

Kinematics

AP Physics 1

Scalars and Vectors in One Dimension



A car speedometer: speed is the magnitude of velocity — a scalar rate of motion

Kinematics 运动学 describes *how* objects move, without asking why. First, two kinds of quantity:

- A **scalar** 标量 has only size (**magnitude** 大小): distance 距离, speed, time, mass.
- A **vector** 矢量 has magnitude **and** direction: displacement, velocity, acceleration, force.

The difference matters. **Distance** is the total path length travelled – a scalar that only grows. **Displacement** is the straight-line change in position, with a direction. Walk 3 m east then 1 m back west: the distance is 4 m, but the displacement is only 2 m east.

In one dimension, direction is just a **sign** (+ or –) along a chosen axis. Choosing the positive direction first is essential – every vector's sign depends on it. A velocity of –5 m/s does not mean "slow"; it means 5 m/s in the negative direction.

Course Companion

AP Physics 1

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exercises.lol

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Displacement, Velocity, and Acceleration



A high-speed train: kinematics describes displacement, velocity and acceleration over time

Three linked vectors describe motion along a line:

- **Displacement** 位移 Δx is the change in position – a vector from start to end (not the total path length, which is distance).
- **Velocity** 速度 is the rate of change of position, $v = \frac{\Delta x}{\Delta t}$. Its sign gives direction; its magnitude is **speed** 速率.
- **Acceleration** 加速度 is the rate of change of velocity, $a = \frac{\Delta v}{\Delta t}$.

Be careful to separate **average velocity** 平均速度 (total displacement over total time) from **instantaneous velocity** 瞬时速度 (the velocity at one instant, the slope of the position–time graph at that point). They are equal only when the velocity is constant.

An object speeds up when v and a have the **same** sign, and slows down (**deceleration** 减速) when they have **opposite** signs. Note that a negative acceleration does not always mean slowing down – a ball falling faster and faster has negative velocity *and* negative acceleration.

For **constant** acceleration, the four kinematic equations (often called SUVAT) apply:

$$v = v_0 + at, \quad \Delta x = v_0 t + \frac{1}{2}at^2, \quad v^2 = v_0^2 + 2a \Delta x, \quad \Delta x = \frac{1}{2}(v_0 + v)t.$$

Pick the equation that contains the three quantities you know plus the one you want, so only one unknown is left. They apply **only** while a is constant.

Worked example. A car starts from rest and accelerates uniformly at 2.0 m/s^2 for 6.0 s . Find its final velocity and the distance it travels.

List what you know: $v_0 = 0$, $a = 2.0 \text{ m/s}^2$, $t = 6.0 \text{ s}$.

$$v = v_0 + at = 0 + 2.0 \times 6.0 = 12 \text{ m/s},$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.0 \times 6.0^2 = 36 \text{ m}.$$

Worked example (free fall). A ball is thrown straight up at 15 m/s . Taking $g = 9.8 \text{ m/s}^2$ and up as positive, how high does it rise, and how long is it in the air before returning to the thrower's hand?

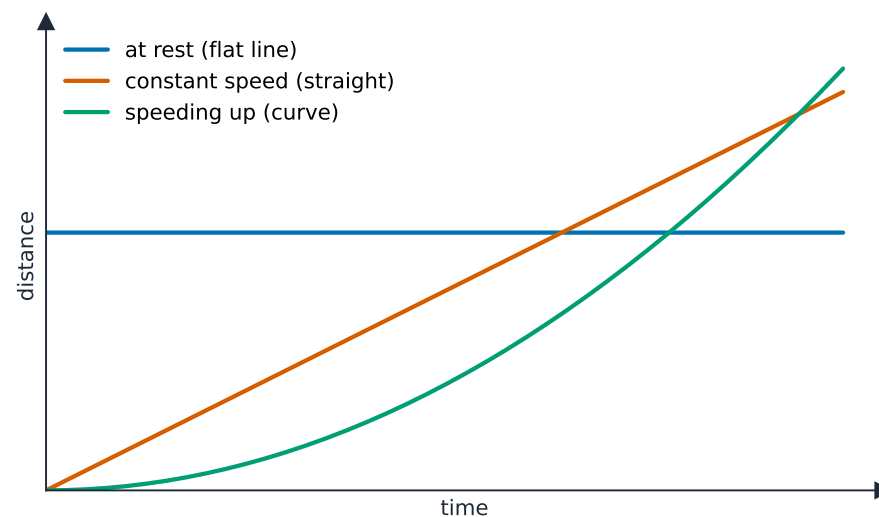
At the highest point the velocity is momentarily zero, and $a = -g = -9.8 \text{ m/s}^2$ throughout (this is **free fall** 自由落体, ignoring air resistance 空气阻力):

$$v^2 = v_0^2 + 2a \Delta x \Rightarrow 0 = 15^2 + 2(-9.8)\Delta x \Rightarrow \Delta x = \frac{225}{19.6} = 11.5 \text{ m}.$$

Time to the top: $0 = 15 - 9.8t \Rightarrow t = 1.53 \text{ s}$. By symmetry the fall takes the same time, so the total is 3.1 s .

Representing Motion

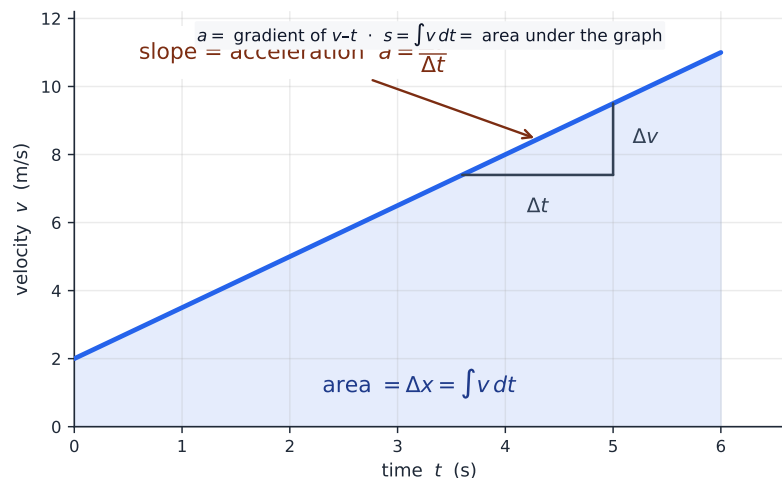
The same motion appears as a **description**, a **graph**, a **table**, or an **equation**, and you should move between them:



Reading a distance-time graph: flat means at rest, a straight slope means constant speed

- On a **position–time** graph, the **slope** is velocity (steeper = faster; a curve = changing velocity).
- On a **velocity–time** graph, the slope is acceleration, and the **area** under the line is displacement.

Reading slopes and areas off graphs is a core exam skill. To get displacement from a velocity–time graph, split the area into triangles and rectangles and add them up; area **below** the time axis counts as **negative** displacement (motion the other way).



On a velocity-time graph the slope is the acceleration and the shaded area is the displacement

Worked example. A cyclist speeds up uniformly from rest to 8.0 m/s in 4.0 s, then holds 8.0 m/s for 6.0 s. Find the total distance from the velocity–time graph.

The area is a triangle followed by a rectangle:

$$\Delta x = \underbrace{\frac{1}{2} \times 4.0 \times 8.0}_{\text{triangle}} + \underbrace{6.0 \times 8.0}_{\text{rectangle}} = 16 + 48 = 64 \text{ m.}$$

Reference Frames and Relative Motion

All motion is measured against a **reference frame** 参考系. Velocities measured in different frames differ, and you combine them by vector addition. The velocity of A relative to C is

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}.$$

A person walking on a moving train has one velocity relative to the train and another relative to the ground – this is **relative motion** 相对运动. A useful shortcut: the velocity of A relative to B is $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$ (subtract B's velocity).

A frame that is not accelerating is an **inertial reference frame** 惯性参考系: one in which a free object (no net force) obeys Newton's first law, staying at rest or moving at constant velocity. A frame that accelerates – a braking car – is **non-inertial**, where objects seem to speed up with no force acting on them.

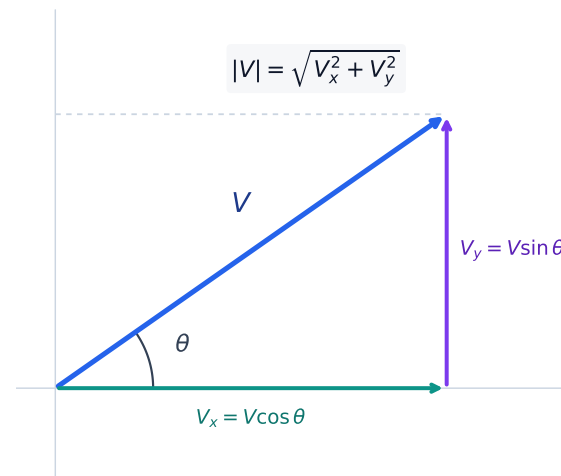
Worked example. A boat points straight across a river and moves at 3.0 m/s relative to the water. The current flows at 4.0 m/s along the river. Find the boat's speed and direction relative to the bank.

The two velocities are perpendicular, so add them as a right triangle:

$$v = \sqrt{3.0^2 + 4.0^2} = 5.0 \text{ m/s}, \quad \theta = \tan^{-1} \frac{4.0}{3.0} = 53^\circ \text{ downstream from straight across.}$$

Vectors and Motion in Two Dimensions

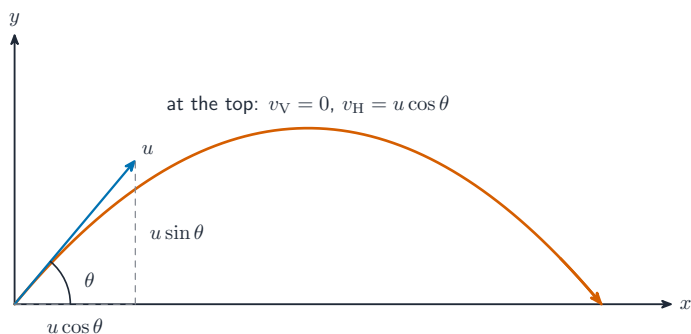
In two dimensions, resolve each vector into **components** 分量 along perpendicular axes (x and y), handle each axis separately, then recombine. A velocity v at angle θ to the horizontal has components $v_x = v \cos \theta$ and $v_y = v \sin \theta$.



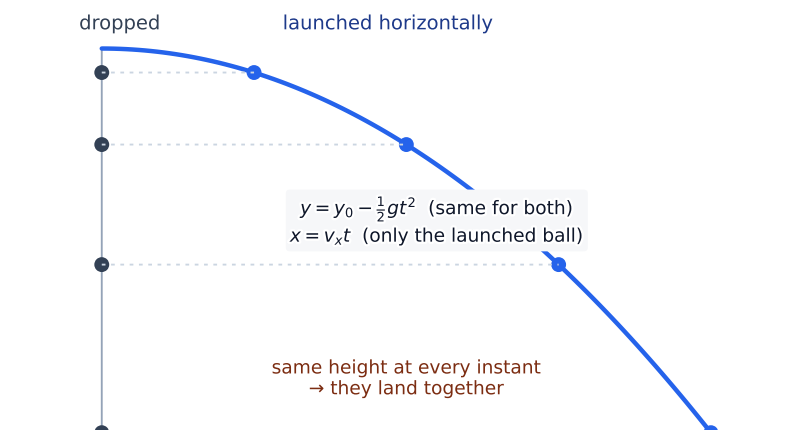
A velocity vector resolved into its horizontal and vertical components

For **projectile motion** 抛体运动 (an object moving under gravity alone): the horizontal and vertical motions are **independent**. Horizontally, velocity is constant

($a_x = 0$); vertically, acceleration is $-g$ (down). The two motions share only the **time**. So a projectile's path (its **trajectory** 轨迹) is a parabola, and you solve it as two one-dimensional problems joined by t .



A projectile launched at an angle: the horizontal and vertical motions are independent



A dropped ball and a horizontally launched ball fall together – the vertical motions are identical

Worked example. A ball is kicked at 20 m/s, 30° above the horizontal. Taking $g = 9.8 \text{ m/s}^2$, find the time of flight, the maximum height, and the horizontal **range** 射程 (assume it lands at launch height).

Split the launch velocity into components:

$$v_{0x} = 20 \cos 30^\circ = 17.3 \text{ m/s}, \quad v_{0y} = 20 \sin 30^\circ = 10 \text{ m/s}.$$

Vertical motion sets the time. At the top $v_y = 0$, so $0 = 10 - 9.8t \Rightarrow t_{\text{up}} = 1.02 \text{ s}$, and the total flight is $2t_{\text{up}} = 2.0 \text{ s}$. The maximum height is

$$\Delta y = \frac{v_{0y}^2}{2g} = \frac{10^2}{19.6} = 5.1 \text{ m}.$$

Horizontal motion runs at constant v_{0x} for the whole flight, so the range is

$$R = v_{0x} \times t_{\text{flight}} = 17.3 \times 2.0 = 35 \text{ m}.$$

A common trap: at the top of the flight the vertical velocity is zero, but the ball is **not** at rest – its horizontal velocity v_{0x} never changes. The speed at the top equals $v_{0x} = 17.3 \text{ m/s}$.

Exam tips

- Choose the right **kinematic equation** by listing the three quantities you know plus the one you want, so only one unknown remains; the SUVAT equations apply **only** while acceleration is constant.
- Fix a **positive direction** first — every displacement, velocity, and acceleration then carries a sign; a negative velocity means “moving the other way”, not “slow”.
- Treat a projectile as **two independent 1-D problems** sharing only the time t : constant velocity horizontally, $a = -g$ vertically. At the top $v_y = 0$ but v_x is unchanged.
- On a **velocity–time graph** the gradient is the acceleration and the **area** is the displacement (area below the axis is negative).
- Distinguish **distance** (scalar, total path) from **displacement** (vector, start-to-end), and **speed** from **velocity**.

Force and Translational Dynamics

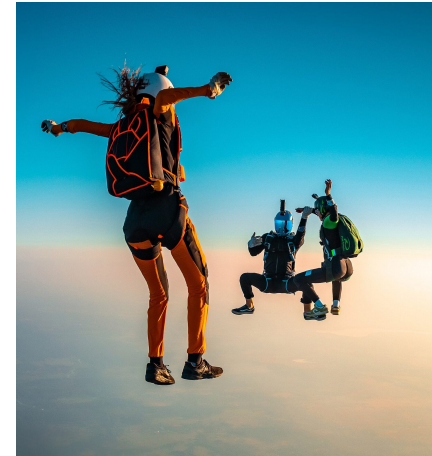
AP Physics 1

Systems and Center of Mass

A **system** 系统 is the object or group of objects you choose to analyze. A system can be treated as a single point at its **center of mass** 质心 – the average position of its mass. External forces change the motion of the center of mass; internal forces (between parts of the system) do not.

This is why a wrench spinning across a table still has its center of mass move in a straight line: the spinning is internal, and only the (near-zero) external force matters for the center of mass. For two masses m_1, m_2 on a line at positions x_1, x_2 , the center of mass sits at $x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ – always closer to the heavier mass.

Forces and Free-Body Diagrams

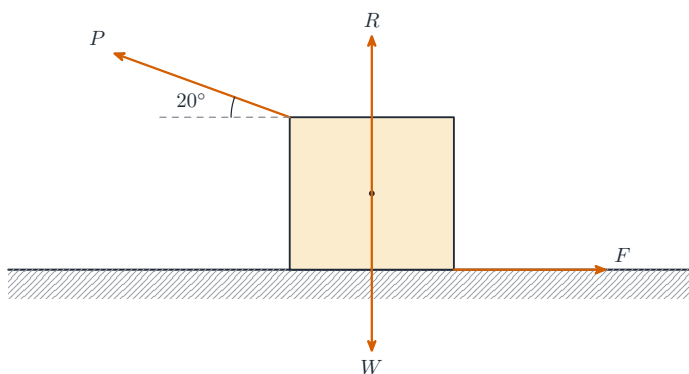


A skydiver: free-body diagrams balance gravity and air resistance at terminal speed



A rocket launch: net force changes momentum — $F_{\text{net}} = ma$ for translational dynamics

A **force** 力 is a push or pull – a vector, measured in newtons (N). A **free-body diagram** 受力图 shows one object as a dot with arrows for **every** force acting **on** it (weight 重力, normal, tension 张力, friction, applied), each labelled and pointing the right way. Draw it before any dynamics problem; it is where most marks are won or lost.

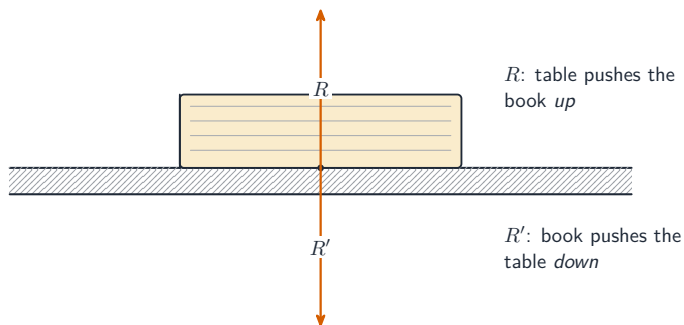


A free-body diagram shows every force acting on one object

Two rules keep free-body diagrams honest: draw only forces acting **on** the chosen object (not forces it exerts on other things), and draw only **real, physical** forces (a rope, a surface, gravity, a hand) – never an “*ma*” arrow, which is the *result* of the forces, not a force itself.

Newton’s Third Law

Newton’s third law 牛顿第三定律: if object A pushes on object B, then B pushes back on A with a force **equal in size and opposite in direction**. These two forces act on **different** objects, so they never cancel each other. Identify third-law pairs by the “A on B / B on A” wording.



A Newton’s third-law pair: equal and opposite forces on two different objects

A classic trap: the weight of a book and the normal force from the table are **not** a

third-law pair – they act on the **same** object (the book). The partner of the book’s weight is the pull the book exerts on the Earth; the partner of the table’s push is the push the book makes on the table.

Newton’s First Law

Newton’s first law 牛顿第一定律 (the law of **inertia** 惯性): an object’s velocity stays constant unless a **net force** 合力 acts on it. So zero net force means constant velocity (including rest) – the object is in **translational equilibrium** 平衡. Inertia is the tendency to resist changes in motion, measured by mass.

Worked example. A 1200 kg car cruises at a steady 25 m/s on a level road. What is the net force on it? Because the velocity is constant, the acceleration is zero, so by the first law the **net** force is zero – the forward drive force exactly balances drag and friction. “Steady speed” always means balanced forces.

Newton’s Second Law

Newton’s second law 牛顿第二定律 relates net force to acceleration:

$$\vec{a} = \frac{\sum \vec{F}}{m}, \quad \text{i.e.} \quad \sum \vec{F} = m\vec{a}.$$

Apply it **one axis at a time**: add the force components along each axis and set the sum equal to ma for that axis. Acceleration points the same way as the net force.

Worked example. A 4.0 kg box is pulled along the floor by a horizontal force of 18 N. Friction on the box is 6.0 N. Find its acceleration. Along the direction of motion the net force is $18 - 6.0 = 12$ N, so

$$a = \frac{\sum F}{m} = \frac{12}{4.0} = 3.0 \text{ m/s}^2.$$

Worked example (incline 斜面). A block of mass m slides down a frictionless ramp tilted at angle θ . Find its acceleration. Resolve gravity into components along and perpendicular to the ramp; only the along-ramp part, $mg \sin \theta$, drives the motion, so

$$a = \frac{mg \sin \theta}{m} = g \sin \theta.$$

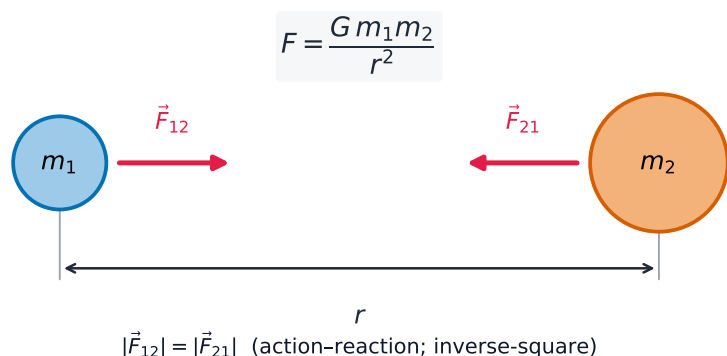
The steeper the ramp, the larger $\sin \theta$ and the faster it accelerates; at $\theta = 90^\circ$ it is free fall.

Gravitational Force

Near a planet's surface, the **gravitational force** (weight) is $F_g = mg$, directed down, where g is the **gravitational field strength** 重力场强度. More generally, **Newton's law of gravitation** 万有引力定律 gives the attraction between any two masses:

$$F_g = \frac{Gm_1m_2}{r^2},$$

directed along the line joining them, weaker as the distance r grows (an inverse-square law). Doubling the separation quarters the force.



Two masses attract each other with equal, opposite, inverse-square forces along the line joining them

Worked example. A 2.0 kg object weighs 19.6 N on Earth ($g = 9.8 \text{ m/s}^2$). On the Moon $g_{\text{Moon}} = 1.6 \text{ m/s}^2$. Its mass is unchanged (2.0 kg), but its weight becomes $F_g = mg = 2.0 \times 1.6 = 3.2 \text{ N}$. Mass measures inertia; weight is a force that depends on where you are.

Mass actually plays two distinct roles. **Inertial mass** 惯性质量 sets how strongly an object resists acceleration ($F = ma$); **gravitational mass** 引力质量 sets how strongly it attracts other masses ($F = Gm_1m_2/r^2$). Experiments confirm the two are **equal** – which is exactly why every object, heavy or light, falls with the same g .

Kinetic and Static Friction

Friction 摩擦力 acts along a surface, opposing relative sliding (or the tendency to slide):

- **Kinetic friction** 动摩擦 (while sliding): $f_k = \mu_k N$.
- **Static friction** 静摩擦 (while not yet sliding): $f_s \leq \mu_s N$ – it adjusts up to a maximum to prevent motion.

Here N is the **normal force** 法向力 (surface push, perpendicular to the surface) and μ is the **coefficient of friction** 摩擦系数.

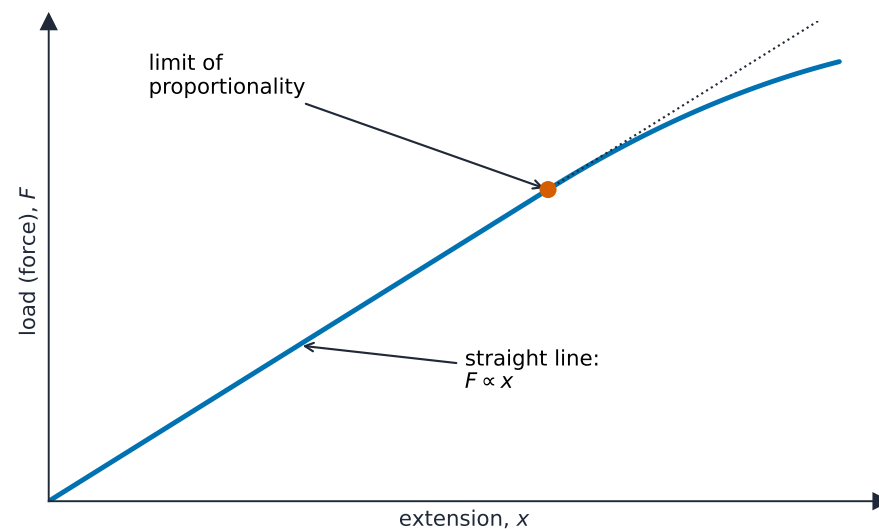
Worked example. A 5.0 kg crate sits on a level floor with $\mu_s = 0.40$. Will a horizontal push of 15 N move it? On a level floor $N = mg = 5.0 \times 9.8 = 49 \text{ N}$, so the largest static friction is $f_{s,\text{max}} = \mu_s N = 0.40 \times 49 = 19.6 \text{ N}$. The 15 N push is smaller than 19.6 N, so friction rises to match it and the crate stays still.

Spring Forces

An ideal spring exerts a **restoring force** 回复力 proportional to its stretch or compression – **Hooke's law** 胡克定律:

$$F_s = -kx,$$

where k is the **spring constant** 弹簧常数 (stiffness) and x is the displacement from the spring's natural length. The minus sign means the force points back toward equilibrium.



Hooke's law: extension is proportional to load up to the limit of proportionality

Worked example. A spring with $k = 200 \text{ N/m}$ hangs vertically and a 0.50 kg mass

is hung on it. How far does it stretch at rest? At rest the spring force balances the weight, $kx = mg$, so

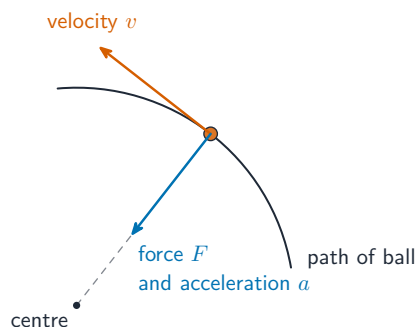
$$x = \frac{mg}{k} = \frac{0.50 \times 9.8}{200} = 0.025 \text{ m} = 2.5 \text{ cm}.$$

Circular Motion

An object moving in a circle at constant speed still **accelerates**, because its velocity direction keeps changing. This **centripetal acceleration** 向心加速度 points toward the center:

$$a_c = \frac{v^2}{r}.$$

It is produced by a **net inward (centripetal) force** 向心力 $F_c = \frac{mv^2}{r}$ – supplied by whatever real force points inward (tension, gravity, friction, normal). The time for one full revolution is the **period** 周期 T , and the number of revolutions per second is the **frequency** 频率 f ; they are reciprocals, $f = 1/T$, and the same definitions describe any rotation or oscillation. There is no separate outward force; “centrifugal” is only an apparent effect.



The velocity points along the tangent; the centripetal force and acceleration point to the centre

Worked example. A 0.30 kg ball on a string is whirled in a horizontal circle of radius 0.80 m at 4.0 m/s. Find the tension in the string. The tension supplies the whole centripetal force:

$$T = \frac{mv^2}{r} = \frac{0.30 \times 4.0^2}{0.80} = 6.0 \text{ N}.$$

If the string can take at most 6.0 N, this is the fastest the ball can go at that radius – spin any faster and the string breaks.

Exam tips

- Always draw a **free-body diagram** first: only real forces **on** the chosen object (weight, normal, tension, friction, applied) — never an “ ma ” arrow.
- Apply $\sum F = ma$ **one axis at a time**; on an incline resolve gravity into $mg \sin \theta$ (along) and $mg \cos \theta$ (perpendicular).
- A **Newton’s third-law pair** acts on **two different objects** — a book’s weight and the table’s normal force are *not* a pair (both act on the book).
- **Static friction** adjusts up to $\mu_s N$ (use it to test whether motion starts); once sliding, use **kinetic** friction $f_k = \mu_k N$.
- Circular motion needs a **net inward (centripetal) force** mv^2/r supplied by a real force — there is no outward “centrifugal” force.

Work, Energy, and Power

AP Physics 1

Translational Kinetic Energy

Energy 能量 is the capacity to do work, measured in **joules** 焦耳 (J). A moving object has **kinetic energy** 动能:

$$K = \frac{1}{2}mv^2.$$

It depends on the **square** of the speed, so doubling the speed **quadruples** the kinetic energy. Kinetic energy is a scalar and is never negative.

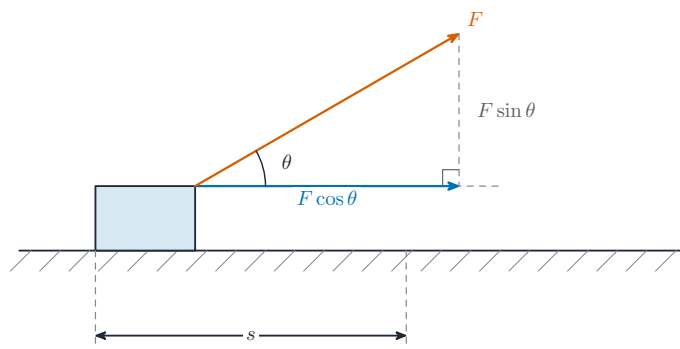
Worked example. A 1500 kg car travels at 20 m/s. Its kinetic energy is $K = \frac{1}{2} \times 1500 \times 20^2 = 3.0 \times 10^5 \text{ J} = 300 \text{ kJ}$. If it speeds up to 40 m/s (double), the kinetic energy becomes $4 \times$ larger, 1200 kJ – which is why stopping distance grows so fast with speed.

Work

Work 功 is energy transferred by a force acting over a displacement:

$$W = F d \cos \theta,$$

where θ is the angle between the force and the displacement. Work is **positive** when the force has a component along the motion (adds energy), **negative** when it opposes the motion (removes energy), and **zero** when the force is perpendicular. On a force–position graph, work is the **area** under the curve. The **work–energy theorem** 动能定理 states that the net work equals the change in kinetic energy: $W_{\text{net}} = \Delta K$.



Only the force component along the displacement does work

Worked example. A 2.0 kg block moving at 3.0 m/s on a frictionless floor is pushed by a 5.0 N force over 4.0 m in the direction of motion. Find its final speed. The net work is $W = Fd = 5.0 \times 4.0 = 20 \text{ J}$, and by the work–energy theorem $W = \frac{1}{2}m(v^2 - v_0^2)$:

$$20 = \frac{1}{2} \times 2.0 \times (v^2 - 3.0^2) \Rightarrow v^2 = 29 \Rightarrow v = 5.4 \text{ m/s}.$$

Potential Energy

Potential energy 势能 is stored energy that depends on position or configuration:

- **Gravitational potential energy** 重力势能 near the surface: $U_g = mgh$ (height h above a reference level).
- **Elastic potential energy** 弹性势能 in a spring: $U_s = \frac{1}{2}kx^2$.

Potential energy is defined only for **conservative forces** 保守力 (gravity, springs), for which the stored energy depends on position, not path. Only *changes* in potential energy matter, so you may put the zero level wherever is convenient.

The $U_g = mgh$ form only holds near a surface where g is roughly constant. The **general** form, for two spherical masses a distance r apart, is

$$U_g = -\frac{Gm_1m_2}{r}.$$

It is **negative** and defined to be **zero at infinite separation**, so gravitational PE rises toward zero as the masses move apart. mgh is simply its near-surface approximation. (This is the form you need for satellites and escape speed.)

Conservation of Energy

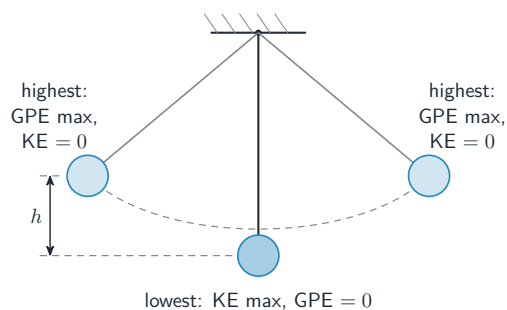


A roller coaster trades energy back and forth: it is highest (most potential energy) at the top and fastest (most kinetic energy) at the bottom

The **total mechanical energy** 机械能 is $E = K + U$. When only conservative forces do work, mechanical energy is **conserved** 守恒:

$$K_1 + U_1 = K_2 + U_2.$$

When friction or other non-conservative forces act, they transfer mechanical energy to **thermal energy** 热能; then the general statement is that total energy (including thermal) is conserved. Energy bar charts are a good way to track where the energy goes.



A swinging pendulum trades gravitational potential energy for kinetic energy and back

Worked example. A ball is released from rest at the top of a frictionless ramp 5.0 m high. Find its speed at the bottom. All the gravitational potential energy becomes kinetic energy:

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5.0} = 9.9 \text{ m/s}.$$

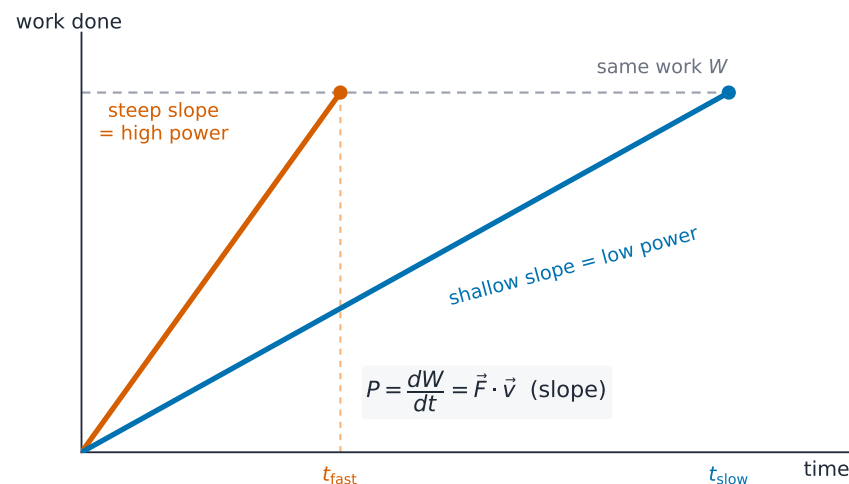
The mass cancels, so every object reaches the same speed – exactly the free-fall result, now got from energy. If instead 30 J were lost to friction, you would subtract it: $mgh - 30 = \frac{1}{2}mv^2$.

Power

Power 功率 is the **rate** of doing work or transferring energy, measured in **watts** 瓦特 (W):

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}, \quad \text{and instantaneously} \quad P = Fv.$$

So the same job done faster requires more power. On an energy–time graph, power is the slope.



Power is the slope of the work–time graph: the same work in less time means more power

Worked example. A motor lifts a 50 kg load at a steady 2.0 m/s. Because it moves at constant speed, the lifting force equals the weight, so

$$P = Fv = mgv = 50 \times 9.8 \times 2.0 = 980 \text{ W}.$$

Real machines waste some energy, so we quote **efficiency** 效率 – useful output power divided by total input power. If this motor draws 1400 W of electrical power to deliver 980 W of useful lifting, its efficiency is $980/1400 = 0.70$, or 70%; the other 30% becomes heat and sound.



A waterfall: power is how quickly energy is transferred — the same drop in less time is more power

Exam tips

- Use $W = Fd \cos \theta$: work is zero when the force is perpendicular to the motion, and negative when it opposes it.
- Reach for the **work–energy theorem** ($W_{\text{net}} = \Delta K$) or **energy conservation** ($K_1 + U_1 = K_2 + U_2$) instead of forces whenever the path is complicated.
- When friction acts, mechanical energy is **not** conserved — subtract the energy lost to heat.
- Remember $K \propto v^2$: doubling the speed **quadruples** the kinetic energy (and the stopping distance).
- Use $P = Fv$ for power at a steady speed; at constant velocity the net force is zero but the power is **not**.

Linear Momentum

AP Physics 1

Linear Momentum

Linear momentum 动量 is mass times velocity – a vector pointing the same way as the velocity:

$$\vec{p} = m\vec{v}.$$

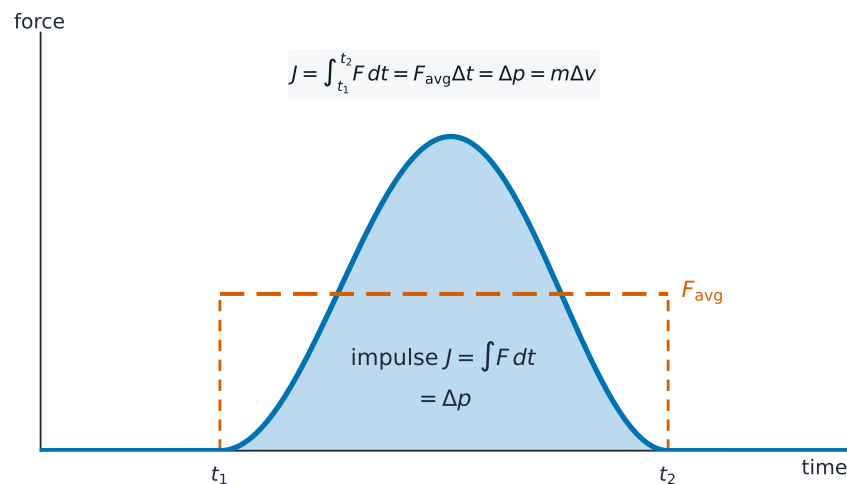
It measures "how hard it is to stop" a moving object. A heavy slow truck and a light fast ball can have the same momentum. Its unit is kg m/s, the same as N s.

Change in Momentum and Impulse

A net force acting over time changes momentum. The **impulse** 冲量 delivered is

$$\vec{J} = \vec{F} \Delta t = \Delta \vec{p}.$$

This is the **impulse–momentum theorem**: impulse equals the change in momentum. On a force–time graph, impulse is the **area** under the curve. It explains why airbags and follow-through help – spreading the same momentum change over a longer time reduces the force.



Impulse is the area under the force–time curve, equal to the average force times the contact time

Worked example. A 0.15 kg ball hits a wall at 20 m/s and bounces straight back at 15 m/s. The contact lasts 0.020 s. Find the average force on the ball. Take the rebound direction as positive, so $u = -20$ m/s and $v = +15$ m/s:

$$\Delta p = m(v - u) = 0.15(15 - (-20)) = 5.25 \text{ kg m/s}, \quad F = \frac{\Delta p}{\Delta t} = \frac{5.25}{0.020} = 260 \text{ N}.$$

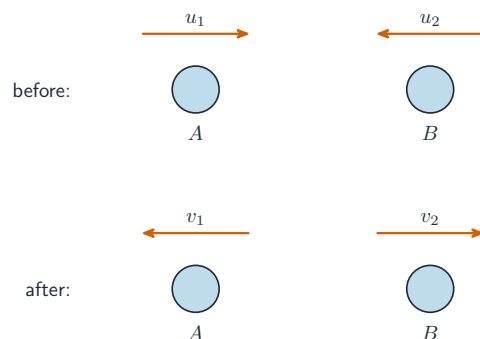
The sign work matters: forgetting that the velocity **reverses** is the most common mistake here.

Conservation of Linear Momentum

If the **net external force** on a system is zero, its total momentum is **conserved** 守恒:

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}.$$

Internal forces (like the push between two colliding carts) come in third-law pairs and cancel, so they cannot change the total momentum. This is the key tool for **collisions** 碰撞 and explosions – apply it separately in the x and y directions.



A head-on collision: total momentum before equals total momentum after

Worked example (recoil 反冲). A 60 kg skater, initially at rest on frictionless ice, throws a 2.0 kg ball at 8.0 m/s. Find her recoil speed. The total momentum starts at zero and stays zero:

$$0 = (60)v + (2.0)(8.0) \Rightarrow v = -\frac{16}{60} = -0.27 \text{ m/s},$$

so she moves at 0.27 m/s in the opposite direction to the ball – the principle behind rockets and guns.

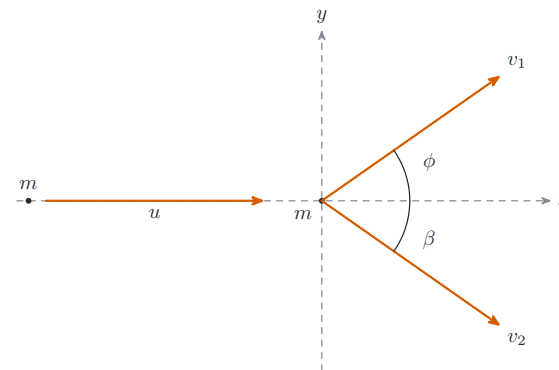
A whole collection of objects can be described by a single **center-of-mass velocity** 质心速度:

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{\sum m_i} = \frac{\vec{p}_{\text{total}}}{M_{\text{total}}}.$$

Since total momentum is conserved when no net external force acts, $\vec{v}_{\text{cm}}$ then stays **constant** – a v_{cm} -versus-time graph is flat unless an outside force acts. An internal collision or explosion never changes it, however violently the parts fly apart.

Elastic and Inelastic Collisions

Momentum is conserved in **every** collision (with no external force). Kinetic energy is not:



A glancing collision, resolved along two perpendicular axes

- In an **elastic collision** 弹性碰撞, kinetic energy is **also** conserved (objects bounce apart cleanly).
- In an **inelastic collision** 非弹性碰撞, some kinetic energy becomes heat or deformation. In a **perfectly inelastic** collision the objects **stick together** and move with one common velocity afterward.

Strategy: always write momentum conservation; add energy conservation only if the collision is stated to be elastic. One elastic fact worth memorising: in a 1D elastic collision between **equal masses**, the two objects simply *swap velocities* (a moving ball striking an identical stationary one stops dead, and the target flies off at the incoming speed).

Worked example. A 1000 kg car moving at 20 m/s runs into a stationary 1500 kg car and they lock together. Find their common speed, and the kinetic energy lost.

Momentum conservation gives

$$1000 \times 20 = (1000 + 1500) v \Rightarrow v = \frac{20000}{2500} = 8.0 \text{ m/s.}$$

Kinetic energy before is $\frac{1}{2}(1000)(20^2) = 2.0 \times 10^5 \text{ J}$; after is $\frac{1}{2}(2500)(8.0^2) = 8.0 \times 10^4 \text{ J}$. So $1.2 \times 10^5 \text{ J}$ (about 60%) is lost to crumpling and heat – momentum is still conserved, but kinetic energy is not.



A Newton's cradle shows momentum and kinetic energy passing through a line of balls in a near-elastic collision

Exam tips

- Momentum is a **vector** — assign $+/-$ signs before adding; a ball that **rebounds** reverses its velocity, giving a large Δp .
- Momentum is conserved in **every** collision (no external force); kinetic energy is conserved **only** if the collision is stated to be **elastic**.
- In a **perfectly inelastic** collision the objects stick and move with one common velocity.
- Impulse $= F \Delta t = \Delta p$ = the **area** under a force–time graph; spreading a collision over a longer time reduces the force (airbags, bending knees).
- For recoil/explosions, set the total momentum equal before and after (often zero before).

Torque and Rotational Dynamics

AP Physics 1

Rotational Kinematics

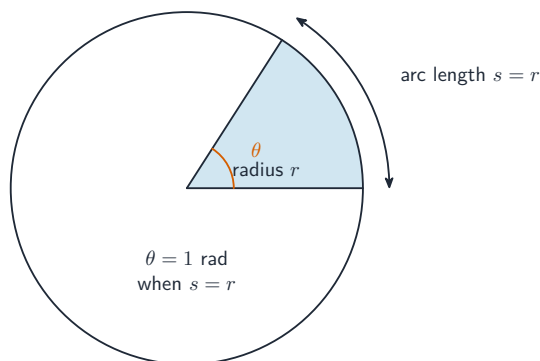


A crash-test collision: large forces and torques produce rapid changes in linear and angular motion



A spring under load: Hooke's law links force and extension — useful context for energy in rotating systems

Rotation is described by angular quantities that mirror the linear ones:



One radian is the angle whose arc length equals the radius

- **angular displacement** 角位移 θ (in **radians** 弧度),
- **angular velocity** 角速度 $\omega = \frac{\Delta\theta}{\Delta t}$,
- **angular acceleration** 角加速度 $\alpha = \frac{\Delta\omega}{\Delta t}$.

For constant α , the rotational kinematic equations have the same form as the linear ones, with θ, ω, α replacing x, v, a : $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, and $\omega^2 = \omega_0^2 + 2\alpha\theta$.

Worked example. A wheel starts from rest and speeds up uniformly to 30 rad/s in 6.0 s. Find its angular acceleration and the total angle turned:

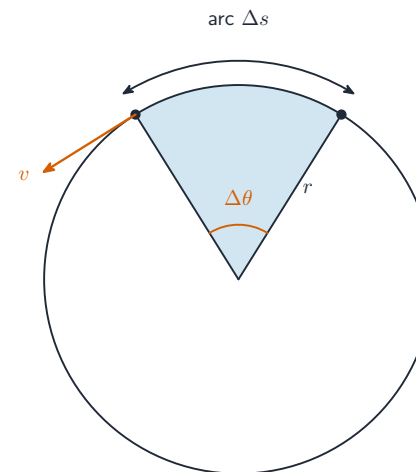
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30}{6.0} = 5.0 \text{ rad/s}^2, \quad \theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 5.0 \times 6.0^2 = 90 \text{ rad}.$$

Connecting Linear and Rotational Motion

A point at radius r from the axis has linear quantities tied to the angular ones:

$$s = r\theta, \quad v = r\omega, \quad a_t = r\alpha.$$

Points farther from the axis move faster. This link lets you switch between “how fast the wheel spins” and “how fast a point on its rim moves.”



As the radius turns through an angle, a point moves along an arc at speed v

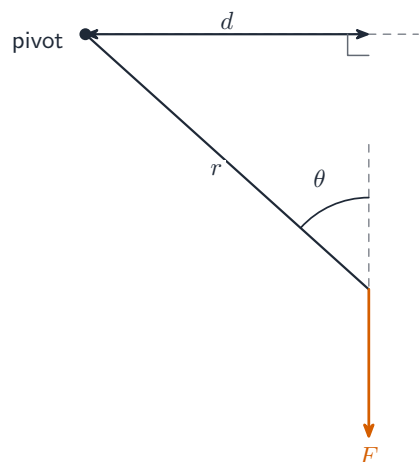
Worked example. A bicycle wheel of radius 0.35 m spins at 12 rad/s. A point on the rim (and so the bike) moves at $v = r\omega = 0.35 \times 12 = 4.2$ m/s. A point halfway to the axis moves at half that speed.

Torque

Torque 力矩 is the rotational effect of a force – how effectively it turns an object about an axis:

$$\tau = rF \sin \theta = F \cdot r_{\perp},$$

where $r_{\perp}$ is the **moment arm** 力臂 (the perpendicular distance from the axis to the force’s line of action). A force applied farther out, or more perpendicular, produces more torque. Torque has a sign (clockwise vs counterclockwise).



The moment of a force depends on the perpendicular distance from the pivot

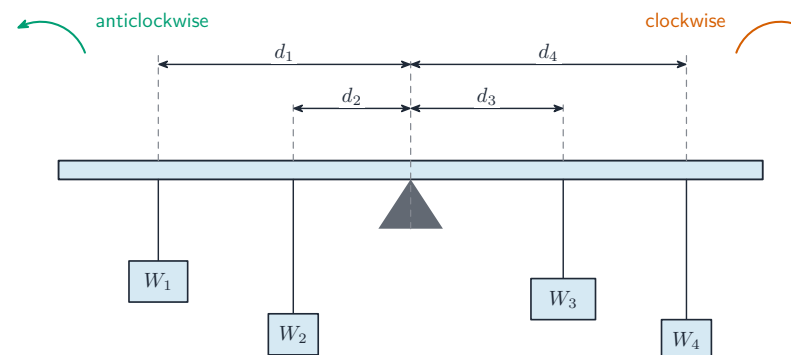
Worked example. You push with 20 N at the end of a 0.30 m wrench. Perpendicular to the wrench the torque is $\tau = rF = 0.30 \times 20 = 6.0 \text{ N m}$. If you push at 60° to the wrench instead, only the perpendicular part counts: $\tau = rF \sin 60^\circ = 0.30 \times 20 \times 0.87 = 5.2 \text{ N m}$ – which is why you push at a right angle for the most turning effect.

Rotational Inertia

Rotational inertia 转动惯量 (moment of inertia) I measures how hard it is to change an object's rotation – the rotational version of mass. It depends on both the mass **and how far** that mass sits from the axis: mass spread farther out gives a larger I . For a point mass, $I = mr^2$; for extended bodies, standard formulas are provided (a hoop is mR^2 , a solid disk $\frac{1}{2}mR^2$). This is why a figure skater spins faster when she pulls her arms in – she reduces I .

Rotational Equilibrium

An object is in **rotational equilibrium** 转动平衡 when the **net torque** is zero, so its angular velocity stays constant. For a balanced (static) object, both the net force **and** the net torque are zero. Choosing the axis at an unknown force's location removes it from the torque equation – a useful trick for beam and ladder problems.



At balance the clockwise and anticlockwise moments about the pivot are equal

Worked example. A 30 kg child sits 2.0 m from the pivot of a seesaw. Where must a 40 kg child sit on the other side to balance it? Set the clockwise torque equal to the anticlockwise torque (the g 's cancel):

$$30 \times 2.0 = 40 \times d \Rightarrow d = \frac{60}{40} = 1.5 \text{ m}.$$

The heavier child sits closer to the pivot – less distance, same torque.

Newton's Second Law in Rotational Form

Net torque produces angular acceleration, in direct analogy with $F = ma$:

$$\sum \tau = I\alpha.$$

So a larger net torque, or a smaller rotational inertia, gives a larger angular acceleration. Solve rotation problems just like translation problems, with $\tau \leftrightarrow F$, $I \leftrightarrow m$, and $\alpha \leftrightarrow a$.

Worked example. A net torque of 12 N m acts on a wheel with rotational inertia $I = 3.0 \text{ kg m}^2$. Its angular acceleration is $\alpha = \tau/I = 12/3.0 = 4.0 \text{ rad/s}^2$ – the exact rotational twin of $a = F/m$.

Exam tips

- Torque $\tau = Fr_{\perp}$ uses the **perpendicular** distance from the pivot; a force through the pivot gives zero torque.
- For balance, set clockwise torque = anticlockwise torque; choosing the pivot at an unknown force removes it from the equation.
- Use the rotational analogues: $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$, so $\sum \tau = I\alpha$ mirrors $\sum F = ma$.
- **Rotational inertia** depends on **how far** the mass sits from the axis, not just its amount — a hoop resists spinning more than a disc of equal mass.
- Convert angles to **radians** and link linear to angular with $v = r\omega$, $a_t = r\alpha$.

Energy and Momentum of Rotating Systems

AP Physics 1

Rotational Kinetic Energy

A spinning object has **rotational kinetic energy** 转动动能, the rotational twin of $\frac{1}{2}mv^2$:

$$K_{\text{rot}} = \frac{1}{2}I\omega^2.$$

An object that both moves and spins (like a rolling ball) has **both** translational and rotational kinetic energy, and its total is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

Torque and Work

A torque acting through an angular displacement does **work**, changing rotational kinetic energy:

$$W = \tau \Delta\theta, \quad P = \tau\omega.$$

This is the rotational form of $W = Fd$ and $P = Fv$, and it extends the work–energy theorem to rotation.

Worked example. A motor applies a steady torque of 8.0 N m to a flywheel while it turns through 10 rad. The work done is $W = \tau \Delta\theta = 8.0 \times 10 = 80$ J, and if the flywheel started from rest this all becomes rotational kinetic energy.

Angular Momentum and Angular Impulse

Angular momentum 角动量 is the rotational version of linear momentum:

$$L = I\omega.$$

A net torque acting over time delivers an **angular impulse** 角冲量 that changes it: $\tau \Delta t = \Delta L$ – the rotational impulse–momentum theorem.

Conservation of Angular Momentum

If the **net external torque** on a system is zero, its total angular momentum is **conserved** 守恒:

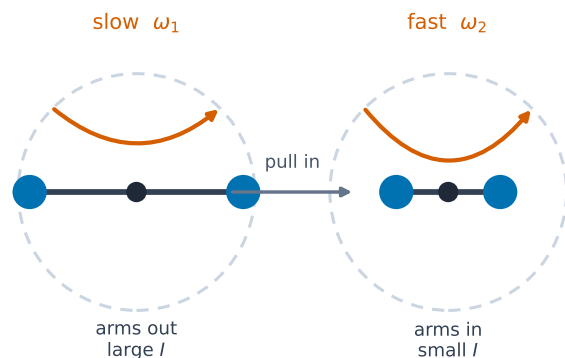
$$I_1\omega_1 = I_2\omega_2.$$

So if I decreases, ω increases to keep L constant – this is why a spinning skater speeds up when pulling their arms in. It applies to collisions and explosions of rotating systems too.

Worked example. A skater spins at 2.0 rev/s with rotational inertia $I_1 = 4.0 \text{ kg m}^2$. She pulls her arms in, dropping her rotational inertia to $I_2 = 1.6 \text{ kg m}^2$. With no external torque, angular momentum is conserved:

$$\omega_2 = \frac{I_1}{I_2} \omega_1 = \frac{4.0}{1.6} \times 2.0 = 5.0 \text{ rev/s}.$$

Her kinetic energy actually *rises* – the extra energy comes from the work her muscles do pulling her arms in against the outward pull.



$L = I\omega$ conserved (no external torque): $I_1\omega_1 = I_2\omega_2$
smaller $I \Rightarrow$ larger ω (angular momentum conserved)

Pulling mass inward lowers I , so ω rises to conserve $L = I\omega$

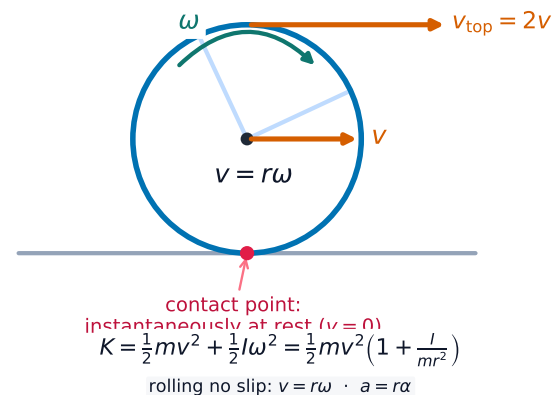
Rolling

Rolling without slipping 纯滚动 links the translational and rotational motions: the contact point is momentarily at rest, so

$$v = r\omega \quad \text{and} \quad a = r\alpha.$$

A rolling object's energy splits between translation and rotation, so on an incline it accelerates **more slowly** than a frictionless sliding object – some energy goes into spin.

Worked example. For a solid disk ($I = \frac{1}{2}mR^2$) that rolls without slipping, what fraction of its kinetic energy is rotational? Using $v = R\omega$, the rotational part is $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mR^2)\omega^2 = \frac{1}{4}mv^2$, while the translational part is $\frac{1}{2}mv^2$. So the total is $\frac{3}{4}mv^2$ and the rotational share is $\frac{1/4}{3/4} = \frac{1}{3}$. A hoop, with its mass farther out, stores half its energy in spin and rolls down a ramp even more slowly.



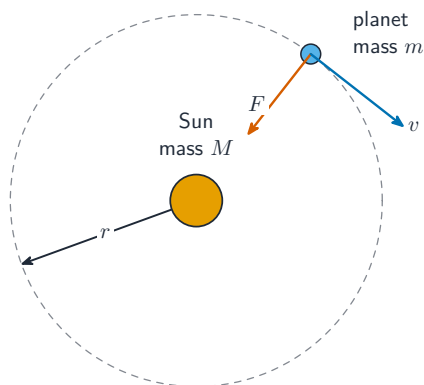
In rolling without slipping the contact point is at rest, so $v = r\omega$

Motion of Orbiting Satellites

A **satellite** 卫星 in orbit is in free fall: gravity provides the exact **centripetal force** needed to curve its path into an orbit. Setting gravity equal to the centripetal requirement,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}.$$

So a larger orbit means a slower speed. For a circular orbit, angular momentum and mechanical energy are both constant; for an elliptical orbit, angular momentum is conserved (no torque about the planet) while speed varies – fastest when closest.



Gravity provides the centripetal force that keeps a satellite in orbit

Worked example. Find the speed of a satellite in a low orbit just above the Earth, radius $r = 6.4 \times 10^6$ m, with $GM = 4.0 \times 10^{14} \text{ m}^3/\text{s}^2$:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{6.4 \times 10^6}} = \sqrt{6.25 \times 10^7} \approx 7.9 \times 10^3 \text{ m/s},$$

about 7.9 km/s – and notice that the satellite’s mass cancels, so all low orbits share this speed.



The International Space Station: a real satellite held in orbit by gravity alone

Escape velocity 逃逸速度 is the launch speed that just lets an object leave for good: its total mechanical energy is exactly zero, so it slows to zero speed only at infinite distance. Setting $\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$ (using the general gravitational PE $U_g = -GMm/r$) and solving for v ,

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}.$$

It is $\sqrt{2}$ times the circular-orbit speed at the same radius, and (like orbital speed) does not depend on the escaping object’s mass.

Exam tips

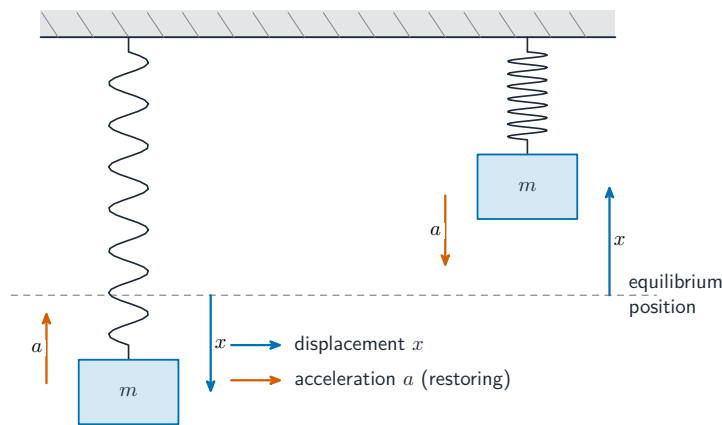
- Conserve **angular momentum** $L = I\omega$ when no external torque acts: a smaller I (arms pulled in) gives a larger ω .
- A rolling object splits its energy between $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$, linked by $v = r\omega$ — so it accelerates down a ramp **more slowly** than a sliding one.
- For a circular orbit set gravity equal to the centripetal requirement: $v = \sqrt{GM/r}$, so a larger orbit is **slower** and the satellite’s mass cancels.
- Pulling in raises the spin **and** the kinetic energy — the extra energy comes from the work done pulling inward; L is unchanged.
- Watch which rotational quantity is conserved: L (no torque) versus energy (no friction) are different conditions.

Oscillations

AP Physics 1

Defining Simple Harmonic Motion

Simple harmonic motion 简谐运动 (SHM) is a back-and-forth **oscillation** 振动 caused by a **restoring force** 回复力 that is proportional to the displacement from equilibrium and always points back toward it: $F = -kx$. A mass on a spring and (for small angles) a pendulum are the standard examples. Because the force grows with displacement, the motion is smooth and repeating, tracing a sine curve in time.



In SHM the acceleration always points back towards equilibrium, opposite the displacement

The test for SHM is exactly this: acceleration proportional to displacement and opposite in direction, $a = -\frac{k}{m}x$. A pendulum only obeys it for **small** swings, where $\sin \theta \approx \theta$; large swings are not quite SHM.



A spring under load: Hooke's law $F = -kx$ is the restoring force that produces SHM

Frequency and Period of SHM

- The **period** 周期 T is the time for one full cycle.
- The **frequency** 频率 $f = \frac{1}{T}$ is cycles per second (hertz).

For SHM these depend only on the system, **not** on the amplitude:

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}, \quad T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}.$$

So a stiffer spring or smaller mass oscillates faster; a longer pendulum swings slower.

Worked example. A 0.25 kg mass hangs on a spring of stiffness $k = 100 \text{ N/m}$. Its period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{100}} = 0.31 \text{ s}, \quad f = \frac{1}{T} = 3.2 \text{ Hz}.$$

Worked example. A pendulum clock ticks with a period of exactly 2.0 s. How long is it? Rearranging $T = 2\pi\sqrt{L/g}$,

$$L = g \left(\frac{T}{2\pi} \right)^2 = 9.8 \times \left(\frac{2.0}{2\pi} \right)^2 = 0.99 \text{ m}.$$

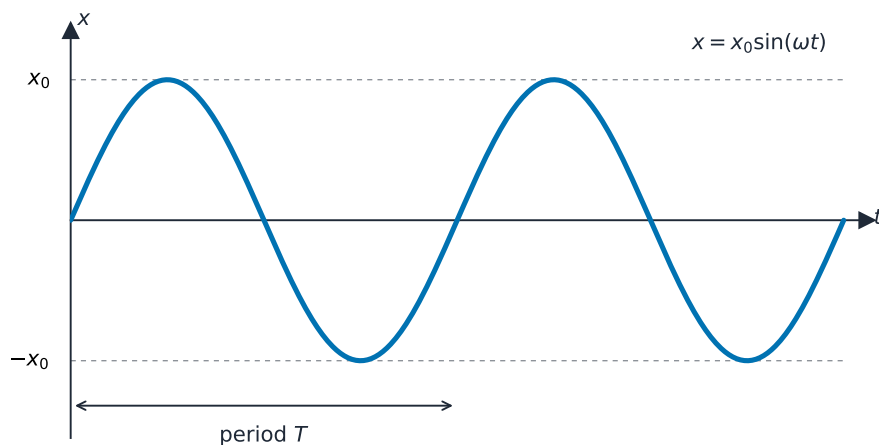
Notice the amplitude never entered – a wide or narrow swing keeps the same time, which is what makes pendulums good clocks.



Tuning forks: a longer, heavier fork vibrates more slowly, giving a lower frequency

Representing and Analyzing SHM

The displacement varies sinusoidally: $x(t) = A \cos(\omega t)$ (or sine), where A is the **amplitude** 振幅 (maximum displacement) and $\omega = 2\pi f$ is the **angular frequency** 角频率. Reading the motion:



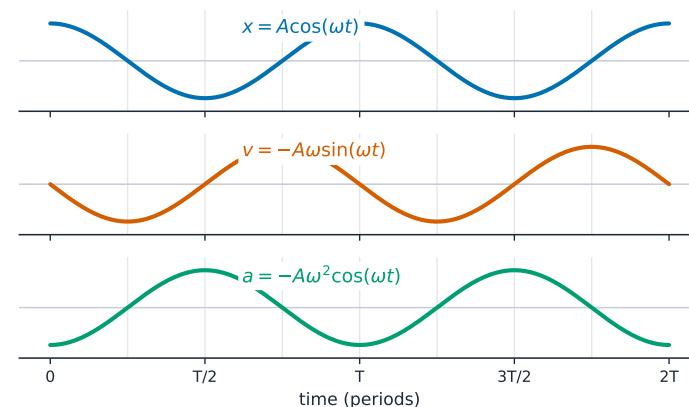
Displacement varies sinusoidally with time in simple harmonic motion

- At the **extremes** ($x = \pm A$): displacement and restoring force are maximum, so

acceleration is maximum, but velocity is **zero**.

- At **equilibrium** 平衡 ($x = 0$): force and acceleration are zero, but **speed is maximum**.

Velocity and acceleration are also sinusoidal, shifted in **phase** 相位 from the displacement – velocity leads displacement by a quarter cycle, and acceleration is exactly opposite to displacement.



$$a = -\omega^2 x \quad \cdot \quad v \text{ leads } x \text{ by } T/4 \quad \cdot \quad a \text{ is in antiphase with } x$$

Displacement, velocity, and acceleration in SHM, each a quarter-cycle apart

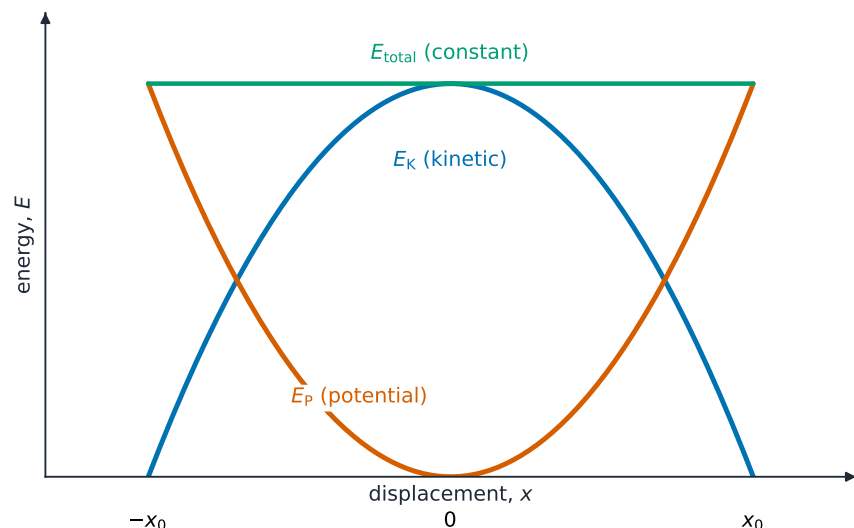
Read the figure as a story: where x is largest (the turning point), v has just fallen to zero and a is at its most negative, hauling the mass back; a quarter-cycle later the mass races through the middle at top speed with zero acceleration.

Energy of Simple Harmonic Oscillators

Energy sloshes between kinetic and potential while the **total** stays constant (no friction):

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2.$$

At the extremes it is **all potential**; at equilibrium it is **all kinetic** (maximum speed). Because $E \propto A^2$, doubling the amplitude quadruples the energy.



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example. A 0.50 kg mass on a spring of stiffness $k = 200 \text{ N/m}$ oscillates with amplitude $A = 0.10 \text{ m}$. Find its maximum speed. All the energy is kinetic at the equilibrium point, so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2$:

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = 0.10 \times \sqrt{\frac{200}{0.50}} = 0.10 \times 20 = 2.0 \text{ m/s}.$$

The energy method is faster than tracking the sine functions when you only need the greatest speed.

Exam tips

- Test for **SHM**: the acceleration must be proportional to the displacement and directed back toward the middle ($a = -\frac{k}{m}x$).
- The **period does not depend on amplitude** — use $T = 2\pi\sqrt{m/k}$ (spring) or $T = 2\pi\sqrt{L/g}$ (pendulum, small angles).
- Speed is **maximum at the middle** (all kinetic) and **zero at the extremes** (all potential); acceleration is largest at the extremes.
- Use energy ($\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$) to find the maximum speed quickly: $v_{\text{max}} = A\sqrt{k/m}$.
- Total energy $\propto A^2$, so doubling the amplitude quadruples the energy.

Fluids

AP Physics 1

Internal Structure and Density

The differences between solids, liquids, and gases come from how strongly their particles interact. A **fluid** 流体 (liquid or gas) has no fixed shape – it flows because its particles move past one another. **Density** 密度 is mass per unit volume:

$$\rho = \frac{m}{V}.$$

It depends on the substance and its state. An object sinks or floats depending on how its density compares with the surrounding fluid's – less dense floats, more dense sinks. An **ideal fluid** 理想流体 is **incompressible** 不可压缩 (constant density, whatever the pressure) and has no **viscosity** 黏度 (internal friction) – the model AP uses throughout.

Pressure

Pressure 压强 is the perpendicular force per unit area, a **scalar** 标量 measured in pascals (Pa):

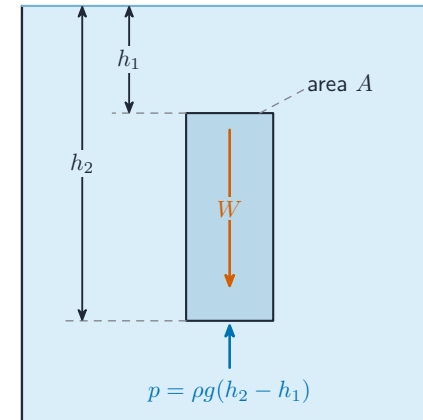
$$P = \frac{F_{\perp}}{A}.$$

In a fluid at rest, pressure increases with **depth** 深度 because of the weight of fluid above:

$$P = P_0 + \rho gh,$$

where P_0 is the pressure at the surface and h is the depth. Pressure acts **equally in all directions** at a point and pushes **perpendicular** to any surface.

Distinguish the two pressures AP asks about: the **gauge pressure** 表压 is the extra pressure the fluid column adds, $P_{\text{gauge}} = \rho gh$, while the **absolute pressure** 绝对压强 is the total, $P = P_0 + P_{\text{gauge}}$ (with P_0 usually atmospheric). A tyre gauge reading "200 kPa" is gauge pressure; the air inside is really at about 300 kPa absolute.



The weight of a liquid column sets the extra pressure at a depth

Worked example. Find the total pressure on a diver 10 m below the surface of water ($\rho = 1000 \text{ kg/m}^3$, surface pressure $P_0 = 1.0 \times 10^5 \text{ Pa}$):

$$P = P_0 + \rho gh = 1.0 \times 10^5 + 1000 \times 9.8 \times 10 = 1.98 \times 10^5 \text{ Pa}.$$

Every 10 m of water adds roughly one extra atmosphere of pressure. Notice the pressure does not depend on the shape or width of the container, only on the depth.



An aneroid barometer measures air pressure with a sealed metal box that flexes as the pressure outside changes

Fluids and Newton's Laws

An object in a fluid feels an upward **buoyant force** 浮力 equal to the weight of the fluid it displaces – **Archimedes' principle** 阿基米德原理:

$$F_b = \rho_{\text{fluid}} g V_{\text{displaced}}$$

Combine this with Newton's laws: the object floats when buoyancy balances weight, sinks when weight wins, and rises when buoyancy wins. A floating object displaces exactly its own weight of fluid.

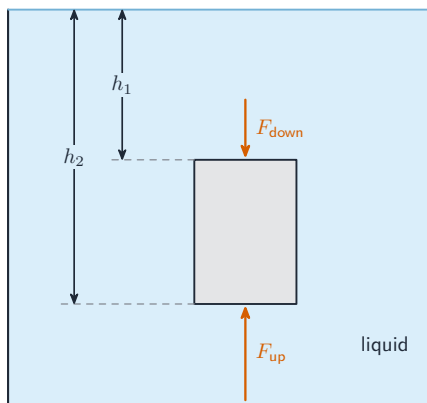
Worked example. A block of density 600 kg/m^3 floats in water (1000 kg/m^3). What fraction is under the surface? For floating, the buoyant force equals the weight, so $\rho_{\text{fluid}} g V_{\text{sub}} = \rho_{\text{object}} g V$:

$$\frac{V_{\text{sub}}}{V} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{600}{1000} = 0.60.$$

So 60% sits below the water – the same reason most of an iceberg (density ≈ 900) hides underwater.

Fluids and Conservation Laws

For an ideal fluid flowing steadily, two conservation ideas apply:



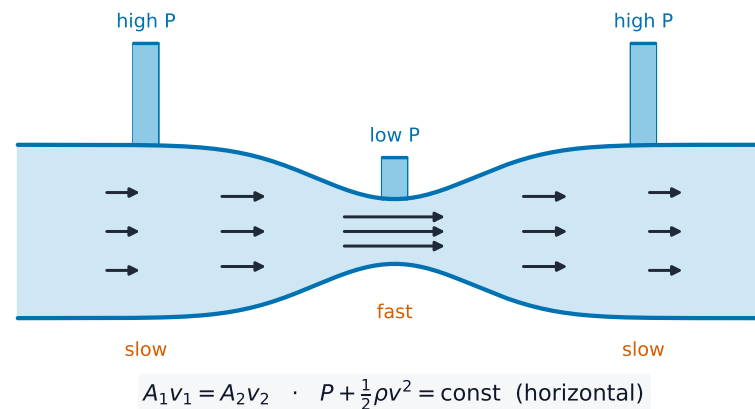
Upthrust arises because the pressure on the bottom of an object exceeds that on the top

- **Continuity** 连续性 (conservation of mass): the volume flow rate is constant, so $A_1 v_1 = A_2 v_2$. A narrower pipe forces **faster** flow.

- **Bernoulli's equation** 伯努利方程 (conservation of energy per volume): along a streamline,

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}.$$

Together they explain why fluid speeds up and its pressure drops where a pipe narrows or where flow is fastest.



Where the pipe narrows the fluid speeds up (continuity) and its pressure drops (Bernoulli)

Worked example. Water flows at 2.0 m/s through a pipe of cross-section 0.010 m^2 , then enters a narrower section of 0.0040 m^2 . By continuity the speed there is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{0.010 \times 2.0}{0.0040} = 5.0 \text{ m/s}.$$

By Bernoulli's equation this faster stream is at **lower** pressure – the effect that lifts an aeroplane wing and pulls two passing ships together.

Exam skill. Choose the right law by what changes. If the pipe changes *width*, start with continuity ($A_1 v_1 = A_2 v_2$) to get the speeds; if you then need a *pressure*, feed those speeds into Bernoulli. Watch the height term ρgy only when the pipe also changes level.

Exam tips

- Pressure with depth is $P = P_0 + \rho gh$ — it depends on **depth only**, not the container's shape or width.
- **Buoyant force** = weight of fluid displaced ($\rho_{\text{fluid}} g V_{\text{disp}}$); a floating object displaces its own weight, so the fraction submerged is $\rho_{\text{object}}/\rho_{\text{fluid}}$.

- Compare densities to predict floating vs sinking; a floating object is in equilibrium (buoyancy = weight), not weightless.
- Use **continuity** $A_1 v_1 = A_2 v_2$: a narrower pipe means faster flow.
- **Bernoulli**: where a fluid flows faster its pressure is lower (wing lift, spray).