



3. (12 points, suggested time 25 minutes)

The left end of a rod of length  $d$  and rotational inertia  $I$  is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point  $C$  marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass  $m_{\text{disk}}$  toward the rod with velocity  $v_0$  perpendicular to the rod, and the disk will stick to the rod a distance  $x$  from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

- (a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point  $C$ , at point  $C$ , or to the right of point  $C$ ?

\_\_\_\_\_ To the left of  $C$       \_\_\_\_\_ At  $C$       \_\_\_\_\_ To the right of  $C$

Briefly explain your reasoning without manipulating equations.

- (b) On the Internet, a student finds the following equation for the postcollision angular speed  $\omega$  of the rod in this situation:  $\omega = \frac{m_{\text{disk}} x v_0}{I}$ . Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for  $\omega$  have the expected dependence as reasoned in part (a)?

\_\_\_\_\_ Yes      \_\_\_\_\_ No

Briefly explain your reasoning without deriving an equation for  $\omega$ .

- (c) Another student deriving an equation for the postcollision angular speed  $\omega$  of the rod makes a mistake and comes up with  $\omega = \frac{I x v_0}{m_{\text{disk}} d^4}$ . Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

- (d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is  $m_{\text{disk}} x^2$  and its angular momentum with respect to the pivot is  $m_{\text{disk}} v_0 x$ . Derive an equation for the postcollision angular speed  $\omega$  of the rod. Express your answer in terms of  $d$ ,  $m_{\text{disk}}$ ,  $I$ ,  $x$ ,  $v_0$ , and physical constants, as appropriate.
- (e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

\_\_\_\_\_ Greater than      \_\_\_\_\_ Less than      \_\_\_\_\_ Equal to

Briefly explain your reasoning.