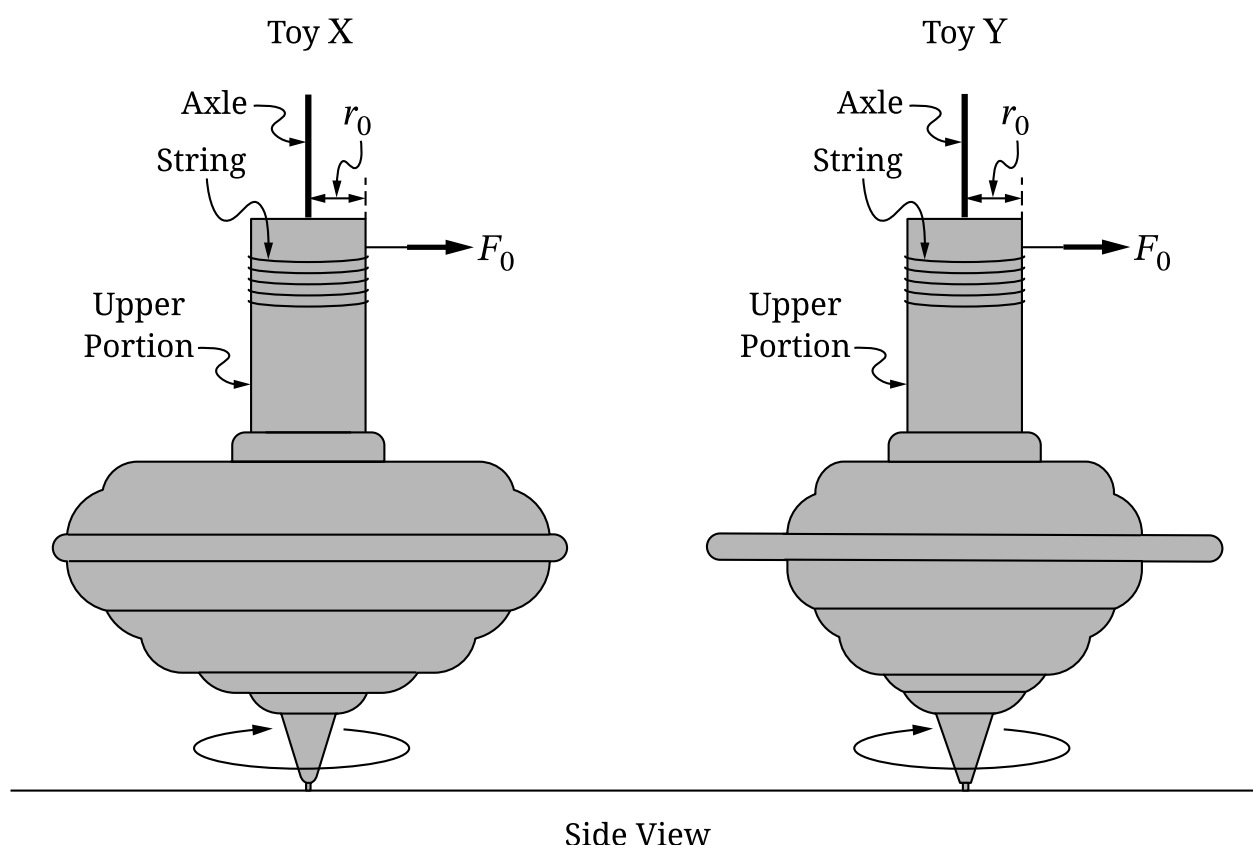


Question 4

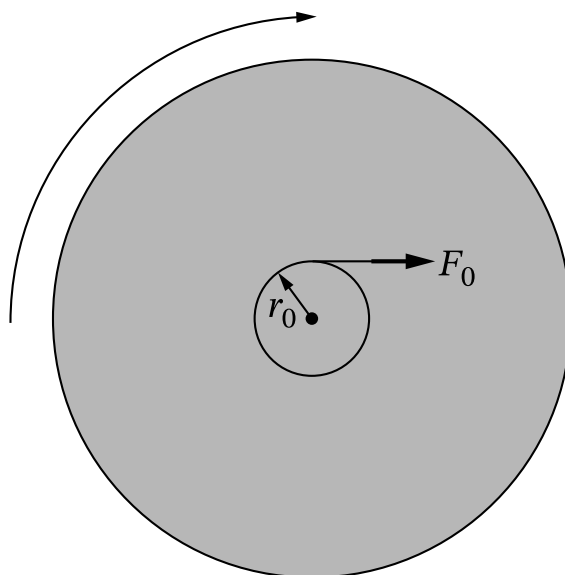
4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0 , as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X , and the rotational inertia of Toy Y is $I_Y = \frac{1}{2}I_X$.

Figure 1

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X .

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y .

Figure 2

Overhead View

Part A

Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

Part B

Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0 , I_X , F_0 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

STOP
END OF EXAM

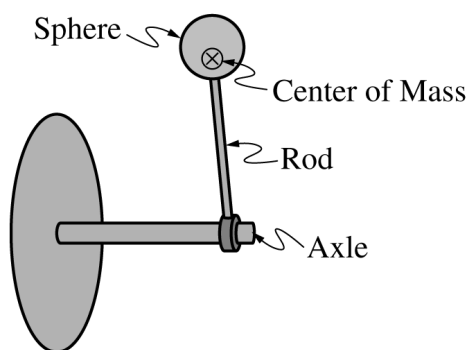


Figure 1

(7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a $\otimes$, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

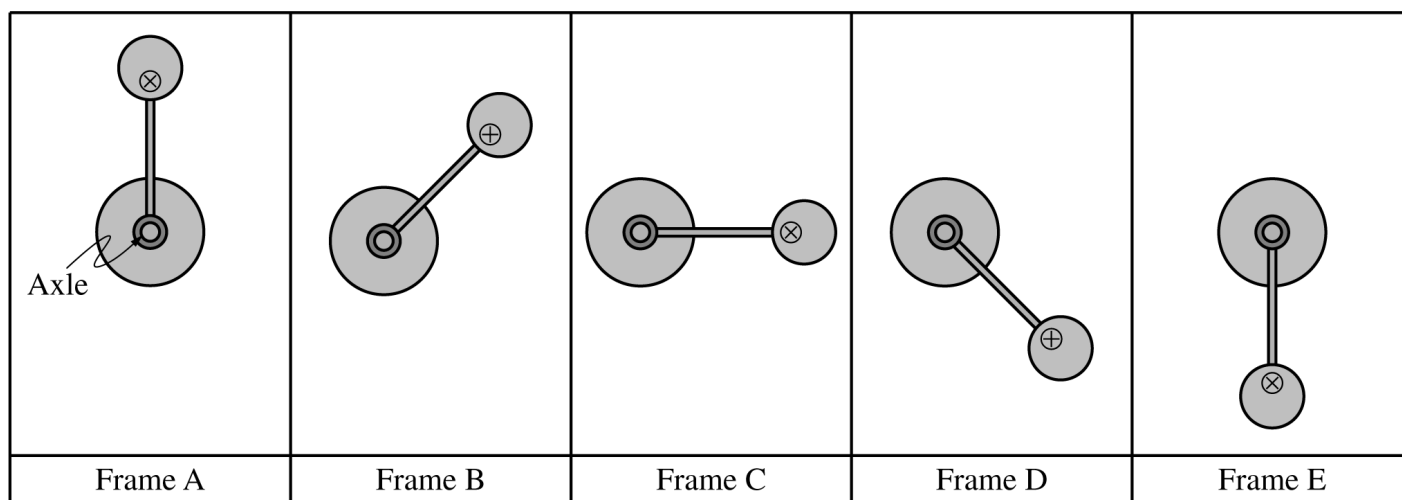


Figure 2

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 5** on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

- i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

- ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

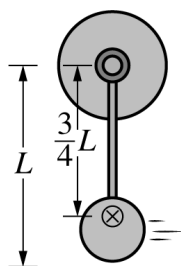


Figure 3

(b) The rod-sphere system has mass M and length L , and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

- i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M , L , and fundamental constants, as appropriate.

- ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM