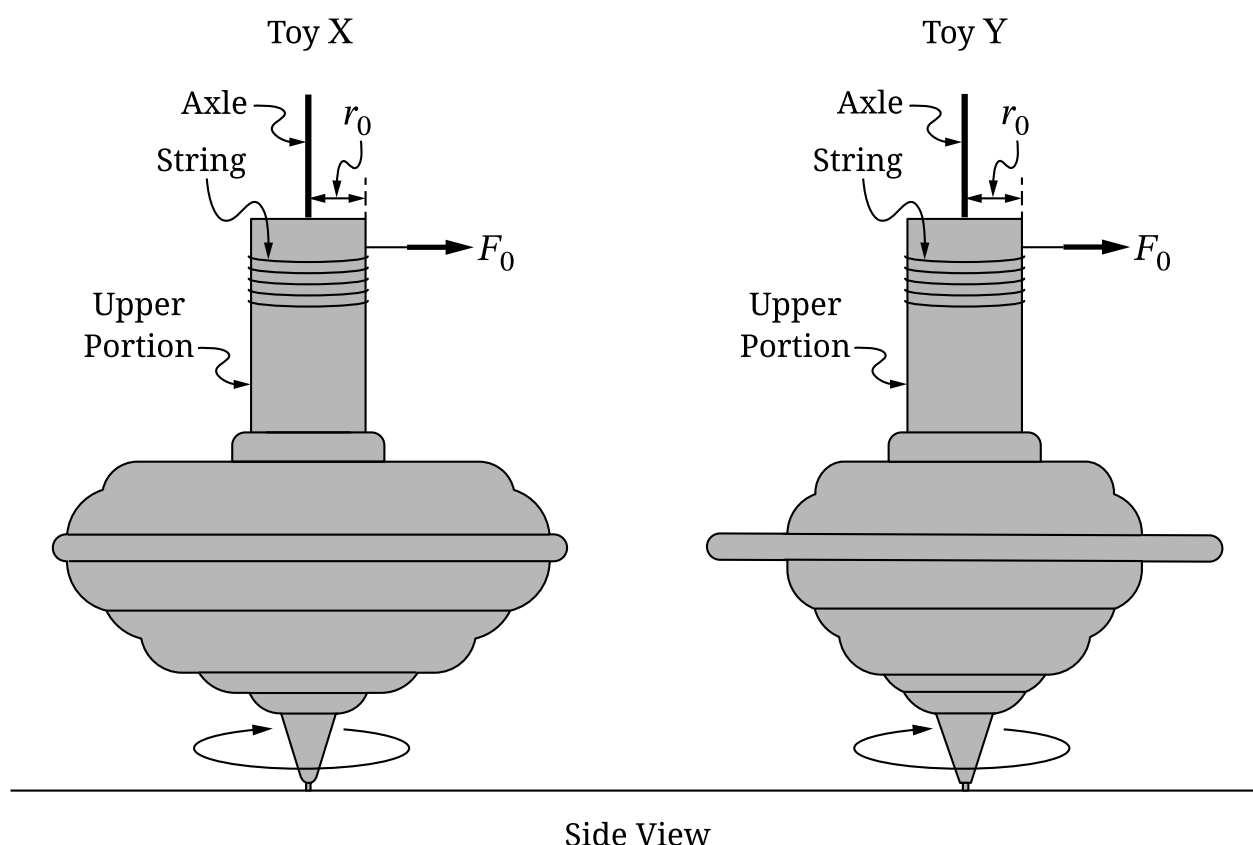


Question 4

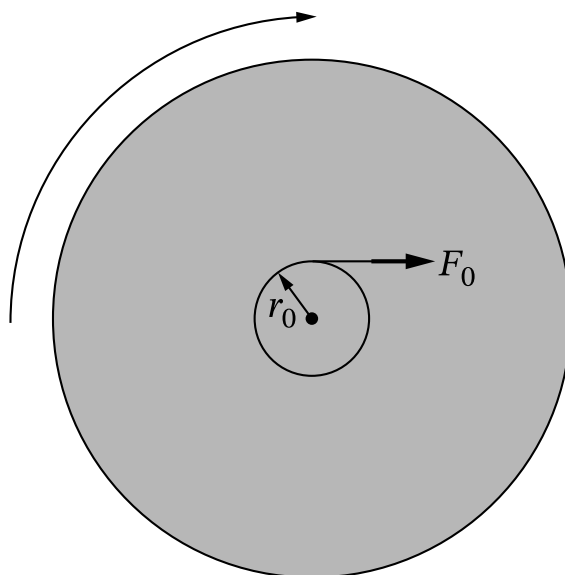
4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0 , as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X , and the rotational inertia of Toy Y is $I_Y = \frac{1}{2}I_X$.

Figure 1

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X .

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y .

Figure 2

Overhead View

Part A

Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

Part B

Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0 , I_X , F_0 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

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END OF EXAM

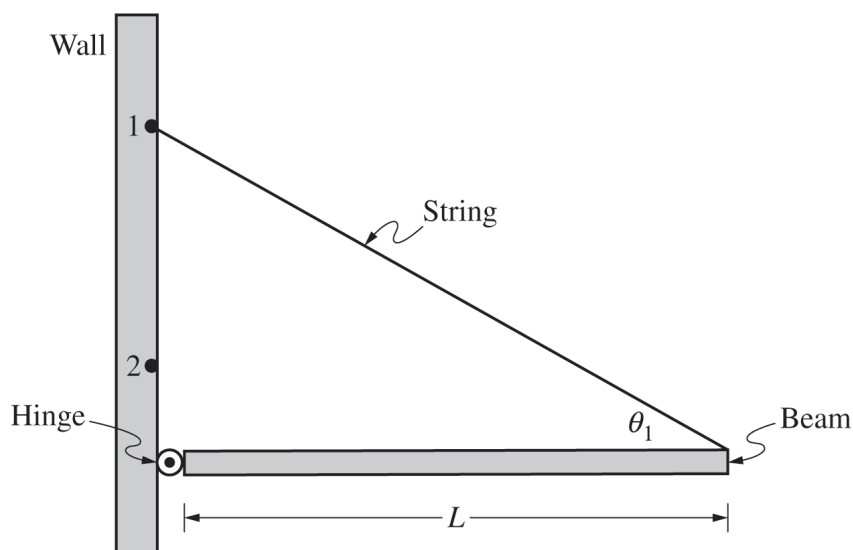
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Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H , and the angle between the beam and the string is $\theta = \theta_1$.

- (a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

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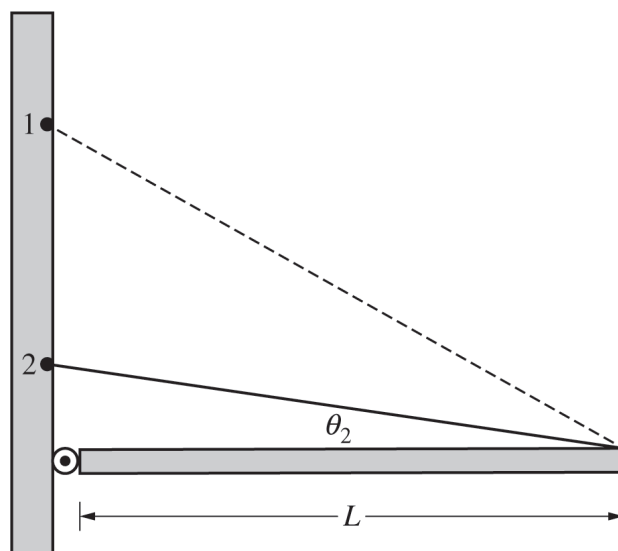
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Figure 2

- (b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is F_{T1} . The magnitude of the tension in the string shown in Figure 2 is F_{T2} . **Indicate** which of the following correctly compares F_{T2} with F_{T1} .

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

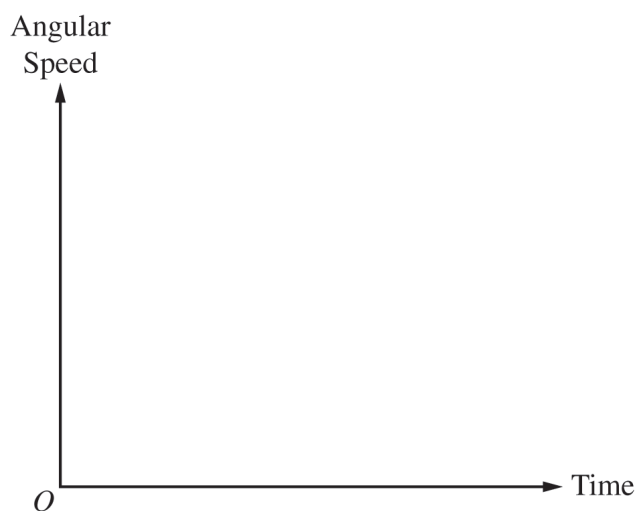
- (c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of M , θ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

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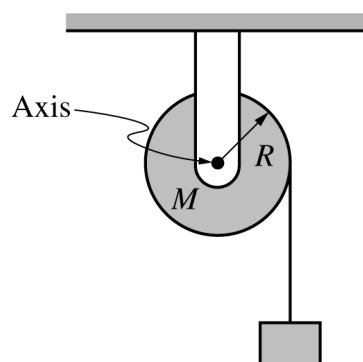
Continue your response to **QUESTION 3** on this page.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.



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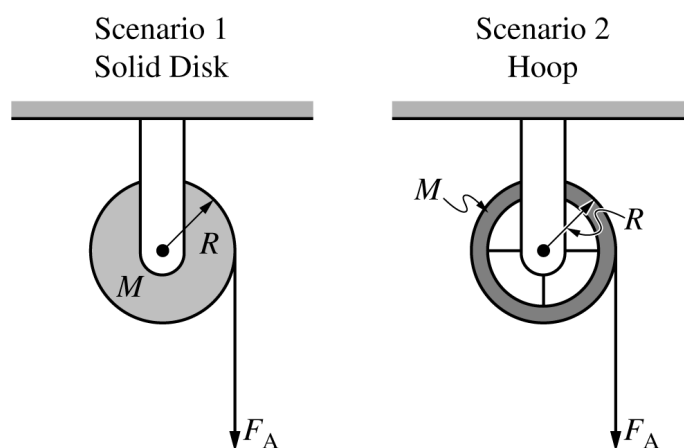
. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R . The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T .

- (a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M , R , F_T , and physical constants as appropriate.

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Continue your response to **QUESTION 4** on this page.

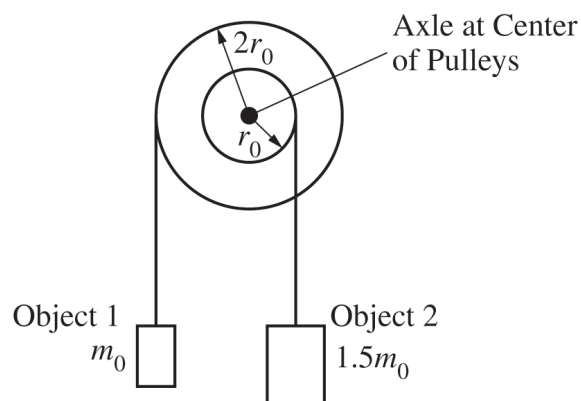
Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt , causing the pulleys to rotate without the string slipping. After time interval Δt , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval Δt . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

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Begin your response to QUESTION 5 on this page.



5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.

$1.5m_0$ is the mass of object 2.

r_0 is the radius of the smaller pulley.

$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0 , r_0 , and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

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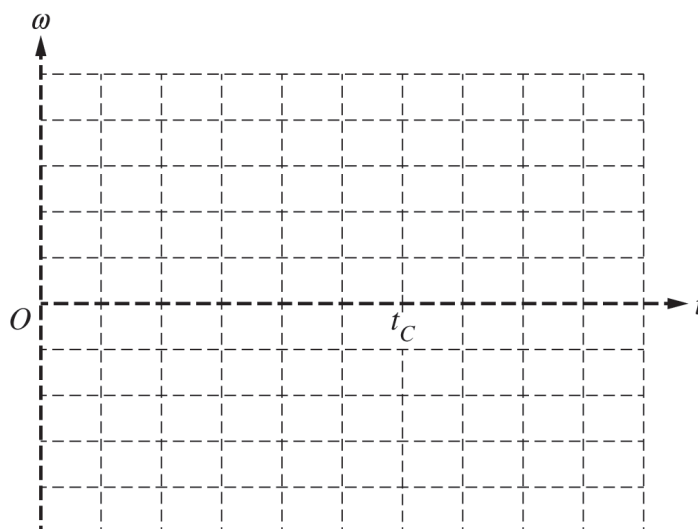
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- (b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction ___ Opposite directions

Briefly explain your reasoning.

- (c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t . Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.

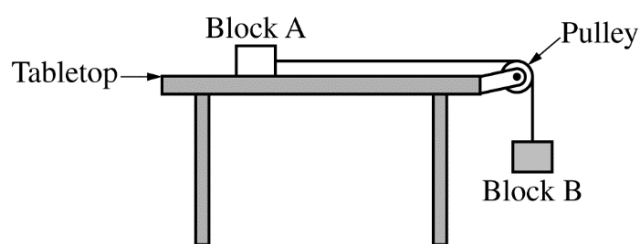


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STOP

END OF EXAM



2. (12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A , rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B , hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

- i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

- ii. Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

- (b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.



Block A



Block B

- (c) Derive an equation for the acceleration of the blocks after release in terms of m_A , m_B , and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).
- (d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B .

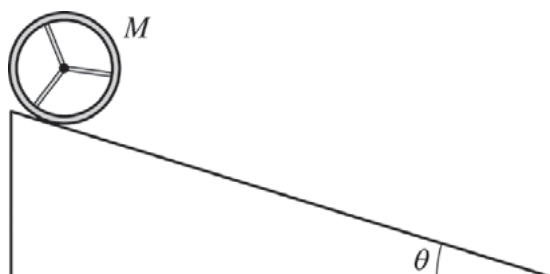
- (e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1 . Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2 . How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.

PHYSICS 1**Section II****5 Questions****Time—90 minutes**

Directions: Questions 1, 4 and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

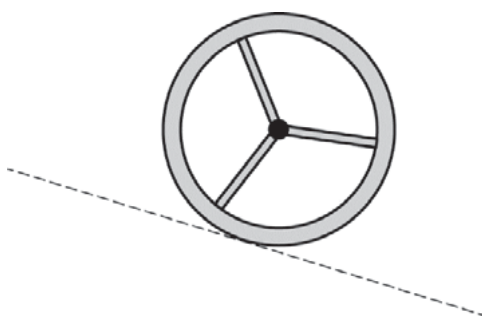


1. (7 points, suggested time 13 minutes)

A wooden wheel of mass M , consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

(a)

- i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



- ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

- (b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M , θ , and physical constants, as appropriate.

- (c) In a second experiment on the same ramp, a block of ice, also with mass M , is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

Briefly explain your answer, reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy.