Energy and Momentum of Rotating Systems

AP Physics 1

Rotational Kinetic Energy

A spinning object has **rotational kinetic energy** 转动动能, the rotational twin of $\frac{1}{2}mv^2$:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2.$$

An object that both moves and spins (like a rolling ball) has **both** translational and rotational kinetic energy, and its total is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

Torque and Work

A torque acting through an angular displacement does **work**, changing rotational kinetic energy:

$$W = \tau\,\Delta\theta, \qquad P = \tau\omega.$$

This is the rotational form of $W = Fd$ and $P = Fv$, and it extends the work–energy theorem to rotation.

Worked example. A motor applies a steady torque of 8.0 N m to a flywheel while it turns through 10 rad. The work done is $W = \tau\,\Delta\theta = 8.0 \times 10 = 80$ J, and if the flywheel started from rest this all becomes rotational kinetic energy.

Angular Momentum and Angular Impulse

Angular momentum 角动量 is the rotational version of linear momentum:

$$L = I\omega.$$

A net torque acting over time delivers an **angular impulse** 角冲量 that changes it: $\tau\,\Delta t = \Delta L$ –the rotational impulse–momentum theorem.

Conservation of Angular Momentum

If the **net external torque** on a system is zero, its total angular momentum is **conserved** 守恒:

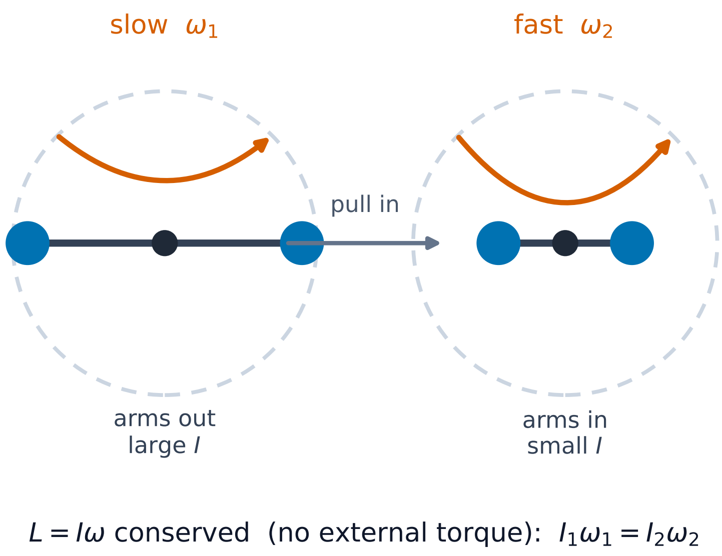
$$I_1\omega_1 = I_2\omega_2.$$

So if I decreases, ω increases to keep L constant –this is why a spinning skater speeds up when pulling their arms in. It applies to collisions and explosions of rotating systems too.

Worked example. A skater spins at 2.0 rev/s with rotational inertia $I_1 = 4.0$ kg m^2. She pulls her arms in, dropping her rotational inertia to $I_2 = 1.6$ kg m^2. With no external torque, angular momentum is conserved:

$$\omega_2 = \frac{I_1}{I_2}\,\omega_1 = \frac{4.0}{1.6} \times 2.0 = 5.0 \text{ rev/s}.$$

Her kinetic energy actually *rises* –the extra energy comes from the work her muscles do pulling her arms in against the outward pull.



Pulling mass inward lowers I, so ω rises to conserve $L = I\omega$

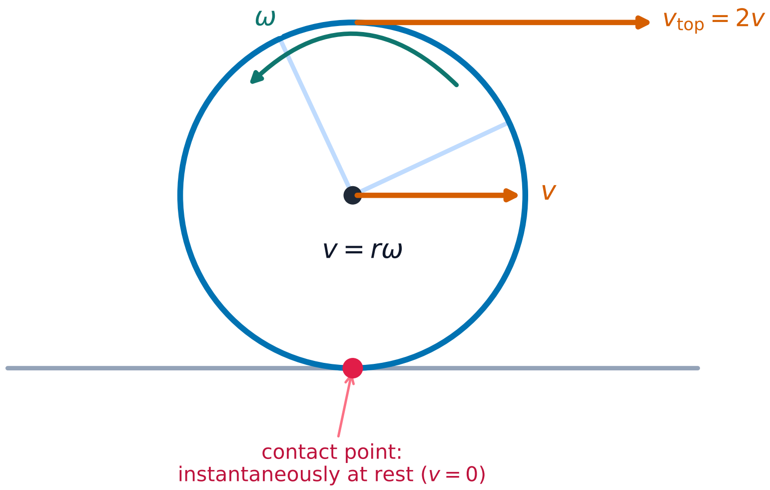
Rolling

Rolling without slipping 纯滚动 links the translational and rotational motions: the contact point is momentarily at rest, so

$$v = r\omega \qquad \text{and} \qquad a = r\alpha.$$

A rolling object's energy splits between translation and rotation, so on an incline it accelerates **more slowly** than a frictionless sliding object –some energy goes into spin.

Worked example. For a solid disk ($I = \frac{1}{2}mR^2$) that rolls without slipping, what fraction of its kinetic energy is rotational? Using $v = R\omega$, the rotational part is $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mR^2)\omega^2 = \frac{1}{4}mv^2$, while the translational part is $\frac{1}{2}mv^2$. So the total is $\frac{3}{4}mv^2$ and the rotational share is $\frac{1/4}{3/4} = \frac{1}{3}$. A hoop, with its mass farther out, stores half its energy in spin and rolls down a ramp even more slowly.



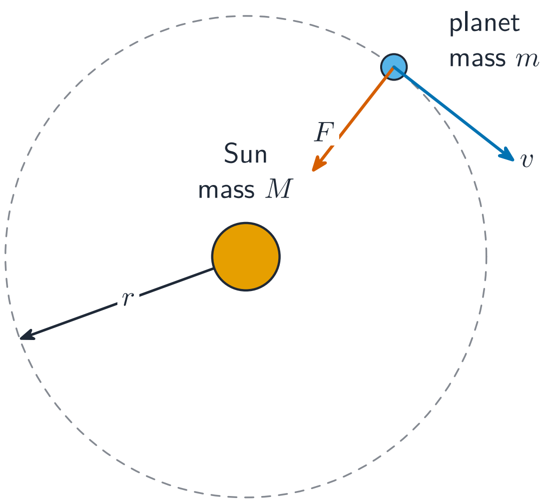
In rolling without slipping the contact point is at rest, so $v = r\omega$

Motion of Orbiting Satellites

A **satellite** 卫星 in orbit is in free fall: gravity provides the exact **centripetal force** needed to curve its path into an orbit. Setting gravity equal to the centripetal requirement,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \;\Rightarrow\; v = \sqrt{\frac{GM}{r}}.$$

So a larger orbit means a slower speed. For a circular orbit, angular momentum and mechanical energy are both constant; for an elliptical orbit, angular momentum is conserved (no torque about the planet) while speed varies –fastest when closest.



Gravity provides the centripetal force that keeps a satellite in orbit

Worked example. Find the speed of a satellite in a low orbit just above the Earth, radius $r = 6.4 \times 10^6$ m, with $GM = 4.0 \times 10^{14}$ m^3/s^2:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{6.4 \times 10^6}} = \sqrt{6.25 \times 10^7} \approx 7.9 \times 10^3 \text{ m/s},$$

about 7.9 km/s –and notice that the satellite's mass cancels, so all low orbits share this speed.

Exam tips

- Conserve **angular momentum** $L = I\omega$ when no external torque acts: a smaller I (arms pulled in) gives a larger ω.
- A rolling object splits its energy between $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$, linked by $v = r\omega$ —so it accelerates down a ramp **more slowly** than a sliding one.
- For a circular orbit set gravity equal to the centripetal requirement: $v = \sqrt{GM/r}$, so a larger orbit is **slower** and the satellite's mass cancels.
- Pulling in raises the spin **and** the kinetic energy —the extra energy comes from the work done pulling inward; L is unchanged.
- Watch which rotational quantity is conserved: L (no torque) versus energy (no friction) are different conditions.