Torque and Rotational Dynamics

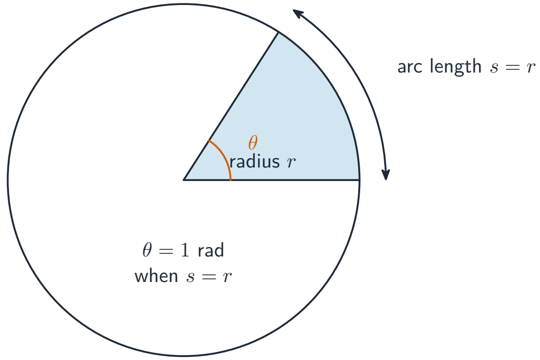
AP Physics 1

Rotational Kinematics

A crash-test collision: large forces and torques produce rapid changes in linear and angular motion

A spring under load: Hooke's law links force and extension — useful context for energy in rotating systems

Rotation is described by angular quantities that mirror the linear ones:



One radian is the angle whose arc length equals the radius

- **angular displacement** 角位移 θ (in **radians** 弧度),
- **angular velocity** 角速度 $\omega = \dfrac{\Delta\theta}{\Delta t}$,
- **angular acceleration** 角加速度 $\alpha = \dfrac{\Delta\omega}{\Delta t}$.

For constant α, the rotational kinematic equations have the same form as the linear ones, with θ, ω, α replacing x, v, a: $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, and $\omega^2 = \omega_0^2 + 2\alpha\theta$.

Worked example. A wheel starts from rest and speeds up uniformly to 30 rad/s in 6.0 s. Find its angular acceleration and the total angle turned:

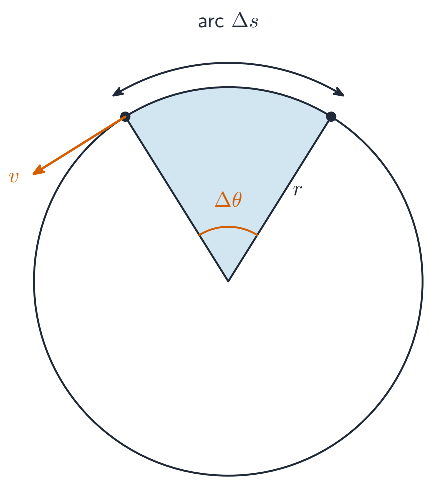
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30}{6.0} = 5.0\ \text{rad/s}^2, \qquad \theta = \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2} \times 5.0 \times 6.0^2 = 90\ \text{rad}.$$

Connecting Linear and Rotational Motion

A point at radius r from the axis has linear quantities tied to the angular ones:

$$s = r\theta, \qquad v = r\omega, \qquad a_t = r\alpha.$$

Points farther from the axis move faster. This link lets you switch between "how fast the wheel spins" and "how fast a point on its rim moves."



As the radius turns through an angle, a point moves along an arc at speed v

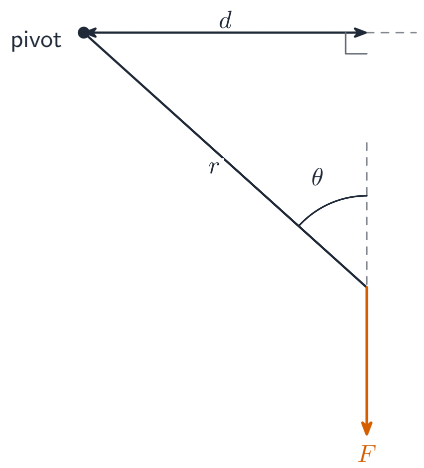
Worked example. A bicycle wheel of radius 0.35 m spins at 12 rad/s. A point on the rim (and so the bike) moves at $v = r\omega = 0.35 \times 12 = 4.2$ m/s. A point halfway to the axis moves at half that speed.

Torque

Torque 力矩 is the rotational effect of a force – how effectively it turns an object about an axis:

$$\tau = rF\sin\theta = F \cdot r_{\perp},$$

where $r_{\perp}$ is the **moment arm** 力臂 (the perpendicular distance from the axis to the force's line of action). A force applied farther out, or more perpendicular, produces more torque. Torque has a sign (clockwise vs counterclockwise).



The moment of a force depends on the perpendicular distance from the pivot

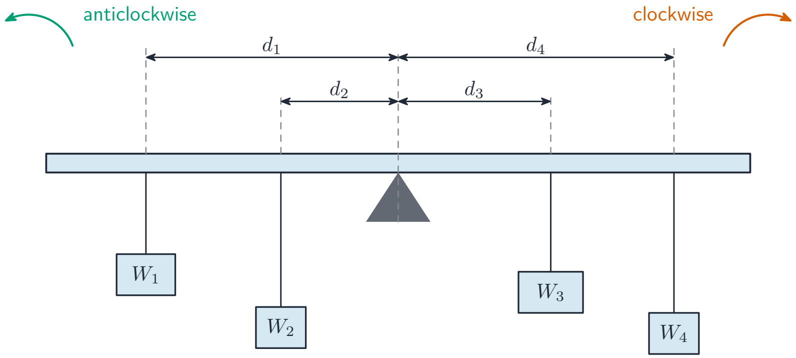
Worked example. You push with 20 N at the end of a 0.30 m wrench. Perpendicular to the wrench the torque is $\tau = rF = 0.30 \times 20 = 6.0$ N m. If you push at 60° to the wrench instead, only the perpendicular part counts: $\tau = rF\sin 60° = 0.30 \times 20 \times 0.87 = 5.2$ N m – which is why you push at a right angle for the most turning effect.

Rotational Inertia

Rotational inertia 转动惯量 (moment of inertia) I measures how hard it is to change an object's rotation – the rotational version of mass. It depends on both the mass **and how far** that mass sits from the axis: mass spread farther out gives a larger I. For a point mass, $I = mr^2$; for extended bodies, standard formulas are provided (a hoop is mR^2, a solid disk $\frac{1}{2}mR^2$). This is why a figure skater spins faster when she pulls her arms in – she reduces I.

Rotational Equilibrium

An object is in **rotational equilibrium** 转动平衡 when the **net torque** is zero, so its angular velocity stays constant. For a balanced (static) object, both the net force **and** the net torque are zero. Choosing the axis at an unknown force's location removes it from the torque equation – a useful trick for beam and ladder problems.



At balance the clockwise and anticlockwise moments about the pivot are equal

Worked example. A 30 kg child sits 2.0 m from the pivot of a seesaw. Where must a 40 kg child sit on the other side to balance it? Set the clockwise torque equal to the anticlockwise torque (the g's cancel):

$$30 \times 2.0 = 40 \times d \;\Rightarrow\; d = \frac{60}{40} = 1.5 \text{ m}.$$

The heavier child sits closer to the pivot – less distance, same torque.

Newton's Second Law in Rotational Form

Net torque produces angular acceleration, in direct analogy with $F = ma$:

$$\sum \tau = I\alpha.$$

So a larger net torque, or a smaller rotational inertia, gives a larger angular acceleration. Solve rotation problems just like translation problems, with $\tau \leftrightarrow F$, $I \leftrightarrow m$, and $\alpha \leftrightarrow a$.

Worked example. A net torque of 12 N m acts on a wheel with rotational inertia $I = 3.0$ kg m^2. Its angular acceleration is $\alpha = \tau/I = 12/3.0 = 4.0$ rad/s^2 – the exact rotational twin of $a = F/m$.

Exam tips

- Torque $\tau = Fr_\perp$ uses the **perpendicular** distance from the pivot; a force through the pivot gives zero torque.
- For balance, set clockwise torque = anticlockwise torque; choosing the pivot at an unknown force removes it from the equation.
- Use the rotational analogues: $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$, so $\sum \tau = I\alpha$ mirrors $\sum F = ma$.
- **Rotational inertia** depends on **how far** the mass sits from the axis, not just its amount — a hoop resists spinning more than a disc of equal mass.
- Convert angles to **radians** and link linear to angular with $v = r\omega$, $a_t = r\alpha$.