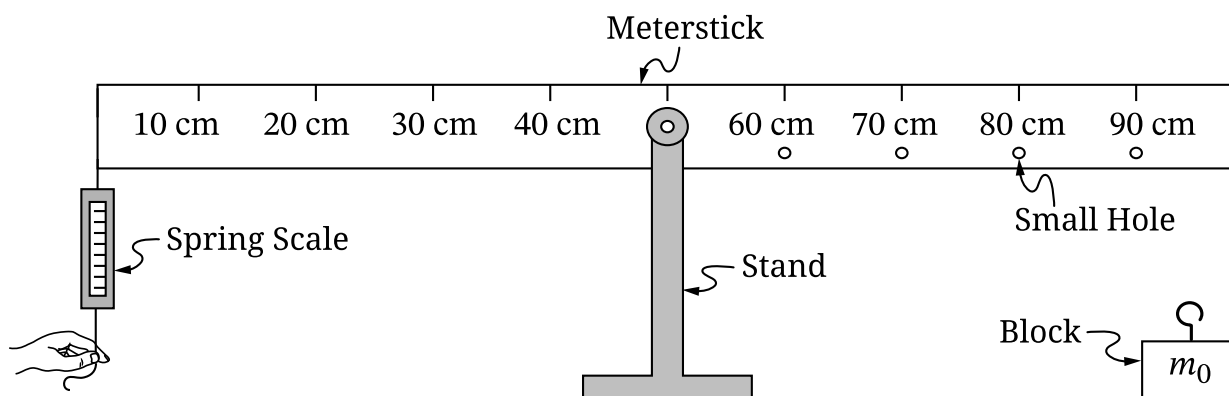


Question 3: Version J

3. Students are investigating balancing systems using the following setup. The students have a spring scale of negligible mass that is fixed to one end of a uniform meterstick. The center of the meterstick is attached to a stand on which the meterstick can pivot. There is a hook of negligible mass fixed to the top of a block of mass m_0 . The hook can be attached to the meterstick through one of the small holes in the meterstick, as shown in Figure 1. The students do not have a direct way to measure the mass of the block. The block cannot be attached to the spring scale.

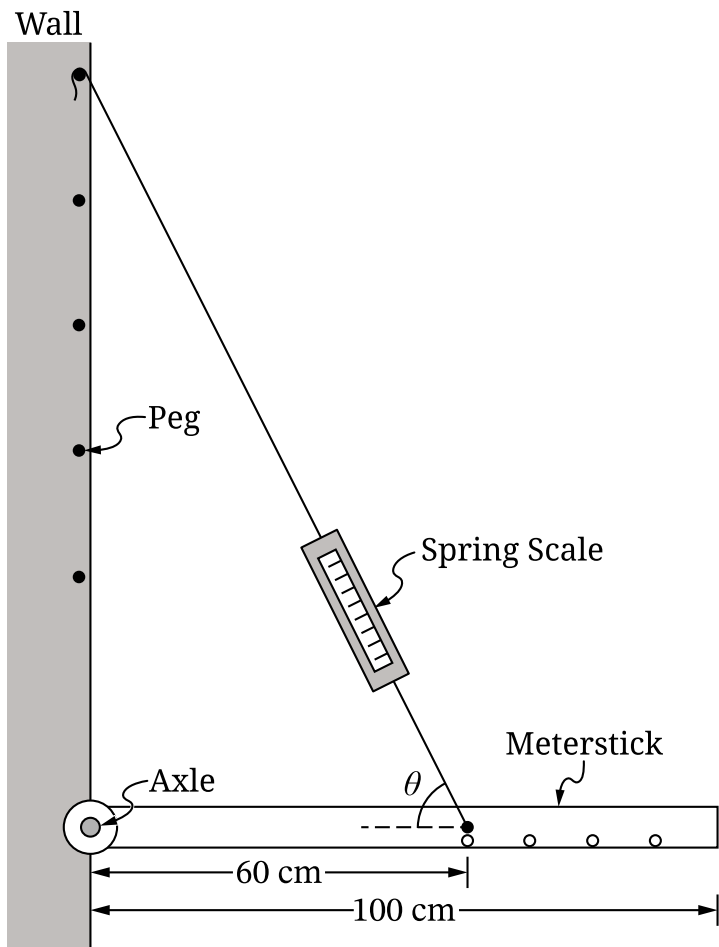
Figure 1

The students are asked to take measurements that will allow the students to create a linear graph whose slope could be used to determine the mass m_0 of the block.

- A. Describe** an experimental procedure to collect data that would allow the students to determine m_0 . Include any steps necessary to reduce experimental uncertainty.
- B. Describe** how the data collected in part A could be graphed and how that graph would be analyzed to determine m_0 .

The students have an identical meterstick of mass M that is now attached to an axle that is fixed to a wall. The meterstick is free to rotate with negligible friction about the axle. The meterstick is suspended horizontally by a string that is connected to a spring scale of negligible mass, as shown in Figure 2.

Figure 2



The angle θ that the string makes with the meterstick can be varied by attaching the string to one of the pegs located along the wall. The students use the spring scale to measure the tension F_T required to hold the meterstick horizontal. Table 1 shows the measured values of θ and F_T .

Table 1

θ (degrees)	F_T (N)
22	21
31	17
36	13
45	12
80	8

The students correctly determine that the relationship between F_T and θ is given by

$$F_T = \frac{5Mg}{6 \sin \theta}.$$

The students create a graph with $\frac{1}{\sin \theta}$ plotted on the horizontal axis.

C.

- i. **Indicate** what measured or calculated quantity could be plotted on the vertical axis to yield a linear graph whose slope can be used to calculate an experimental value for the mass M of the meterstick.

Vertical axis: _____ Horizontal axis: $\frac{1}{\sin \theta}$

- ii. On the blank grid provided, create a graph of the quantities indicated in part C (i) that can be used to determine M .

- Use Table 2 to record the data points or calculated quantities that you will plot.
- Clearly **label** the vertical axis, including units as appropriate.
- **Plot** the points you recorded in Table 2.

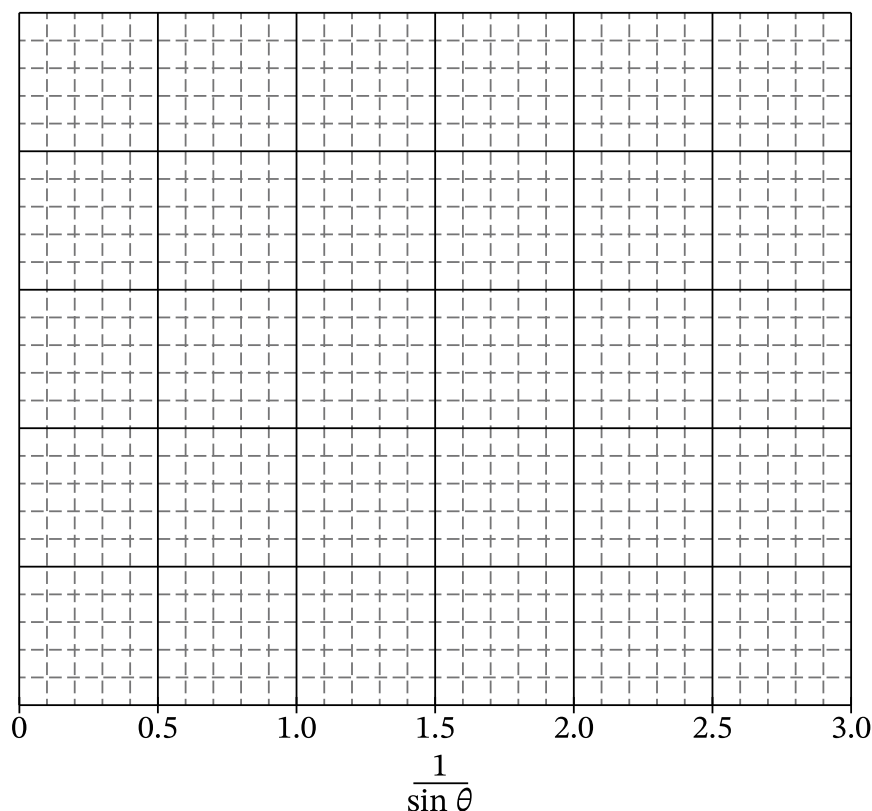


Figure 3

- iii. **Draw** a straight best-fit line for the data graphed in part C (ii).

- D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for the mass M of the meterstick.

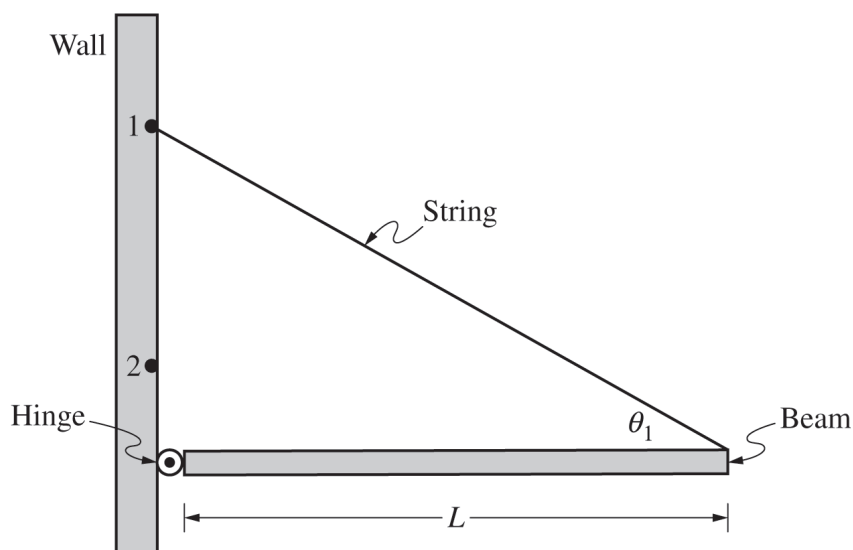
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Figure 1

3. (12 points, suggested time 25 minutes)

The left end of a uniform beam of mass M and length L is attached to a wall by a hinge, as shown in Figure 1. One end of a string with negligible mass is attached to the right end of the beam. The other end of the string is attached to the wall above the hinge at Point 1. The beam remains horizontal. The hinge exerts a force on the beam of magnitude F_H , and the angle between the beam and the string is $\theta = \theta_1$.

- (a) The following rectangle represents the beam in Figure 1. On the rectangle, **draw** and **label** the forces (not components) exerted on the beam. Draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted.

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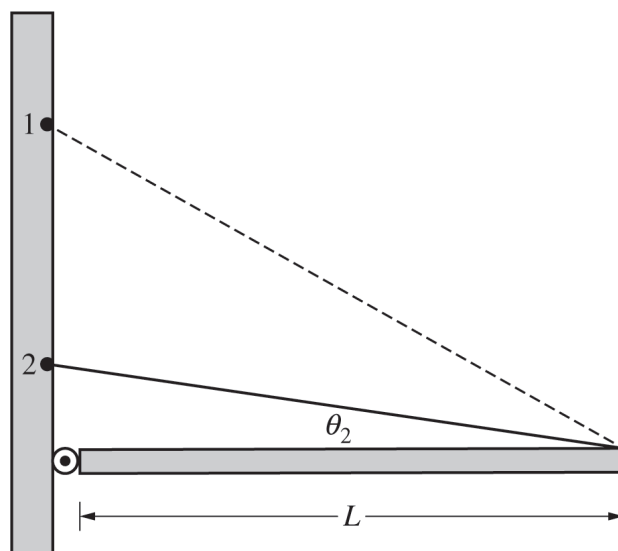
Continue your response to **QUESTION 3** on this page.

Figure 2

- (b) The string is then attached lower on the wall, at Point 2, and the beam remains horizontal, as shown in Figure 2. The angle between the beam and the string is $\theta = \theta_2$. The dashed line represents the string shown in Figure 1.

The magnitude of the tension in the string shown in Figure 1 is F_{T1} . The magnitude of the tension in the string shown in Figure 2 is F_{T2} . **Indicate** which of the following correctly compares F_{T2} with F_{T1} .

_____ $F_{T2} > F_{T1}$ _____ $F_{T2} < F_{T1}$ _____ $F_{T2} = F_{T1}$

Briefly **justify** your answer, using qualitative reasoning beyond referencing equations.

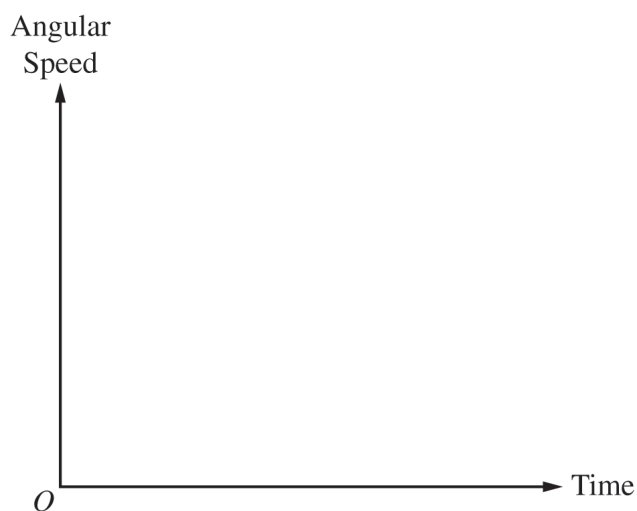
- (c) Starting with Newton's second law in rotational form, **derive** an expression for the magnitude of the tension in the string. Express your answer in terms of M , θ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference book.

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Continue your response to **QUESTION 3** on this page.

(d) Is your derived equation in part (c) consistent with your justification in part (b) ? **Explain** your reasoning.

(e) The string is cut, and the beam begins to rotate about the hinge with negligible friction. On the following axes, **sketch** the angular speed of the beam as a function of time for the time interval while the beam falls but before the beam becomes vertical.



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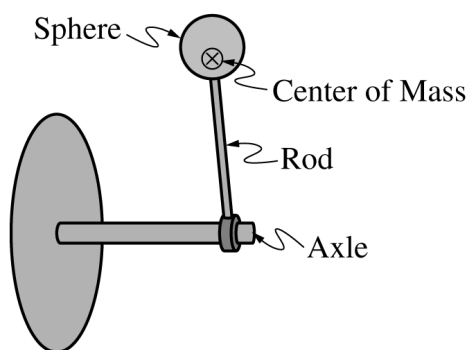


Figure 1

(7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a $\otimes$, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

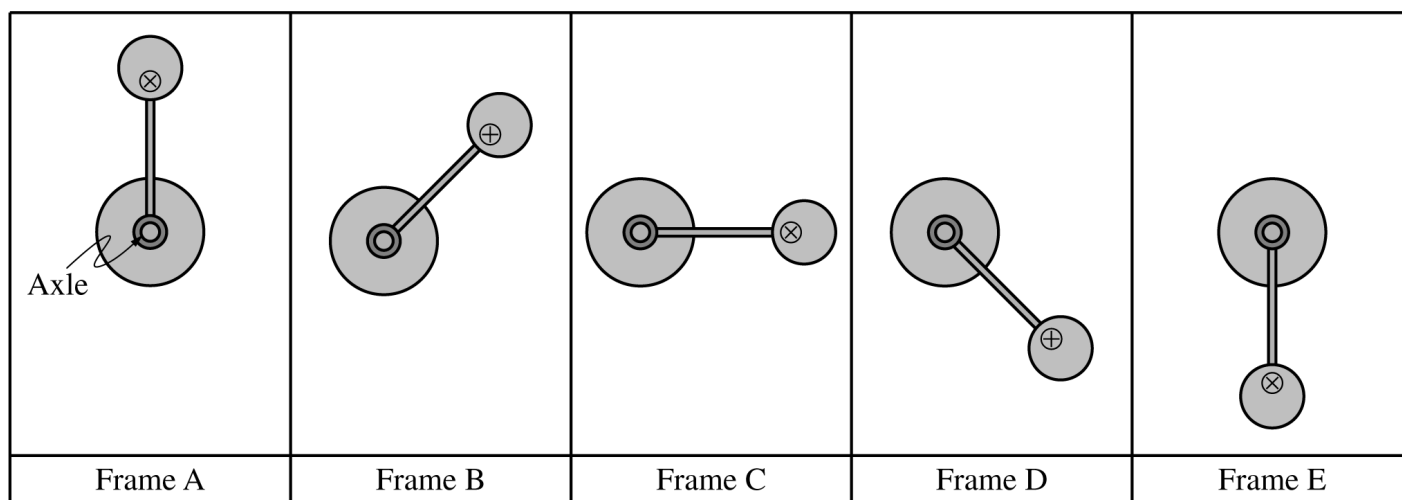


Figure 2

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Continue your response to **QUESTION 5** on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

- i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

- ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

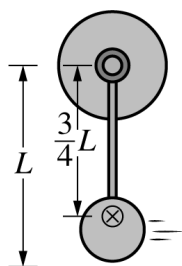


Figure 3

(b) The rod-sphere system has mass M and length L , and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

- i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M , L , and fundamental constants, as appropriate.

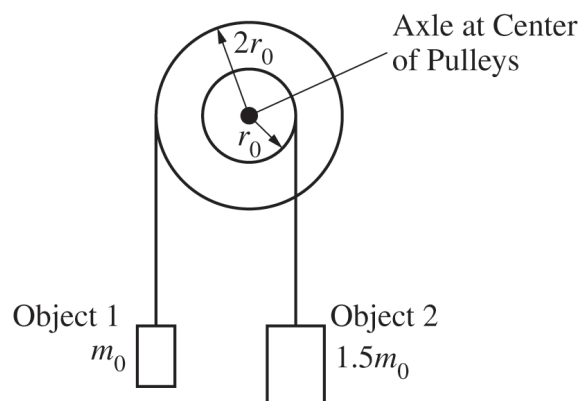
- ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

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STOP

END OF EXAM

Begin your response to QUESTION 5 on this page.



5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.

$1.5m_0$ is the mass of object 2.

r_0 is the radius of the smaller pulley.

$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0 , r_0 , and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

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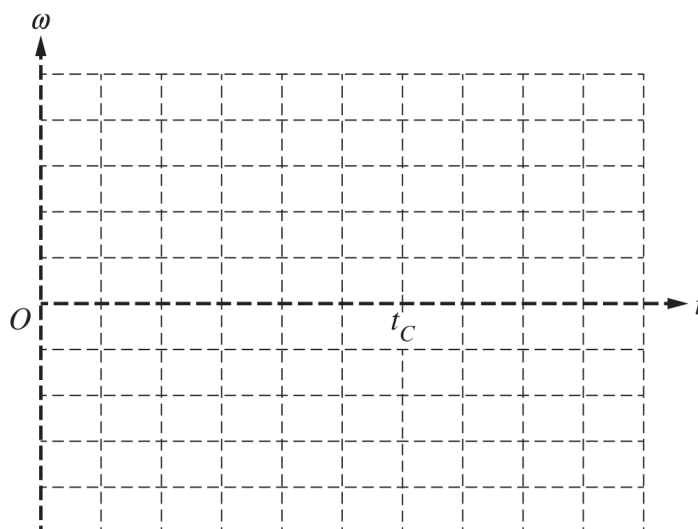
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- (b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

☐ Same direction ☐ Opposite directions

Briefly explain your reasoning.

- (c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t . Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.

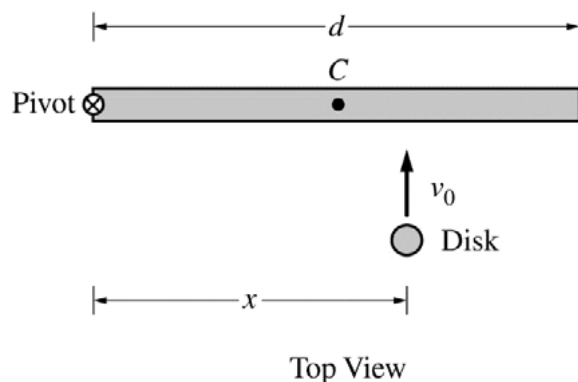


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END OF EXAM



3. (12 points, suggested time 25 minutes)

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

- (a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C , at point C , or to the right of point C ?

_____ To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

- (b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

_____ Yes _____ No

Briefly explain your reasoning without deriving an equation for ω .

- (c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

- (d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}} x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}} v_0 x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d , m_{disk} , I , x , v_0 , and physical constants, as appropriate.
- (e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.