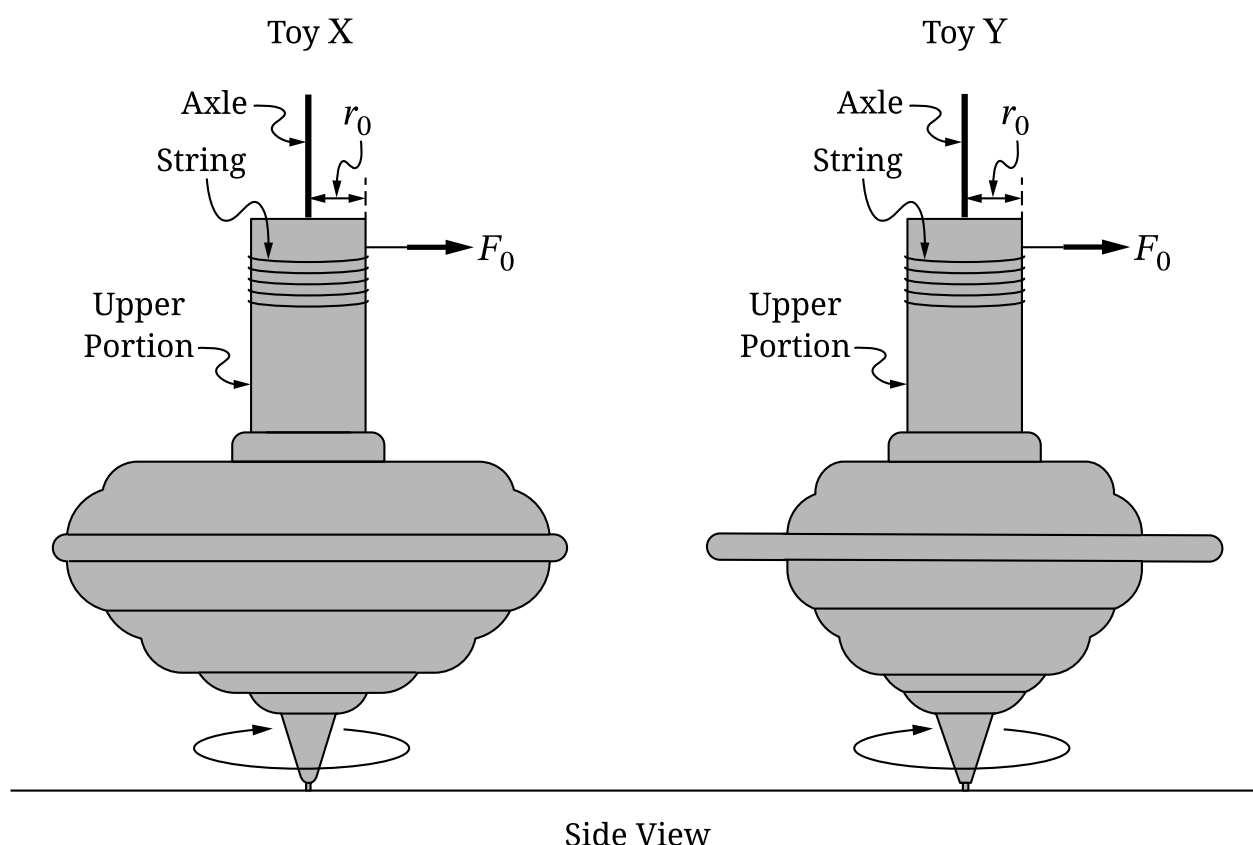


Question 4

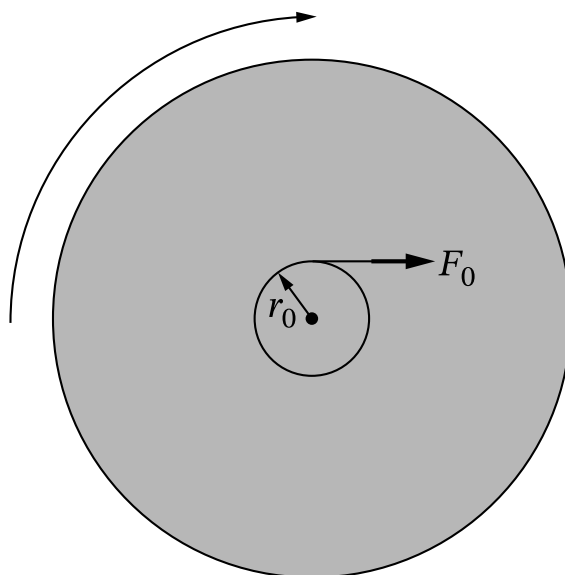
4. Toys X and Y are free to rotate with negligible friction about a vertical axle through the center of each toy. The upper portion of each toy has a radius r_0 , as shown in the side view in Figure 1. A string of length ℓ_0 is completely wrapped around the upper portion of each toy. The rotational inertia of Toy X is I_X , and the rotational inertia of Toy Y is $I_Y = \frac{1}{2}I_X$.

Figure 1

Toys X and Y are initially at rest. A constant horizontal force of magnitude F_0 is then exerted on the string of each toy, which causes the toys to rotate, as shown in the overhead view in Figure 2. The axles of the toys remain vertical as the force is exerted on the strings. The strings do not slip as the force is exerted on the strings.

When the string loses contact with the axle of Toy X, the angular speed of Toy X is ω_X .

When the string loses contact with the axle of Toy Y, the angular speed of Toy Y is ω_Y .

Figure 2

Overhead View

Part A

Indicate whether ω_Y is greater than, less than, or equal to ω_X by writing one of the following.

- $\omega_Y > \omega_X$
- $\omega_Y < \omega_X$
- $\omega_Y = \omega_X$

Justify your answer using qualitative reasoning beyond referencing equations.

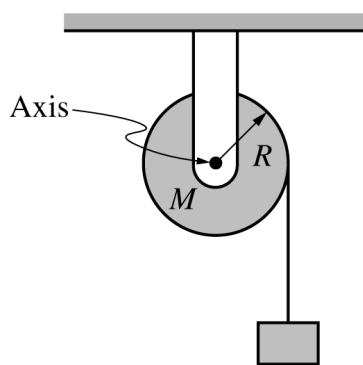
Part B

Starting with the work-energy theorem or Newton's second law in rotational form, **derive** an expression for the angular speed ω_X of Toy X at the instant the string loses contact with the axle. Express your final answer in terms of ℓ_0 , I_X , F_0 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Justify how your derived equation in part B is or is not consistent with your reasoning in part A.

STOP
END OF EXAM



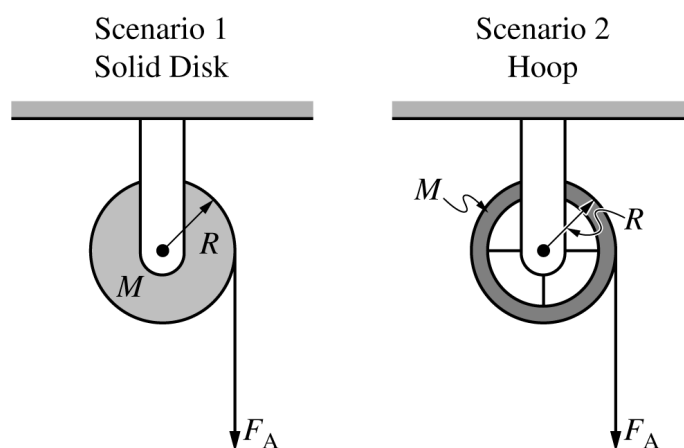
. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R . The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T .

- (a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M , R , F_T , and physical constants as appropriate.

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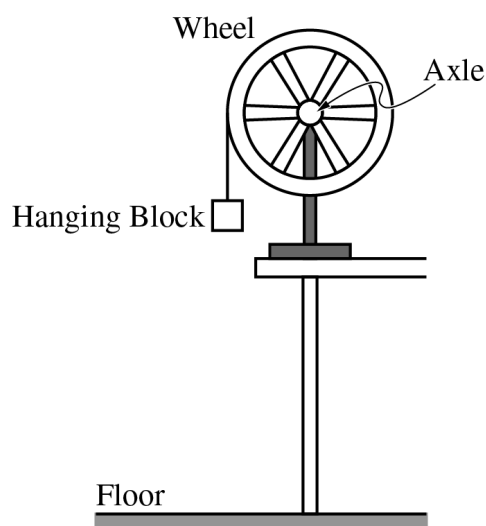
Continue your response to **QUESTION 4** on this page.

Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt , causing the pulleys to rotate without the string slipping. After time interval Δt , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval Δt . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

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. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

- (a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

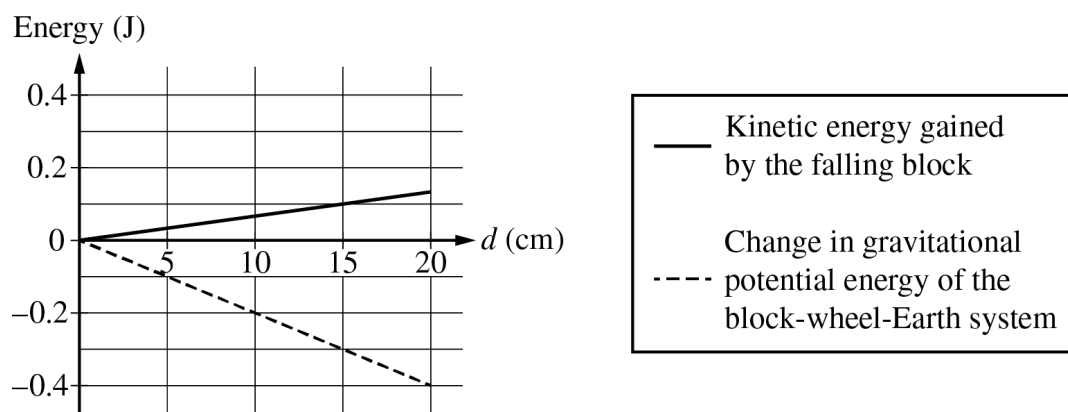
In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

If needed, you may include a simple diagram of the setup with your procedure.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement	Procedure (and diagram, if needed)

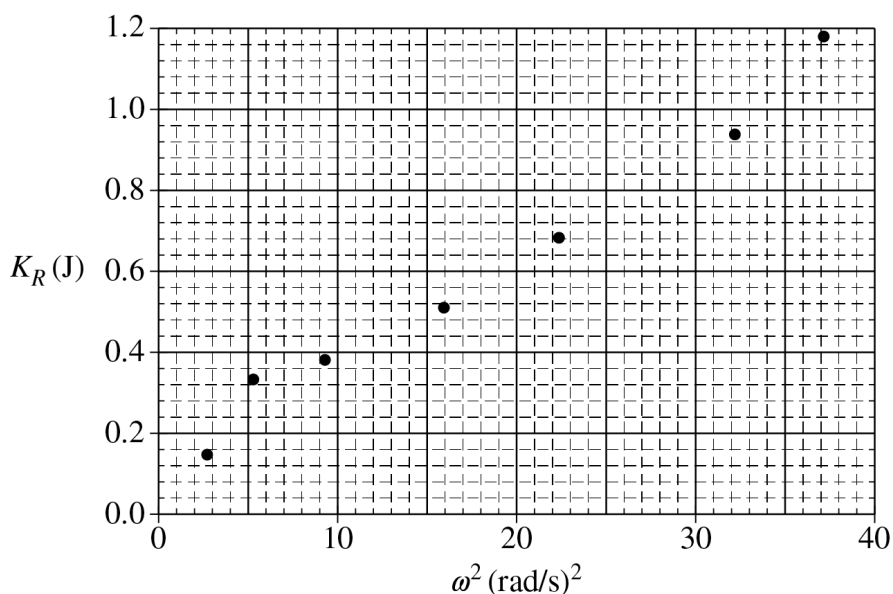
- (b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

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Continue your response to **QUESTION 3** on this page.

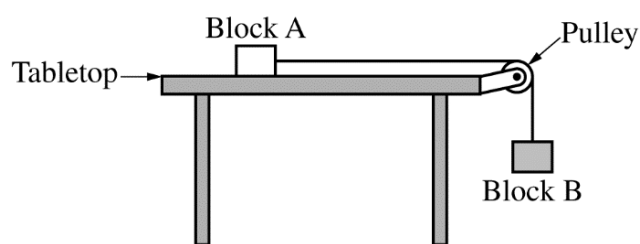
(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance d . On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance d .

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2 , as shown.



- On the above graph, draw a straight line that best represents the data.
- Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

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2. (12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A , rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B , hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

- i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

- ii. Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

- (b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.



Block A



Block B

- (c) Derive an equation for the acceleration of the blocks after release in terms of m_A , m_B , and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).
- (d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

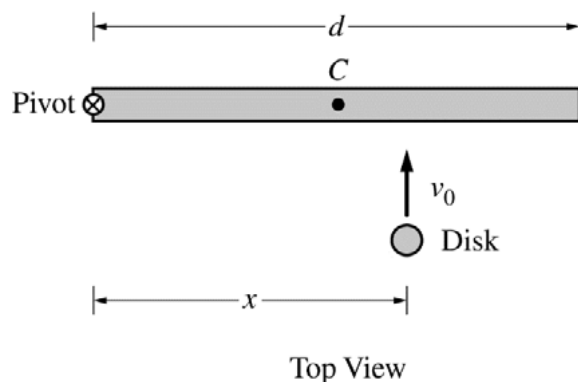
_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B .

- (e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1 . Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2 . How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.



3. (12 points, suggested time 25 minutes)

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

- (a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C , at point C , or to the right of point C ?

_____ To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

- (b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

_____ Yes _____ No

Briefly explain your reasoning without deriving an equation for ω .

- (c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

- (d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}} x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}} v_0 x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d , m_{disk} , I , x , v_0 , and physical constants, as appropriate.
- (e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.