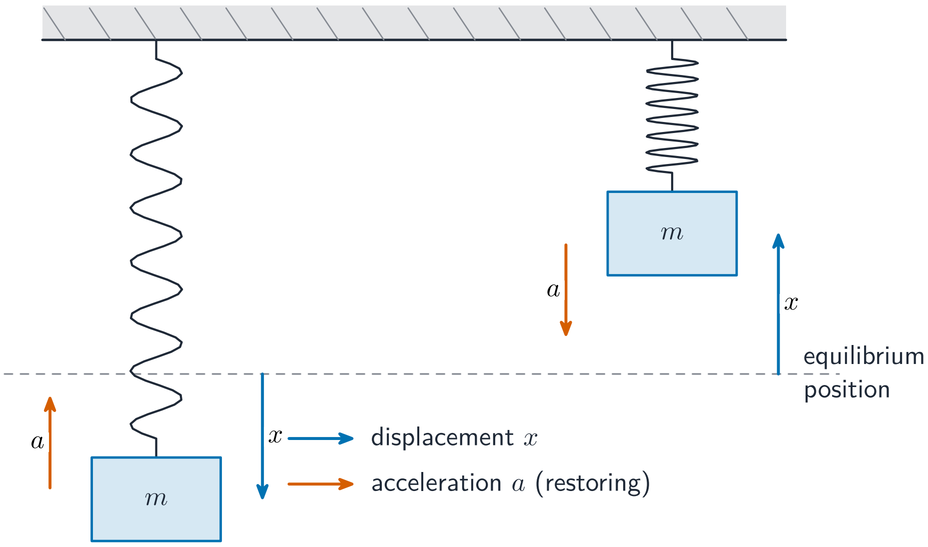
Oscillations

AP Physics 1

Defining Simple Harmonic Motion

Simple harmonic motion 简谐运动 (SHM) is a back-and-forth **oscillation** 振动 caused by a **restoring force** 回复力 that is proportional to the displacement from equilibrium and always points back toward it: $F = -kx$. A mass on a spring and (for small angles) a pendulum are the standard examples. Because the force grows with displacement, the motion is smooth and repeating, tracing a sine curve in time.



In SHM the acceleration always points back towards equilibrium, opposite the displacement

The test for SHM is exactly this: acceleration proportional to displacement and opposite in direction, $a = -\frac{k}{m}x$. A pendulum only obeys it for **small** swings, where $\sin\theta \approx \theta$; large swings are not quite SHM.

Frequency and Period of SHM

- The **period** 周期 T is the time for one full cycle.
- The **frequency** 频率 $f = \frac{1}{T}$ is cycles per second (hertz).

For SHM these depend only on the system, **not** on the amplitude:

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}, \qquad T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}.$$

So a stiffer spring or smaller mass oscillates faster; a longer pendulum swings slower.

Worked example. A 0.25 kg mass hangs on a spring of stiffness $k = 100$ N/m. Its period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{100}} = 0.31 \text{ s}, \qquad f = \frac{1}{T} = 3.2 \text{ Hz}.$$

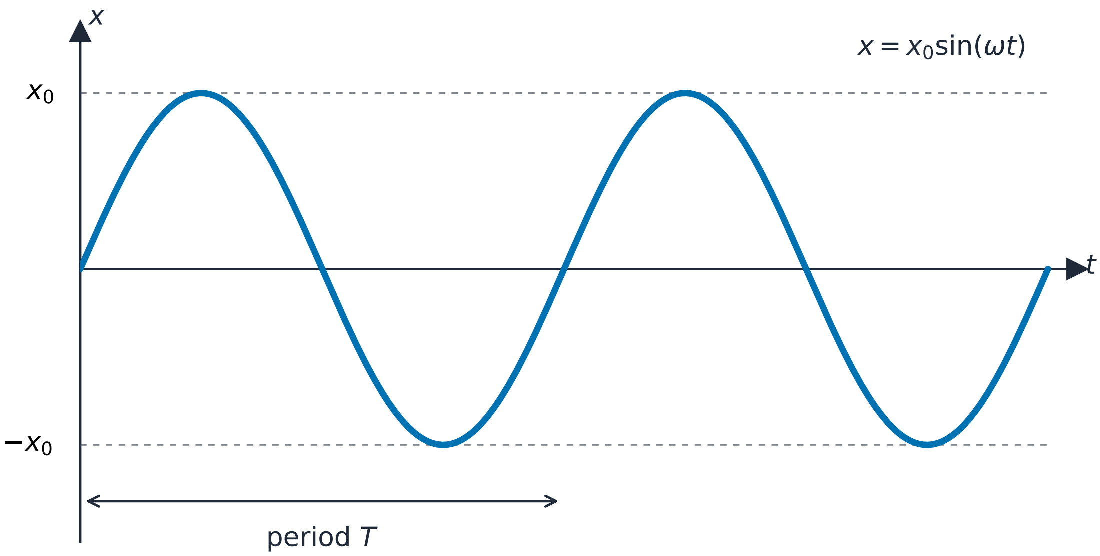
Worked example. A pendulum clock ticks with a period of exactly 2.0 s. How long is it? Rearranging $T = 2\pi\sqrt{L/g}$,

$$L = g\left(\frac{T}{2\pi}\right)^2 = 9.8 \times \left(\frac{2.0}{2\pi}\right)^2 = 0.99 \text{ m}.$$

Notice the amplitude never entered –a wide or narrow swing keeps the same time, which is what makes pendulums good clocks.

Representing and Analyzing SHM

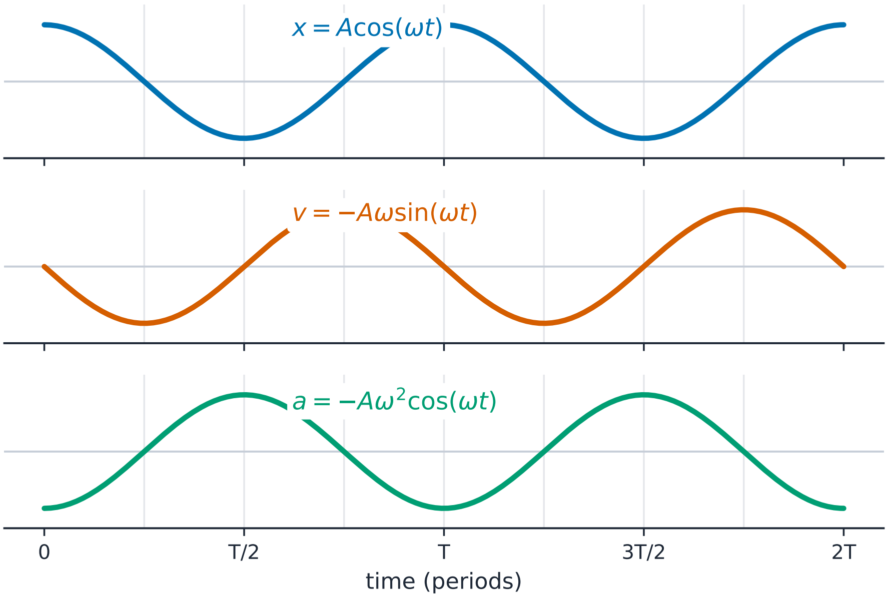
The displacement varies sinusoidally: $x(t) = A\cos(\omega t)$ (or sine), where A is the **amplitude** 振幅 (maximum displacement) and $\omega = 2\pi f$ is the **angular frequency** 角频率. Reading the motion:



Displacement varies sinusoidally with time in simple harmonic motion

- At the **extremes** ($x = \pm A$): displacement and restoring force are maximum, so acceleration is maximum, but velocity is **zero**.
- At **equilibrium** 平衡位置 ($x = 0$): force and acceleration are zero, but **speed is maximum**.

Velocity and acceleration are also sinusoidal, shifted in **phase** 相位 from the displacement –velocity leads displacement by a quarter cycle, and acceleration is exactly opposite to displacement.



Displacement, velocity, and acceleration in SHM, each a quarter-cycle apart

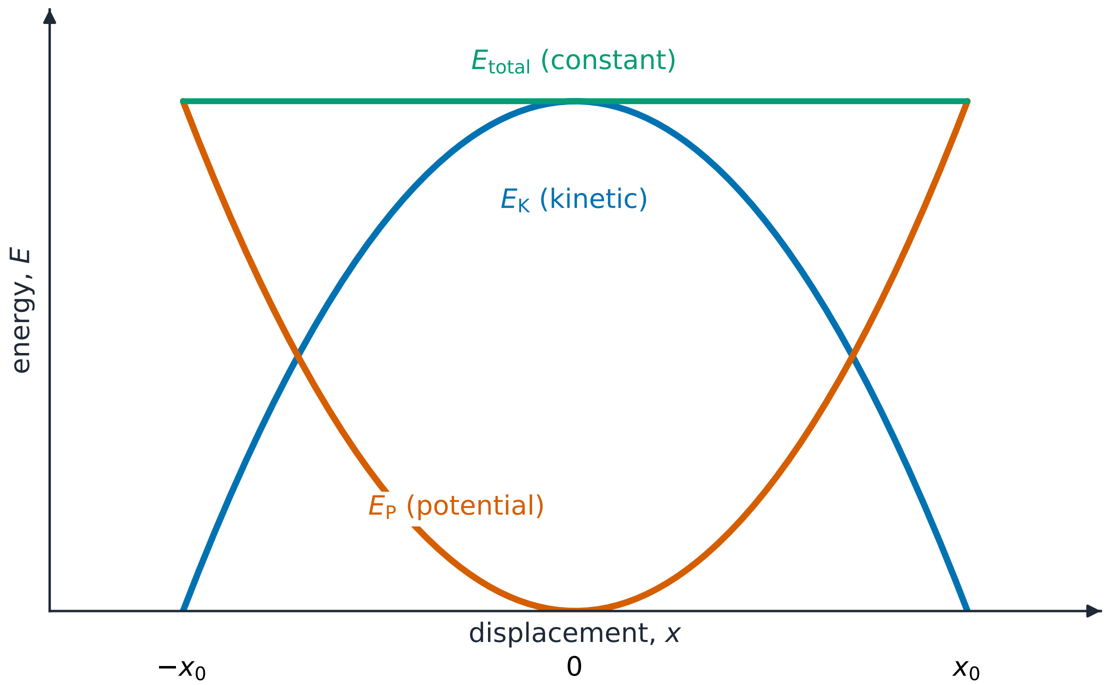
Read the figure as a story: where x is largest (the turning point), v has just fallen to zero and a is at its most negative, hauling the mass back; a quarter-cycle later the mass races through the middle at top speed with zero acceleration.

Energy of Simple Harmonic Oscillators

Energy sloshes between kinetic and potential while the **total** stays constant (no friction):

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2.$$

At the extremes it is **all potential**; at equilibrium it is **all kinetic** (maximum speed). Because $E \propto A^2$, doubling the amplitude quadruples the energy.



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example. A 0.50 kg mass on a spring of stiffness $k = 200$ N/m oscillates with amplitude $A = 0.10$ m. Find its maximum speed. All the energy is kinetic at the equilibrium point, so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$:

$$v_{max} = A\sqrt{\frac{k}{m}} = 0.10 \times \sqrt{\frac{200}{0.50}} = 0.10 \times 20 = 2.0 \text{ m/s}.$$

The energy method is faster than tracking the sine functions when you only need the greatest speed.

Exam tips

- Test for **SHM**: the acceleration must be proportional to the displacement and directed back toward the middle ($a = -\frac{k}{m}x$).
- The **period does not depend on amplitude** —use $T = 2\pi\sqrt{m/k}$ (spring) or $T = 2\pi\sqrt{L/g}$ (pendulum, small angles).
- Speed is **maximum at the middle** (all kinetic) and **zero at the extremes** (all potential); acceleration is largest at the extremes.
- Use energy ($\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$) to find the maximum speed quickly: $v_{max} = A\sqrt{k/m}$.
- Total energy $\propto A^2$, so doubling the amplitude quadruples the energy.