Kinematics

AP Physics 1

Scalars and Vectors in One Dimension

Kinematics 运动学 describes *how* objects move, without asking why. First, two kinds of quantity:

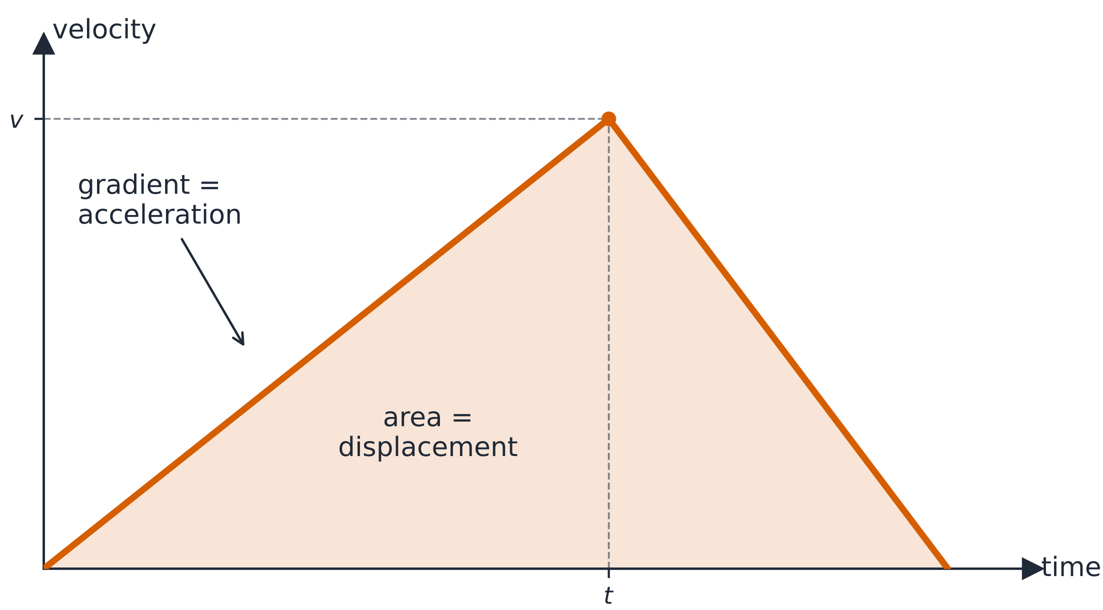
- A **scalar** 标量 has only size (**magnitude** 大小): distance 距离, speed, time, mass.
- A **vector** 矢量 has magnitude **and** direction: displacement, velocity, acceleration, force.

The difference matters. **Distance** is the total path length travelled –a scalar that only grows. **Displacement** is the straight-line change in position, with a direction. Walk 3 m east then 1 m back west: the distance is 4 m, but the displacement is only 2 m east.

In one dimension, direction is just a **sign** (+ or −) along a chosen axis. Choosing the positive direction first is essential –every vector's sign depends on it. A velocity of −5 m/s does not mean "slow"; it means 5 m/s in the negative direction.

Displacement, Velocity, and Acceleration

Three linked vectors describe motion along a line:



On a velocity-time graph the gradient is the acceleration and the area is the displacement

- **Displacement** 位移 Δx is the change in position –a vector from start to end (not the total path length, which is distance).

- **Velocity** 速度 is the rate of change of position, $v = \dfrac{\Delta x}{\Delta t}$. Its sign gives direction; its magnitude is **speed** 速率.
- **Acceleration** 加速度 is the rate of change of velocity, $a = \dfrac{\Delta v}{\Delta t}$.

Be careful to separate **average velocity** 平均速度 (total displacement over total time) from **instantaneous velocity** 瞬时速度 (the velocity at one instant, the slope of the position–time graph at that point). They are equal only when the velocity is constant.

An object speeds up when v and a have the **same** sign, and slows down (**deceleration** 减速) when they have **opposite** signs. Note that a negative acceleration does not always mean slowing down –a ball falling faster and faster has negative velocity *and* negative acceleration.

For **constant** acceleration, the four kinematic equations (often called SUVAT) apply:

$$v = v_0 + at, \qquad \Delta x = v_0 t + \tfrac{1}{2}at^2, \qquad v^2 = v_0^2 + 2a\,\Delta x, \qquad \Delta x = \tfrac{1}{2}(v_0 + v)\,t.$$

Pick the equation that contains the three quantities you know plus the one you want, so only one unknown is left. They apply **only** while a is constant.

Worked example. A car starts from rest and accelerates uniformly at 2.0 m/s^2 for 6.0 s. Find its final velocity and the distance it travels.

List what you know: $v_0 = 0$, $a = 2.0$ m/s^2, $t = 6.0$ s.

$$v = v_0 + at = 0 + 2.0 \times 6.0 = 12 \text{ m/s},$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2} \times 2.0 \times 6.0^2 = 36 \text{ m}.$$

Worked example (free fall). A ball is thrown straight up at 15 m/s. Taking $g = 9.8$ m/s^2 and up as positive, how high does it rise, and how long is it in the air before returning to the thrower's hand?

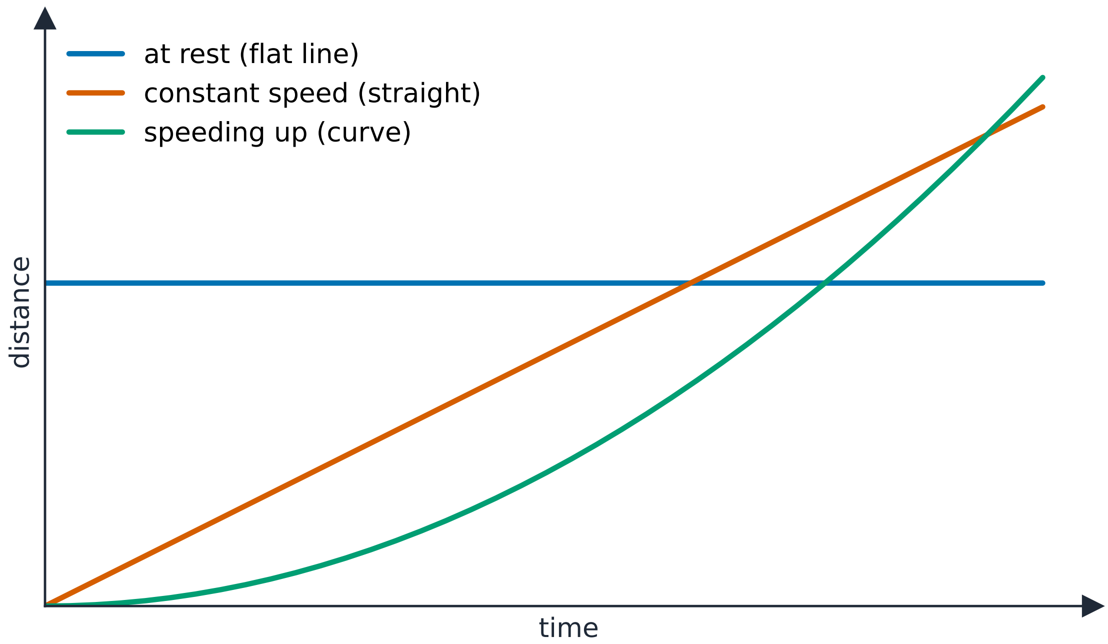
At the highest point the velocity is momentarily zero, and $a = -g = -9.8$ m/s^2 throughout (this is **free fall** 自由落体, ignoring air resistance 空气阻力):

$$v^2 = v_0^2 + 2a\,\Delta x \;\Rightarrow\; 0 = 15^2 + 2(-9.8)\Delta x \;\Rightarrow\; \Delta x = \frac{225}{19.6} = 11.5 \text{ m}.$$

Time to the top: $0 = 15 - 9.8\,t \Rightarrow t = 1.53$ s. By symmetry the fall takes the same time, so the total is 3.1 s.

Representing Motion

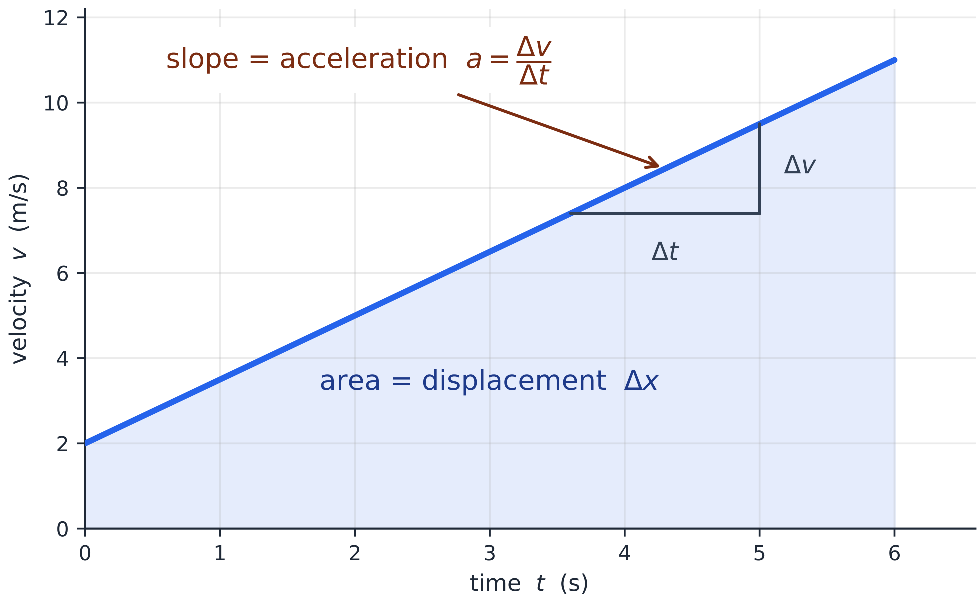
The same motion appears as a **description**, a **graph**, a **table**, or an **equation**, and you should move between them:



Reading a distance-time graph: flat means at rest, a straight slope means constant speed

- On a **position–time** graph, the **slope** is velocity (steeper = faster; a curve = changing velocity).
- On a **velocity–time** graph, the slope is acceleration, and the **area** under the line is displacement.

Reading slopes and areas off graphs is a core exam skill. To get displacement from a velocity–time graph, split the area into triangles and rectangles and add them up; area **below** the time axis counts as **negative** displacement (motion the other way).



On a velocity-time graph the slope is the acceleration and the shaded area is the displacement

Worked example. A cyclist speeds up uniformly from rest to 8.0 m/s in 4.0 s, then holds 8.0 m/s for 6.0 s. Find the total distance from the velocity–time graph.

The area is a triangle followed by a rectangle:

$$\Delta x = \underbrace{\tfrac{1}{2} \times 4.0 \times 8.0}_{\text{triangle}} + \underbrace{6.0 \times 8.0}_{\text{rectangle}} = 16 + 48 = 64 \text{ m}.$$

Reference Frames and Relative Motion

All motion is measured against a **reference frame** 参考系. Velocities measured in different frames differ, and you combine them by vector addition. The velocity of A relative to C is

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}.$$

A person walking on a moving train has one velocity relative to the train and another relative to the ground –this is **relative motion** 相对运动. A useful shortcut: the velocity of A relative to B is $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$ (subtract B's velocity).

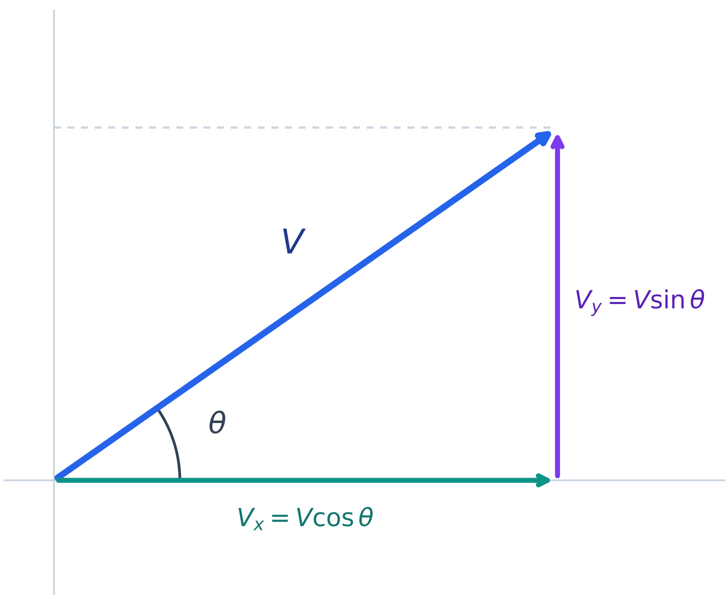
Worked example. A boat points straight across a river and moves at 3.0 m/s relative to the water. The current flows at 4.0 m/s along the river. Find the boat's speed and direction relative to the bank.

The two velocities are perpendicular, so add them as a right triangle:

$$v = \sqrt{3.0^2 + 4.0^2} = 5.0 \text{ m/s}, \qquad \theta = \tan^{-1}\frac{4.0}{3.0} = 53^\circ \text{ downstream from straight across.}$$

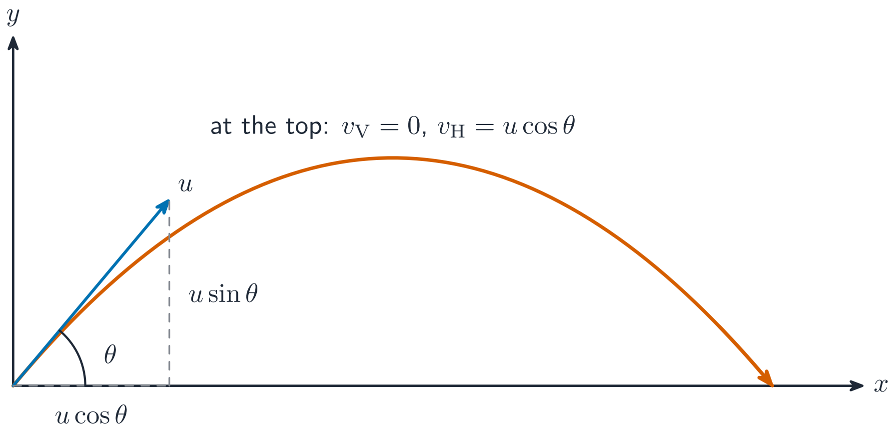
Vectors and Motion in Two Dimensions

In two dimensions, resolve each vector into **components** 分量 along perpendicular axes (x and y), handle each axis separately, then recombine. A velocity v at angle θ to the horizontal has components $v_x = v\cos\theta$ and $v_y = v\sin\theta$.

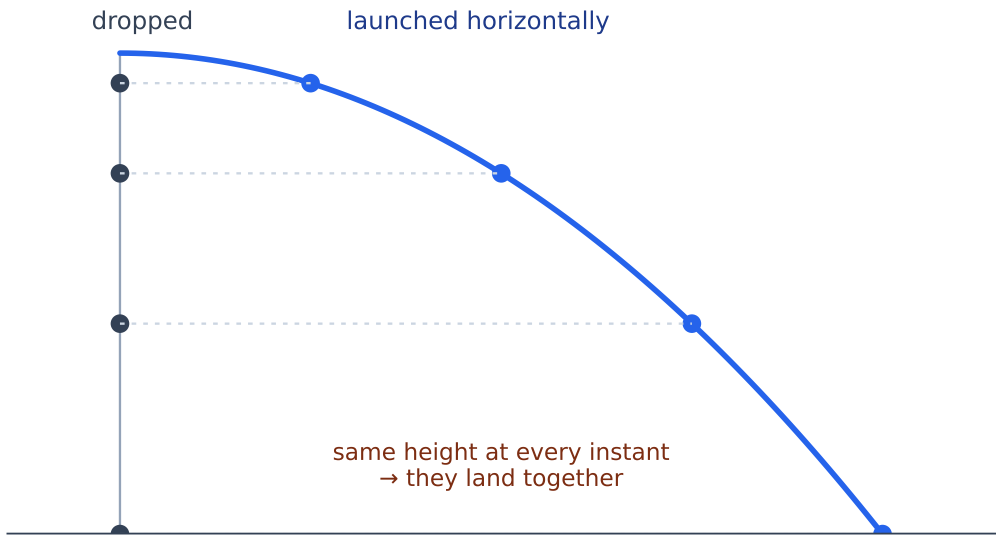


A velocity vector resolved into its horizontal and vertical components

For **projectile motion** 抛体运动 (an object moving under gravity alone): the horizontal and vertical motions are **independent**. Horizontally, velocity is constant ($a_x = 0$); vertically, acceleration is $-g$ (down). The two motions share only the **time**. So a projectile's path (its **trajectory** 轨迹) is a parabola, and you solve it as two one-dimensional problems joined by t.



A projectile launched at an angle: the horizontal and vertical motions are independent



A dropped ball and a horizontally launched ball fall together –the vertical motions are identical

Worked example. A ball is kicked at 20 m/s, 30° above the horizontal. Taking $g = 9.8$ m/s^2, find the time of flight, the maximum height, and the horizontal **range** 射程 (assume it lands at launch height).

Split the launch velocity into components:

$$v_{0x} = 20\cos 30^\circ = 17.3 \text{ m/s}, \qquad v_{0y} = 20\sin 30^\circ = 10 \text{ m/s}.$$

Vertical motion sets the time. At the top $v_y = 0$, so $0 = 10 - 9.8\,t \Rightarrow t_{\text{up}} = 1.02$ s, and the total flight is $2t_{\text{up}} = 2.0$ s. The maximum height is

$$\Delta y = \frac{v_{0y}^2}{2g} = \frac{10^2}{19.6} = 5.1 \text{ m.}$$

Horizontal motion runs at constant v_{0x} for the whole flight, so the range is

$$R = v_{0x} \times t_{\text{flight}} = 17.3 \times 2.0 = 35 \text{ m.}$$

A common trap: at the top of the flight the vertical velocity is zero, but the ball is **not** at rest –its horizontal velocity v_{0x} never changes. The speed at the top equals $v_{0x} = 17.3$ m/s.

Exam tips

- Choose the right **kinematic equation** by listing the three quantities you know plus the one you want, so only one unknown remains; the SUVAT equations apply **only** while acceleration is constant.
- Fix a **positive direction** first —every displacement, velocity, and acceleration then carries a sign; a negative velocity means ”moving the other way”, not ”slow”.
- Treat a projectile as **two independent 1-D problems** sharing only the time t: constant velocity horizontally, $a = -g$ vertically. At the top $v_y = 0$ but v_x is unchanged.
- On a **velocity–time graph** the gradient is the acceleration and the **area** is the displacement (area below the axis is negative).
- Distinguish **distance** (scalar, total path) from **displacement** (vector, start-to-end), and **speed** from **velocity**.