum is conserved; kinetic energy is not.

Exam tips

- Conserve **angular momentum** $L = I\omega$ when no external torque acts: a smaller I (arms pulled in) gives a larger ω.
- A rolling object splits its energy between $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$, linked by $v = r\omega$ – so it accelerates down a ramp **more slowly** than a sliding one.
- For a circular orbit set gravity equal to the centripetal requirement: $v = \sqrt{GM/r}$, so a larger orbit is **slower** and the satellite's mass cancels.
- Pulling in raises the spin **and** the kinetic energy – the extra energy comes from the work done pulling inward; L is unchanged.
- Watch which rotational quantity is conserved: L (no torque) versus energy (no friction) are different conditions.

Common mistakes

- Leaving out rotational kinetic energy for a rolling object. Its total kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, so it reaches the bottom of a ramp slower than a sliding block.
- Using ω in rev/s inside $\frac{1}{2}I\omega^2$. Change to rad/s first ($\times 2\pi$); in $I_1\omega_1 = I_2\omega_2$ any unit works, because it cancels.
- Saying a skater's kinetic energy stays the same when she pulls their arms in. Angular momentum stays the same; kinetic energy rises, supplied by the work her muscles do.
- Conserving linear momentum when an object hits a pivoted rod. The pivot exerts an external force; conserve angular momentum about the pivot instead.
- Thinking a heavier ball rolls down faster. For rolling from rest, only the shape factor $I/(mR^2)$ decides the speed.
- Writing that "there is no gravity" on an orbiting satellite. Gravity is the centripetal force; the satellite is in free fall.
- Saying static friction takes energy from a rolling object. Without slipping, the contact point does not move, so static friction does no work.
- Thinking that the same angular impulse gives the same kinetic energy. It gives the same angular momentum; the object with the smaller I gets more energy.
- Saying a disk that sticks to a rod gives it more spin than one that bounces. Bouncing reverses the disk's angular momentum, so it gives the larger angular impulse.
- Saying the potential energy of a satellite in an elliptical orbit stays constant. Only the total energy and the angular momentum are constant.

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Oscillations

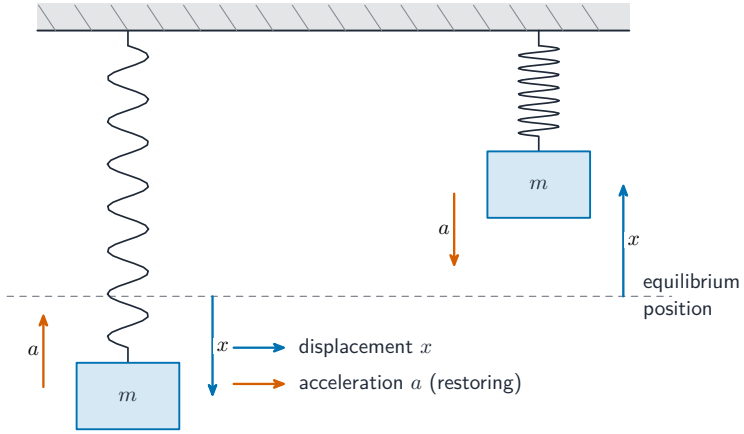
AP Physics 1

Defining Simple Harmonic Motion

Simple harmonic motion 简谐运动 (SHM) is a back-and-forth **oscillation** 振动 caused by a **restoring force** 回复力 that is **proportional** 成正比 to the **displacement** 位移 from equilibrium and always points back toward it: $F = -kx$. A mass on a spring and (for small angles) a **pendulum** 单摆 are the standard examples. Because the force grows with displacement, the motion is smooth and repeating, tracing a sine curve in time.

SHM is a special case of **periodic motion** 周期性运动, any motion that repeats itself. A bouncing ball repeats, but it is not SHM: the force on it (gravity) does not grow with the distance from the middle. Two words matter here. The **equilibrium position** is where the net force is zero. A **restoring force** is one that always points back toward it, opposite to the displacement.

Here is the surprising part this unit explains. For small swings, changing the amplitude barely changes the time a child's swing takes to come back. In the SHM approximation, a bigger swing travels further and faster, and the two effects cancel; at large angles, a pendulum's period increases.



In SHM the acceleration always points back towards equilibrium, opposite the displacement

The test for SHM is exactly this: acceleration proportional to displacement and opposite in direction, $a = -\dfrac{k}{m}x$. A pendulum only obeys it for **small** swings, where $\sin\theta \approx \theta$; large swings are not quite SHM.

For a pendulum, think in terms of torque. About the pivot, gravity's restoring torque is $\tau = -mgL\sin\theta$. For small angles $\sin\theta \approx \theta$ (in radians), so the restoring torque is proportional to the angular displacement, $\tau \approx -mgL\,\theta$: that is the rotational version of $F = -kx$.



A spring under load: Hooke's law $F = -kx$ is the restoring force that produces SHM

Frequency and Period of SHM

- The **period** 周期 T is the time for one full **cycle** 循环.
- The **frequency** 频率 $f = \dfrac{1}{T}$ is cycles per second (**hertz** 赫兹).

For SHM these depend only on the system, **not** on the amplitude:

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}, \qquad T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}.$$

So a stiffer spring or smaller mass oscillates faster; a longer pendulum swings slower.

Worked example. A 0.25 kg mass hangs on a spring of **stiffness** 劲度 $k = 100$ N/m. Its period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{100}} = 0.31 \text{ s}, \qquad f = \frac{1}{T} = 3.2 \text{ Hz}.$$

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Worked example. A pendulum clock ticks with a period of exactly 2.0 s. How long is it? Rearranging $T = 2\pi\sqrt{L/g}$,

$$L = g\left(\frac{T}{2\pi}\right)^2 = 9.8 \times \left(\frac{2.0}{2\pi}\right)^2 = 0.99 \text{ m}.$$

Notice the amplitude never entered – small swings keep nearly the same time as their amplitude changes, which is what makes pendulums good clocks.

Worked example (a stretchy pendulum on another planet). A pendulum hangs from a slightly **elastic** string. It is taken to Planet Y, which has the same radius as Earth but more mass, so g is larger there. Can its period be longer **and** can it be shorter than on Earth?

- A larger g on its own makes the period **shorter**, because $T \propto 1/\sqrt{g}$.
- But the bob now pulls harder on the string. The elastic string stretches, so L grows, and a longer pendulum has a **longer** period, because $T \propto \sqrt{L}$.
- Compare the ratios: $T_Y/T_E = \sqrt{(L_Y/L_E)/(g_Y/g_E)}$. The period is longer only if the length increases by a larger factor than g. For a Hookean string of relaxed length L_0 and stiffness k, $L = L_0 + mg/k$, so $T^2 = 4\pi^2(L_0/g + m/k)$: increasing g always shortens the period in this model. A longer period would require a different stretching law that meets the ratio condition.

On a planet with the same mass but a **larger** radius, g is smaller, so gravity does less work on the bob as it swings down from the same angle: $W = mgL(1 - \cos\theta)$.

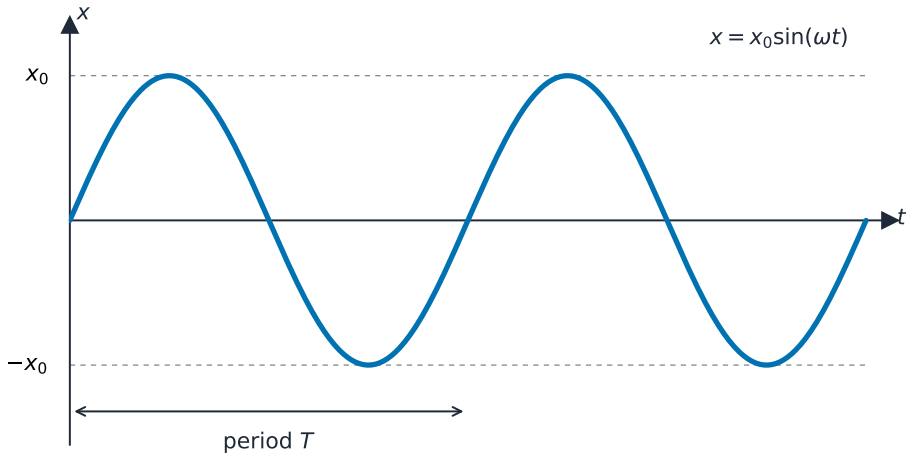


Tuning forks: a longer, heavier fork vibrates more slowly, giving a lower frequency

Representing and Analyzing SHM

The displacement varies **sinusoidally** 按正弦规律: $x(t) = A\cos(\omega t)$ (or sine), where A is the **amplitude** 振幅 (maximum displacement) and $\omega = 2\pi f$ is the **angular frequency** 角频率. Reading the motion:

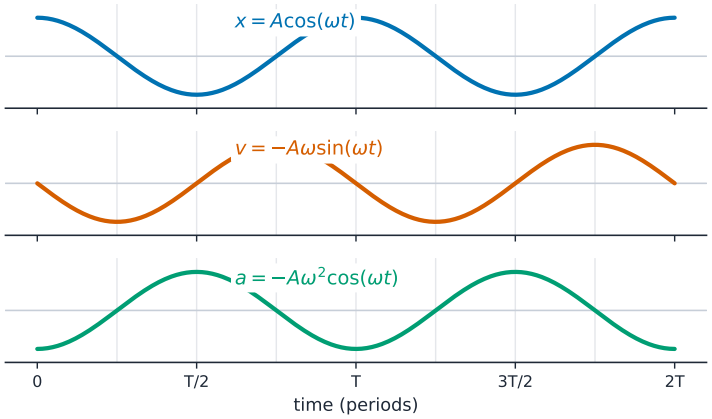
- At the **extremes** ($x = \pm A$): displacement and restoring force are maximum, so acceleration is maximum, but velocity is **zero**.
- At **equilibrium** 平衡 ($x = 0$): force and acceleration are zero, but **speed is maximum**.



Displacement varies sinusoidally with time in simple harmonic motion

Velocity and acceleration are also sinusoidal, shifted in **phase** 相位 from the displacement – velocity leads displacement by a quarter cycle, and acceleration is exactly opposite to displacement.

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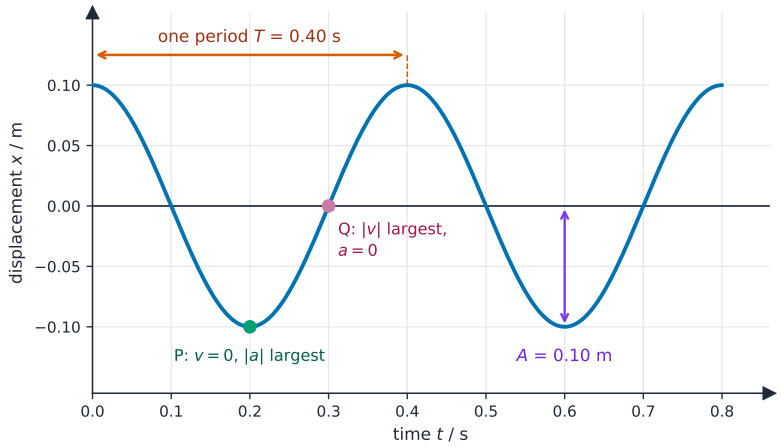


$a = -\omega^2 x$ · v leads x by $T/4$ · a is in antiphase with x

SHM phase: velocity leads displacement by a quarter cycle; acceleration is half a cycle from displacement

Read the figure as a story: where x is largest (the turning point), v has just fallen to zero and a is at its most negative, hauling the mass back; a quarter-cycle later the mass races through the middle at top speed with zero acceleration.

> **Worked example (reading an x–t graph).** The graph shows a 0.50 kg cart oscillating on a horizontal spring.



Read T from crest to crest and A from the axis to a crest; the slope shows where the speed is greatest

- **Read the graph.** Crest to crest takes $T = 0.40$ s, so $f = 1/T = 2.5$ Hz. The axis-to-crest height is $A = 0.10$ m (not the crest-to-trough height of 0.20 m).
- **Spring constant.** From $T = 2\pi\sqrt{m/k}$: $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(0.50)}{0.40^2} = 1.2 \times 10^2$ N/m.
- **Maximum speed.** $v_{\max} = \omega A = \dfrac{2\pi A}{T} = \dfrac{2\pi(0.10)}{0.40} = 1.6$ m/s, at Q, where the graph is steepest. Energy agrees: $\frac{1}{2}kA^2 = \frac{1}{2}(123)(0.10)^2 = 0.62$ J $= \frac{1}{2}(0.50)v_{\max}^2$.
- **At P** ($x = -A$) the slope is zero, so $v = 0$; the spring pulls the cart back toward equilibrium, so the acceleration is **positive** and largest, $a = \dfrac{kA}{m} = 25$ m/s^2.
- **At Q** the slope is positive, so the cart moves in the $+x$ direction at top speed with $a = 0$.

Reading a motion-detector graph. A 0.50 kg object hangs from a vertical spring above a motion detector, is pulled down and released. The detector records its height against time, a sine curve about the equilibrium height.

- **The spring constant.** If the graph shows crests 0.75 s apart, then $T = 0.75$ s and $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(0.50)}{0.75^2} = 35$ N/m.
- **Same kinetic energy at two times.** Where the curve has the **same steepness** (same size of slope), the object has the same speed, and so the same kinetic energy.
- **Same potential energy at those times.** If the object is at the **same height** at both times, the spring has the same stretch and the object the same height, so both the elastic and the gravitational potential energy are the same.
- **A spring four times stiffer.** The period halves, $T \propto 1/\sqrt{k}$. The new equilibrium stretch mg/k is a quarter as large, so the object hangs **higher**. Pulled down the same distance d_0, it has the same amplitude about this higher center line.

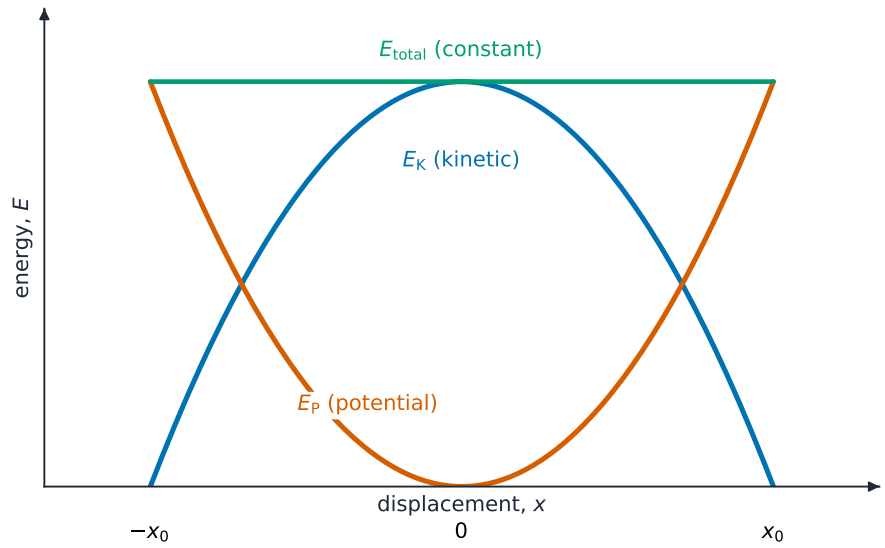
Energy of Simple Harmonic Oscillators

Energy sloshes between **kinetic energy** 动能 and **potential energy** 势能 while the **total** stays constant (no friction):

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2.$$

For a vertical spring, $\frac{1}{2}kx^2$ here is the combined spring and gravitational potential energy above its minimum at equilibrium, not the spring energy alone. At the extremes it is **all potential**; at equilibrium it is **all kinetic** (maximum speed). Because $E \propto A^2$, doubling the amplitude quadruples the energy.

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Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example. A 0.50 kg mass on a spring of stiffness $k = 200$ N/m oscillates with amplitude $A = 0.10$ m. Find its maximum speed. All the energy is kinetic at the equilibrium point, so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$:

$$v_{\max} = A\sqrt{\frac{k}{m}} = 0.10 \times \sqrt{\frac{200}{0.50}} = 0.10 \times 20 = 2.0 \text{ m/s}.$$

The energy method is faster than tracking the sine functions when you only need the greatest speed.

Worked example (speed at any position). For the same oscillator, find the speed at $x = 0.050$ m, half the amplitude. Energy is shared between the spring and the mass:

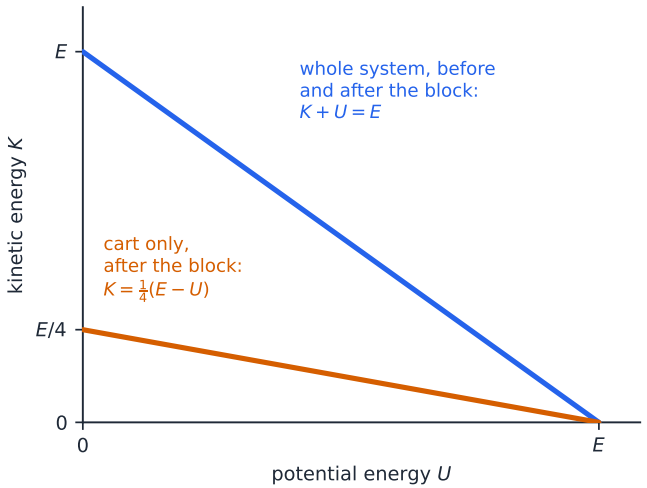
$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 \;\Rightarrow\; v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = 20\sqrt{0.0100 - 0.0025} = 1.7 \text{ m/s}.$$

At half the amplitude the speed is already 87% of the maximum: the mass slows down sharply only near the **turning points** 折返点.

Notice what the amplitude does and does not change. A larger amplitude means more energy, $E = \frac{1}{2}kA^2$, and a larger maximum speed, but the **same** period. The kinetic energy always falls to **zero** at the turning points, whatever the amplitude.

Energy against energy. A graph of K against U for an oscillator is a straight line

of slope -1: $K = E - U$ at every point. It crosses **both** axes at E, because when $U = 0$ all the energy is kinetic, and when $K = 0$ all of it is potential.



K against U is a straight line of slope −1 for the whole system; for the cart alone after the block is added, the kinetic energy is a quarter of that

Worked example (a block dropped onto a cart: at the end, or in the middle?). A cart of mass m_0 oscillates on a spring between $x = +L$ and $x = -L$.

- *Block of mass $3m_0$ dropped on at $x = +L$, where the cart is momentarily at rest.* There is no horizontal motion, so no horizontal kinetic energy is lost when the block sticks; any energy from its vertical fall is separate. The amplitude stays L, the energy $E = \frac{1}{2}kL^2$ is the same, and the system's K–U line does not change. The mass is four times larger, so $\dfrac{f_2}{f_1} = \sqrt{\dfrac{m_0}{4m_0}} = \frac{1}{2}$. The cart carries only a quarter of the moving mass, so its own kinetic energy is $\frac{1}{4}$ of the total at every point: the lower line in the figure.
- *Block of mass $2m_0$ dropped on at $x = 0$, where the cart moves fastest.* This is a sticking collision. Horizontal momentum is conserved: $m_0 v = 3m_0 v'$, so $v' = \frac{1}{3}v$. The kinetic energy becomes $\frac{1}{2}(3m_0)\left(\frac{1}{3}v\right)^2 = \frac{1}{3}\left(\frac{1}{2}m_0 v^2\right)$: two thirds of the horizontal oscillation energy is lost. Since $E = \frac{1}{2}kA^2$, the new amplitude is $A/\sqrt{3}$, **smaller** than before. The period grows by $\sqrt{3}$.

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Vertical springs

Hang a 0.50 kg mass on a spring with $k = 200$ N/m. It stretches until the spring force balances the weight: $k\,\Delta y = mg$, so $\Delta y = \dfrac{(0.50)(9.8)}{200} = 0.025$ m. Pull it down and let go, and it oscillates about this **new** equilibrium position.

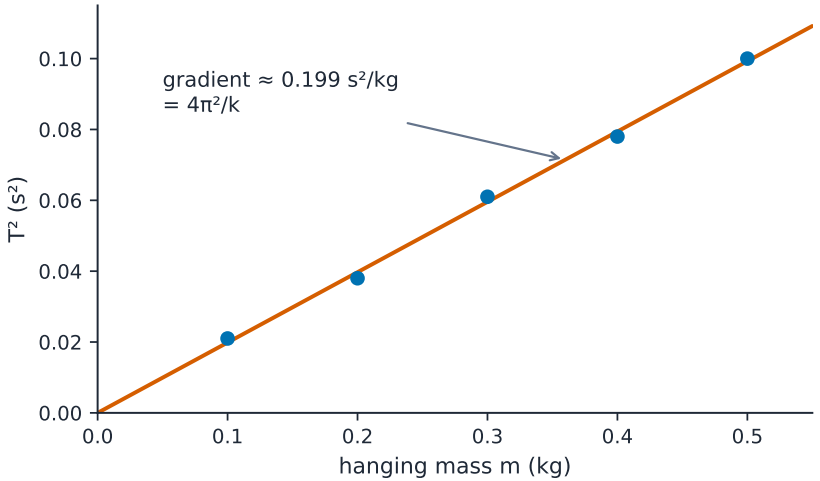
- Gravity only **moves** the center of the oscillation. It does not change the period: $T = 2\pi\sqrt{m/k} = 0.31$ s, exactly as on a horizontal track.
- Measure x from the new equilibrium, not from the spring's **unstretched length** 原长, and every formula in this unit still works.

Finding k from an experiment

A laboratory question rarely hands you k. Instead you time oscillations for several hanging masses. Squaring the period formula gives

$$T^2 = \frac{4\pi^2}{k}\,m,$$

so a graph of T^2 against m should be a straight line through the **origin** 原点 with **gradient** 斜率 $4\pi^2/k$.



A straight line through the origin confirms $T^2 \propto m$; the gradient gives the spring constant

Read the gradient from the **best-fit line** 最佳拟合线, not from one data point: 0.199 s^2/kg, so $k = \dfrac{4\pi^2}{0.199} = 198$ N/m. Time 10 oscillations and divide by 10: your **reaction time** 反应时间 is then spread across ten periods instead of one.

Worked example (an experimental design question). The third free-response question, *Experimental Design and Analysis*, can ask you to plan the measurement yourself, for example: *describe a procedure that could be used to determine the spring constant k, including the equipment and the measurements you would take.* A full-mark plan names what is changed, what is measured and with what, and how the data give k:

1. Hang the spring from a clamp stand and attach a mass hanger.
2. Hang a known mass m (measured on a balance), pull it down a few centimetres and release it.
3. Use a stopwatch to time 10 full oscillations; divide by 10 to get T.
4. Repeat for at least five different masses, for example 0.10 kg to 0.50 kg.
5. Plot T^2 on the vertical axis against m on the horizontal axis, draw the best-fit straight line, and use $k = \dfrac{4\pi^2}{\text{gradient}}$.

Two follow-up parts are common. *Why plot T^2 and not T?* Because T^2 is proportional to m, so the graph is a straight line whose gradient can be measured. *The best-fit line crosses the m-axis slightly to the left of the origin. Suggest a reason.* The spring has mass of its own, which also oscillates, so the moving mass is a little larger than the hanging mass m; the gradient, and so k, is still valid.

Proportional reasoning

Many multiple-choice items change one quantity and ask what happens to the period. Read the power in the formula.

Change	Formula	Effect on T
mass on a spring ×4	$T \propto \sqrt{m}$	×2
spring constant ×4	$T \propto 1/\sqrt{k}$	×½
pendulum length ×4	$T \propto \sqrt{L}$	×2
pendulum on the Moon ($g \div 6$)	$T \propto 1/\sqrt{g}$	$\times\sqrt{6} \approx 2.4$
mass of a pendulum **bob** 摆锤 ×4	m is not in the formula	no change
amplitude ×2	A is not in the formula	no change

Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2.

1. A 0.20 kg mass oscillates on a spring with $k = 80$ N/m. Find the period.
2. The mass in question 1 is replaced by a 0.80 kg mass. Find the new period without recalculating from the start.
3. A pendulum has a period of 1.5 s on Earth. Find its period on the Moon, where $g = 1.6$ m/s^2.
4. A 0.50 kg mass on a spring with $k = 50$ N/m oscillates with amplitude 0.20 m. Find its maximum speed and its speed at $x = 0.10$ m.
5. A student says: "If I double the amplitude, the mass travels twice as far, so the period doubles." Correct the student.

Answers

1. $T = 2\pi\sqrt{0.20/80} = 2\pi(0.050) = 0.31$ s.
2. The mass is 4 times larger and $T \propto \sqrt{m}$, so the period doubles: 0.63 s.
3. $T \propto 1/\sqrt{g}$, so $T = 1.5\sqrt{9.8/1.6} = 1.5(2.47) = 3.7$ s.
4. $v_{\max} = A\sqrt{k/m} = 0.20\sqrt{100} = 2.0$ m/s. At $x = 0.10$ m: $v = \sqrt{100\,(0.040 - 0.010)} = \sqrt{3.0} = 1.7$ m/s.
5. The distance doubles, but so does the speed at every point, because $v_{\max} = A\sqrt{k/m}$. The two effects cancel, so the period, $2\pi\sqrt{m/k}$, does not change. Amplitude is not in the formula.

Exam tips

- Test for **SHM**: the acceleration must be proportional to the displacement and directed back toward the middle ($a = -\frac{k}{m}x$).
- The **period does not depend on amplitude** – use $T = 2\pi\sqrt{m/k}$ (spring) or $T = 2\pi\sqrt{L/g}$ (pendulum, small angles).
- Speed is **maximum at the middle** (all kinetic) and **zero at the extremes** (all potential); acceleration is largest at the extremes.
- Use energy ($\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$) to find the maximum speed quickly: $v_{\max} = A\sqrt{k/m}$.
- Total energy $\propto A^2$, so doubling the amplitude quadruples the energy.

Common mistakes

- Saying the acceleration is zero at the turning points because the velocity is zero there. At $x = \pm A$ the restoring force, and so the acceleration, is largest.
- Making the period depend on amplitude, or on the mass of a pendulum bob. Neither appears in $T = 2\pi\sqrt{m/k}$ or $T = 2\pi\sqrt{L/g}$.
- Using the pendulum formula for a mass on a spring, or the reverse. A spring's period depends on m and k; a pendulum's on L and g.
- Measuring a vertical spring's displacement from its unstretched length. Measure from the new equilibrium position, where the spring force balances the weight.

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- Calling one swing from one side to the other a full period. A period is a complete cycle: there and back.
- Reading k from one data point instead of the gradient of a T^2–m best-fit line.
- Assuming the amplitude stays the same whenever a block is added to an oscillating cart. Added at a turning point, it does; added at equilibrium, it is a sticking collision that loses energy, so the amplitude falls.
- Calling a bouncing ball SHM. It repeats, but its restoring force is not proportional to displacement.
- Saying a pendulum's period on a heavier planet must be shorter. Compare L/g: the period is longer only if the length increases by a larger factor than g. A Hookean string lengthens, but not enough to reverse the decrease in period.

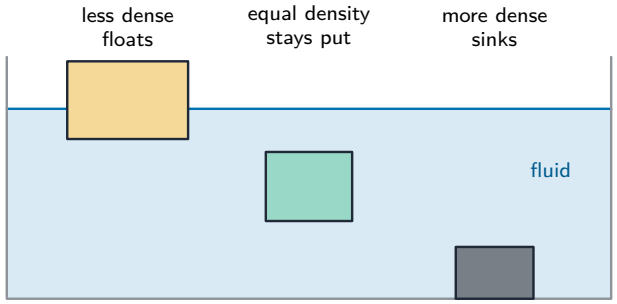
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Fluids

AP Physics 1

Internal Structure and Density

A steel ship weighing many thousands of tonnes floats, while a steel bolt sinks at once. A mechanic lifts a car with one hand on a pump handle. Both follow from one idea: a fluid pushes back with pressure, and pressure depends on depth and area in ways this unit makes exact.



$\rho = m/V$ compared with the fluid's density

Compare the object's density with the fluid's.

The differences between solids, liquids, and gases come from how strongly their particles interact. A **fluid** 流体 (liquid or gas) has no fixed shape – it flows because its particles move past one another. **Density** 密度 is mass per unit volume:

$$\rho = \frac{m}{V}.$$

It depends on the substance and its state. An object sinks or floats depending on how its density compares with the surrounding fluid's – less dense floats, more dense sinks. An **ideal fluid** 理想流体 is **incompressible** 不可压缩 (constant density, whatever the pressure) and has no **viscosity** 黏度 (internal friction) – the model AP uses throughout.

> **Worked example (density).** A 0.27 kg metal block measures 5.0 cm × 4.0 cm × 5.0 cm. Its volume is 100 $\text{cm}^3 = 1.0 \times 10^{-4}$ m^3, so $\rho = \dfrac{0.27}{1.0 \times 10^{-4}} = 2700$ kg/m^3: aluminium. Cut the block in half and the density stays the same: density belongs to the material, not to the size of the piece.

Pressure

Pressure 压强 is the **perpendicular** 垂直 force per unit area, a **scalar** 标量 measured in **pascals** 帕斯卡 (Pa):

$$P = \frac{F_\perp}{A}.$$

In a fluid at rest, pressure increases with **depth** 深度 because of the weight of fluid above:

$$P = P_0 + \rho g h,$$

where P_0 is the pressure at the surface and h is the depth. Pressure acts **equally in all directions** at a point and pushes **perpendicular** to any surface.

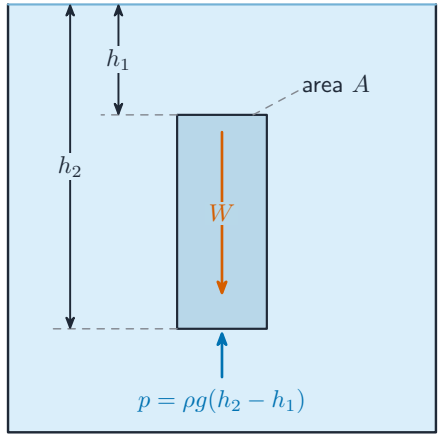
Where pressure comes from. A fluid is made of particles moving in all directions. Each one that hits a wall bounces off and gives the wall a tiny push. Pressure is the total effect of all those collisions, spread over the area. Because the pushes come from every direction, pressure has **no direction** of its own: it is a scalar, and the *force* it makes on a surface is always perpendicular to that surface.

Squeezing a liquid hardly changes its volume. An ideal (incompressible) fluid keeps the **same volume and density** whatever the pressure, which is why ρ in $\rho g h$ can be treated as constant all the way down.



Hydraulic jacks: pressure applied to a small piston is transmitted undiminished through the fluid to a large one, so a small force lifts a car

Distinguish the two pressures AP asks about: the **gauge pressure** 表压 is pressure above the surrounding **atmospheric pressure** 大气压强. For a stationary liquid with its surface open to the atmosphere, $P_{\text{gauge}} = \rho g h$. The **absolute pressure** 绝对压强 is measured from vacuum, $P = P_{\text{atm}} + P_{\text{gauge}}$. A tyre gauge reading "200 kPa" is gauge pressure; the air inside is really at about 300 kPa absolute.



The weight of a liquid column sets the extra pressure at a depth

Worked example. Find the total pressure on a diver 10 m below the surface of water ($\rho = 1000$ kg/m^3, surface pressure $P_0 = 1.0 \times 10^5$ Pa):

$$P = P_0 + \rho g h = 1.0 \times 10^5 + 1000 \times 9.8 \times 10 = 1.98 \times 10^5 \text{ Pa}.$$

Every 10 m of water adds roughly one extra atmosphere of pressure. Notice the pressure does not depend on the shape or width of the container, only on the depth.



An aneroid barometer measures air pressure with a sealed metal box that flexes as the pressure outside changes

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Worked example (a hydraulic lift). Pressure applied to an enclosed fluid is passed on equally throughout it (**Pascal's principle** 帕斯卡原理). Ignoring piston weight, friction and hydrostatic pressure differences between the pistons, a hydraulic lift has a small piston of area 0.0010 m^2 and a large piston of area 0.10 m^2. What force on the small piston holds up a 1500 kg car?

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \;\Rightarrow\; F_1 = \frac{A_1}{A_2}F_2 = \frac{0.0010}{0.10}(1500)(9.8) = 147 \text{ N}.$$

The force is 100 times smaller, but nothing is free: to raise the car 0.10 m, the small piston must move 10 m. The work done on each piston is the same, 1470 J.

Fluids and Newton's Laws

An object in a fluid feels an upward **buoyant force** 浮力 equal to the weight of the fluid it displaces – **Archimedes' principle** 阿基米德原理:

$$F_b = \rho_{\text{fluid}}\, g\, V_{\text{displaced}}.$$

Combine this with Newton's laws: the object floats when buoyancy balances weight, sinks when weight wins, and rises when buoyancy wins. A floating object displaces exactly its own weight of fluid.

Worked example. A block of density 600 kg/m^3 floats in water (1000 kg/m^3). What fraction is under the surface? For floating, the buoyant force equals the weight, so $\rho_{\text{fluid}}\, g\, V_{\text{sub}} = \rho_{\text{object}}\, g\, V$:

$$\frac{V_{\text{sub}}}{V} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{600}{1000} = 0.60.$$

So 60% sits below the water – the same reason most of an iceberg (density ≈ 900) hides underwater.

Worked example (apparent weight). A 2.0 kg rock with volume 8.0×10^{-4} m^3 hangs from a spring scale, fully under water. Its weight is 19.6 N. The buoyant force is $(1000)(9.8)(8.0 \times 10^{-4}) = 7.8$ N. The rock is at rest, so the scale reads $19.6 - 7.8 = 11.8$ N: this is its **apparent weight** 视重. The rock seems lighter, though its weight has not changed.

The buoyant force is not a new kind of force. The fluid particles push on every surface of the object, and the pushes on the **bottom** are larger than those on the **top**, because the bottom is deeper. The net result of all those pushes is an upward force: the buoyant force.

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Worked example (a block released under water). A swimmer holds a block of mass m and volume V fully under water of density ρ and lets go. Two forces act on it: the weight mg down and the buoyant force $\rho V g$ up. Newton's second law, with up positive:

$$ma = \rho V g - mg \;\Rightarrow\; a = g\left(\frac{\rho V}{m} - 1\right).$$

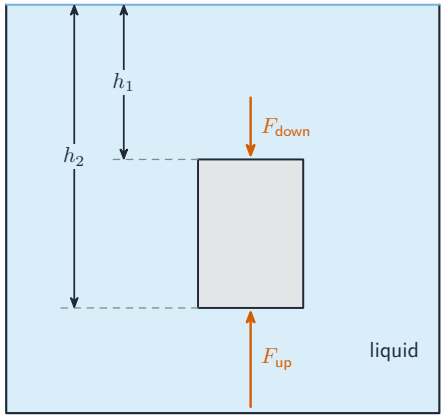
In **salt water**, which is denser than fresh water, the buoyant force is larger but the weight is the same. So the net upward force is larger, giving a larger upward acceleration (or a smaller downward acceleration): $a_{\text{salt}} > a_{\text{fresh}}$. The formula agrees: a larger ρ gives a larger a.

Fluids and Conservation Laws

For an ideal fluid flowing steadily, two conservation ideas apply:



A dam is thickest at the bottom because water pressure grows with depth, P = P0 + ρgh



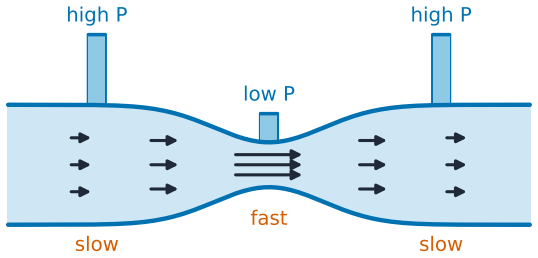
Upthrust arises because the pressure on the bottom of an object exceeds that on the top

- **A pressure difference can accelerate fluid.** In horizontal flow, the pressure force pushes from higher toward lower pressure. If the height changes, gravity also contributes; fluid can flow toward higher pressure as it falls.
- **Continuity** 连续性 (**conservation of mass** 质量守恒): for steady incompressible flow, the **volume flow rate** 体积流量 is constant, so $A_1v_1 = A_2v_2$. A narrower pipe forces **faster** flow.
- **Bernoulli's equation** 伯努利方程 (**conservation of energy** 能量守恒 per volume): for steady, incompressible, non-viscous flow along a **streamline** 流线,

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}.$$

Together they explain why fluid speeds up and its pressure drops where a level pipe narrows. Compare speeds and pressures along the same streamline at the same height; a height change also changes the gravitational energy term.

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$$A_1v_1 = A_2v_2 \quad \cdot \quad P + \tfrac{1}{2}\rho v^2 = \text{const} \ \text{(horizontal)}$$

Where the pipe narrows the fluid speeds up (continuity) and its pressure drops (Bernoulli)

Worked example. Water flows at 2.0 m/s through a pipe of cross-section 0.010 m^2, then enters a narrower section of 0.0040 m^2. By continuity the speed there is

$$v_2 = \frac{A_1v_1}{A_2} = \frac{0.010 \times 2.0}{0.0040} = 5.0 \text{ m/s}.$$

If the two pipe sections are at the same height, Bernoulli's equation puts this faster stream at **lower** pressure – the effect that lifts an aeroplane wing and pulls two passing ships together.

Why continuity holds. In a pipe that is full of an incompressible fluid, every kilogram that enters one end must leave the other: the fluid cannot pile up or thin out inside. In time Δt, a length $v\,\Delta t$ of fluid passes any point, a volume $Av\,\Delta t$. So the **volume flow rate** is Av, and the mass flow rate ρAv, and both are the same everywhere along the pipe.

Worked example (a fountain). Water leaves a nozzle of radius r_0 at angle θ_0 above the horizontal, and a droplet rises a height h_1. Find the volume flow rate.

- At the top, the vertical velocity is zero, so the vertical part of the launch velocity is $v_0 \sin\theta_0 = \sqrt{2gh_1}$.
- Launch speed: $v_0 = \dfrac{\sqrt{2gh_1}}{\sin\theta_0}$.
- Flow rate: $Q = Av_0 = \dfrac{\pi r_0^2\sqrt{2gh_1}}{\sin\theta_0}$.

If the nozzle is made **narrower** with the same flow rate and launch angle, the water must leave **faster** (Av is fixed and A is smaller), so it rises **higher**: $h_2 > h_1$. This is why putting your thumb over the end of a garden hose makes the water spray farther.

Return to the narrowing pipe example with Bernoulli's equation. The pipe is level, so the height terms cancel:

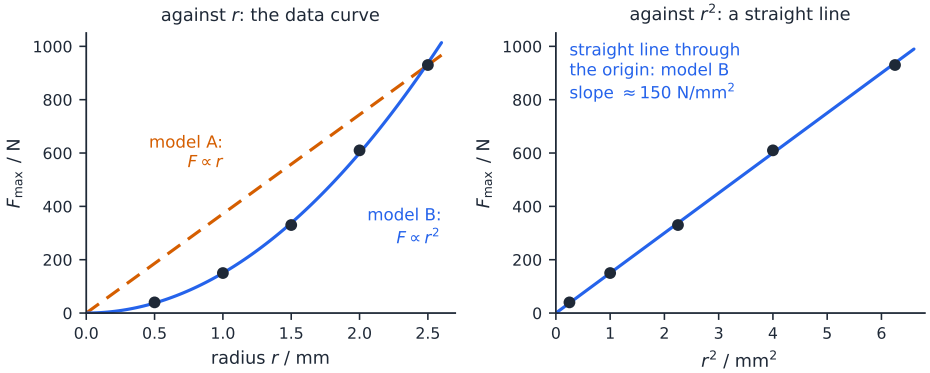
$$P_1 - P_2 = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \tfrac{1}{2}(1000)\left(5.0^2 - 2.0^2\right) = 1.05 \times 10^4 \text{ Pa}.$$

The pressure in the narrow section is about 10 kPa lower, roughly a tenth of an atmosphere.

> **Worked example (a draining tank).** Water leaves a small hole 1.8 m below the surface of an open tank. The surface and the jet are both at atmospheric pressure, and the surface barely moves, so Bernoulli's equation reduces to $\rho g h = \tfrac{1}{2}\rho v^2$:
>
> $$v = \sqrt{2gh} = \sqrt{2(9.8)(1.8)} = 5.9 \text{ m/s}.$$
>
> This is **Torricelli's result** 托里拆利定理: the water leaves as fast as if it had fallen freely from the surface.

Exam skill. Choose the right law by what changes. If the pipe changes *width*, start with continuity ($A_1 v_1 = A_2 v_2$) to get the speeds; if you then need a *pressure*, feed those speeds into Bernoulli. Watch the height term $\rho g y$ only when the pipe also changes level.

Testing a model with a straight-line graph. Many lab questions give two models and ask which one fits. A group measures the force $F_{\max}$ that breaks plastic rods of different radius r. Model A says $F_{\max} \propto r$; model B says $F_{\max} \propto r^2$ (proportional to the cross-sectional area πr^2).



Plot the data against the quantity each model says it is proportional to: the straight line through the origin picks the model

- Plot $F_{\max}$ against r: the points curve upward, so model A does not fit.
- Plot $F_{\max}$ against r^2: the points lie on a straight line **through the origin**, which is exactly what model B predicts.

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- In words: doubling the radius makes the rod about **four** times as strong, not twice.

To get good data, test several rods of each radius and average, measure each radius with calipers in a few places, and pull slowly with a force sensor so the reading at the moment of breaking is not missed.

Practice

Answer each before reading the answers below. Take $g = 9.8$ m/s^2 and $P_0 = 1.0 \times 10^5$ Pa.

1. Find the gauge pressure and the absolute pressure 25 m below the surface of sea water, density 1030 kg/m^3.
2. Ice of density 920 kg/m^3 floats in sea water of density 1030 kg/m^3. What fraction of the ice is under the surface?
3. A hydraulic press has pistons of area 2.0 cm^2 and 400 cm^2. What force can a 50 N push on the small piston produce?
4. Water flows through a pipe whose radius halves at a narrowing. By what factor does the speed change?
5. A student says: "Steel is denser than water, so a steel ship should sink." Correct the student.

Answers

1. Gauge: $(1030)(9.8)(25) = 2.5 \times 10^5$ Pa. Absolute: $1.0 \times 10^5 + 2.5 \times 10^5 = 3.5 \times 10^5$ Pa.
2. $920/1030 = 0.89$, so about 89% is under the surface.
3. $F_2 = F_1 \dfrac{A_2}{A_1} = 50 \times \dfrac{400}{2.0} = 1.0 \times 10^4$ N.
4. Halving the radius divides the area by 4, so by continuity the speed is 4 times larger.
5. What matters is the **average** density of the whole ship, including the air inside the hull. The hull pushes aside a large volume of water; the ship floats when the weight of that displaced water equals its own weight. A solid steel bolt cannot displace enough water, so it sinks.

Exam tips

- Pressure with depth is $P = P_0 + \rho g h$ – it depends on **depth only**, not the container's shape or width.
- **Buoyant force** = weight of fluid displaced ($\rho_{\text{fluid}}\, g V_{\text{disp}}$); a floating object displaces its own weight, so the fraction submerged is $\rho_{\text{object}}/\rho_{\text{fluid}}$.
- Compare densities to predict floating vs sinking; a floating object is in **equilibrium** 平衡 (buoyancy = weight), not weightless.

- Use **continuity** $A_1 v_1 = A_2 v_2$: a narrower pipe means faster flow.
- **Bernoulli**: along the same streamline at the same height, faster flow has lower pressure; include the height term when levels differ.

■ Common mistakes

- Using the volume of the whole object in $F_b = \rho g V$ for a floating object. Use the *submerged* volume, and the density of the *fluid*, not of the object.
- Mixing gauge and absolute pressure. $\rho g h$ alone is the gauge pressure; add P_0 for the absolute pressure.
- Making the pressure at a depth depend on the width or shape of the container. Only the depth below the surface (and the fluid's density) matter.
- Writing $A_1 v_1 = A_2 v_2$ with radii instead of areas. Area goes as r^2, so halving the radius makes the flow 4 times faster.
- Saying that faster-moving fluid has higher pressure. In Bernoulli's equation, where the speed rises (same height), the pressure falls.
- Saying a hydraulic lift multiplies energy. It multiplies force; the small piston moves further, so the work in equals the work out.
- Saying a denser fluid pushes a submerged block up with the same force. The buoyant force is $\rho V g$, so it is larger in a denser fluid, while the weight stays the same.
- Thinking a narrower nozzle gives less water. With the same flow rate, a smaller area means a faster jet, not less water per second.
- Giving pressure a direction. Pressure is a scalar; the force it makes on a surface is perpendicular to that surface.

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