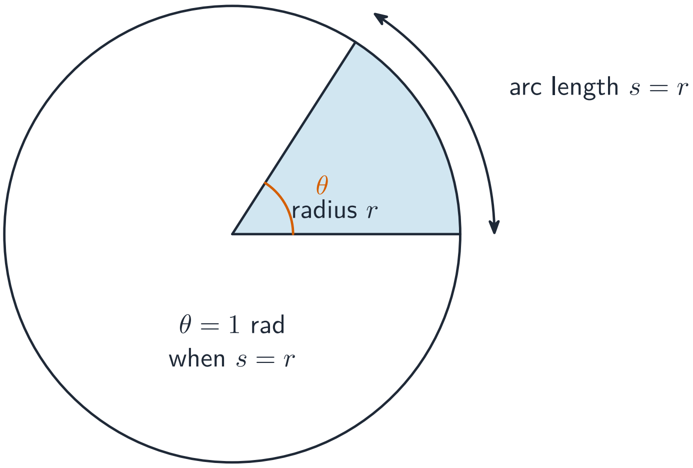
# Torque and Rotational Dynamics

AP Physics 1

## Rotational Kinematics

Rotation is described by angular quantities that mirror the linear ones:




*One radian is the angle whose arc length equals the radius*

- **angular displacement** 角位移 $\theta$ (in **radians** 弧度),
- **angular velocity** 角速度 $\omega = \dfrac{\Delta\theta}{\Delta t}$,
- **angular acceleration** 角加速度 $\alpha = \dfrac{\Delta\omega}{\Delta t}$.

For constant $\alpha$, the rotational kinematic equations have the same form as the linear ones, with $\theta, \omega, \alpha$ replacing $x, v, a$: $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, and $\omega^2 = \omega_0^2 + 2\alpha\theta$.

**Worked example.** A wheel starts from rest and speeds up uniformly to 30 rad/s in 6.0 s. Find its angular acceleration and the total angle turned:

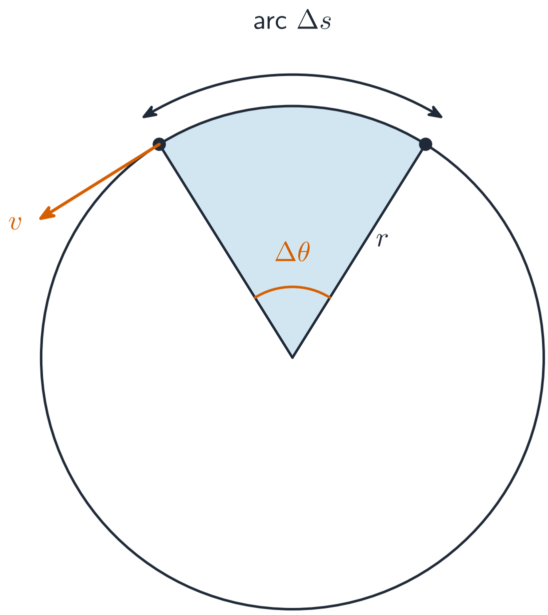
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30}{6.0} = 5.0\ \text{rad/s}^2, \qquad \theta = \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2} \times 5.0 \times 6.0^2 = 90\ \text{rad}.$$

## Connecting Linear and Rotational Motion

A point at radius $r$ from the axis has linear quantities tied to the angular ones:

$$s = r\theta, \qquad v = r\omega, \qquad a_t = r\alpha.$$

Points farther from the axis move faster. This link lets you switch between "how fast the wheel spins" and "how fast a point on its rim moves."




*As the radius turns through an angle, a point moves along an arc at speed $v$*

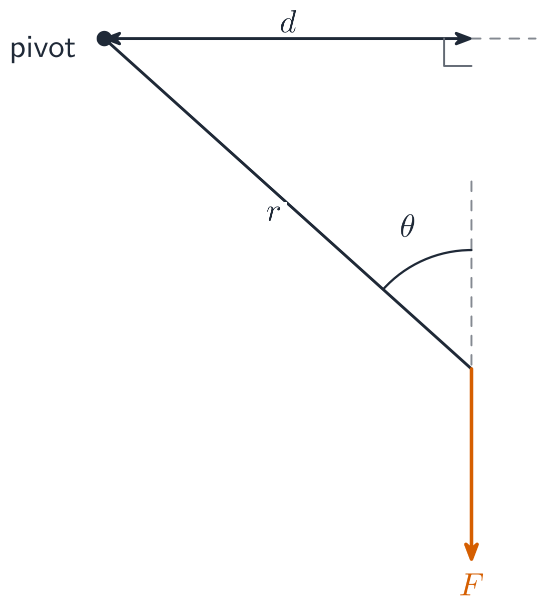
**Worked example.** A bicycle wheel of radius 0.35 m spins at 12 rad/s. A point on the rim (and so the bike) moves at $v = r\omega = 0.35 \times 12 = 4.2$ m/s. A point halfway to the axis moves at half that speed.

## Torque

**Torque 力矩** is the rotational effect of a force –how effectively it turns an object about an axis:

$$\tau = rF\sin\theta = F \cdot r_\perp,$$

where $r_\perp$ is the **moment arm 力臂** (the perpendicular distance from the axis to the force's line of action). A force applied farther out, or more perpendicular, produces more torque. Torque has a sign (clockwise vs counterclockwise).




*The moment of a force depends on the perpendicular distance from the pivot*

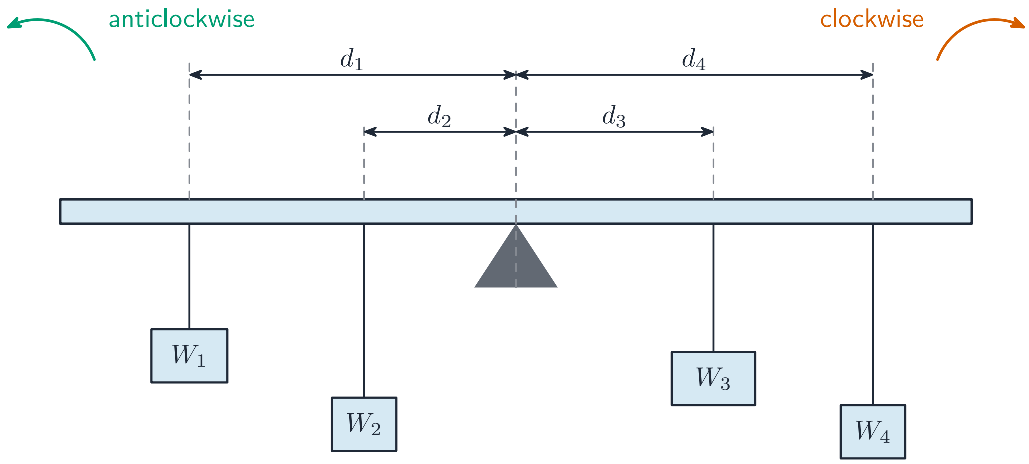
**Worked example.** You push with 20 N at the end of a 0.30 m wrench. Perpendicular to the wrench the torque is $\tau = rF = 0.30 \times 20 = 6.0$ N m. If you push at 60° to the wrench instead, only the perpendicular part counts: $\tau = rF \sin 60° = 0.30 \times 20 \times 0.87 = 5.2$ N m –which is why you push at a right angle for the most turning effect.

# Rotational Inertia

**Rotational inertia** 转动惯量 (moment of inertia) $I$ measures how hard it is to change an object's rotation –the rotational version of mass. It depends on both the mass **and how far** that mass sits from the axis: mass spread farther out gives a larger $I$. For a point mass, $I = mr^2$; for extended bodies, standard formulas are provided (a hoop is $mR^2$, a solid disk $\frac{1}{2}mR^2$). This is why a figure skater spins faster when she pulls her arms in –she reduces $I$.

# Rotational Equilibrium

An object is in **rotational equilibrium** 转动平衡 when the **net torque** is zero, so its angular velocity stays constant. For a balanced (static) object, both the net force **and** the net torque are zero. Choosing the axis at an unknown force's location removes it from the torque equation –a useful trick for beam and ladder problems.




*At balance the clockwise and anticlockwise moments about the pivot are equal*

**Worked example.** A 30 kg child sits 2.0 m from the pivot of a seesaw. Where must a 40 kg child sit on the other side to balance it? Set the clockwise torque equal to the anticlockwise torque (the $g$'s cancel):

$$30 \times 2.0 = 40 \times d \;\Rightarrow\; d = \frac{60}{40} = 1.5 \text{ m}.$$

The heavier child sits closer to the pivot –less distance, same torque.

## Newton's Second Law in Rotational Form

Net torque produces angular acceleration, in direct analogy with $F = ma$:

$$\sum \tau = I\alpha.$$

So a larger net torque, or a smaller rotational inertia, gives a larger angular acceleration. Solve rotation problems just like translation problems, with $\tau \leftrightarrow F$, $I \leftrightarrow m$, and $\alpha \leftrightarrow a$.

**Worked example.** A net torque of 12 N m acts on a wheel with rotational inertia $I = 3.0$ kg m$^2$. Its angular acceleration is $\alpha = \tau/I = 12/3.0 = 4.0$ rad/s$^2$ –the exact rotational twin of $a = F/m$.

## Exam tips

- Torque $\tau = Fr_\perp$ uses the **perpendicular** distance from the pivot; a force through the pivot gives zero torque.
- For balance, set clockwise torque = anticlockwise torque; choosing the pivot at an unknown force removes it from the equation.
- Use the rotational analogues: $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$, so $\sum \tau = I\alpha$ mirrors $\sum F = ma$.
- **Rotational inertia** depends on **how far** the mass sits from the axis, not just its amount —a hoop resists spinning more than a disc of equal mass.
- Convert angles to **radians** and link linear to angular with $v = r\omega$, $a_t = r\alpha$.