Question 1: Mathematical Routines (MR) — 10 points

A **(i)** For sketching **one** of the following: **Point A1**

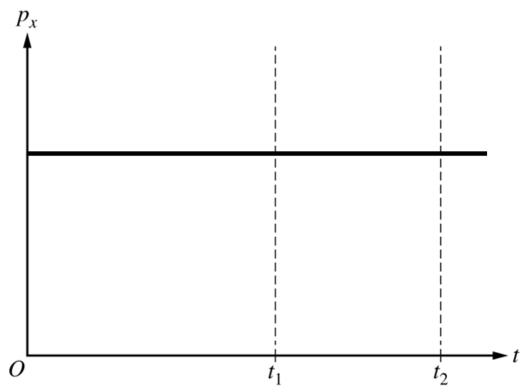
- A constant p_x for $t < t_1$
- A constant p_x for $t > t_1$

For sketching a line that demonstrates momentum is constant **Point A2**

Accept **one** of the following:

- A continuous, nonzero, horizontal line from $t = 0$ to $t = t_2$
- A continuous, nonzero, horizontal line from $t = 0$ to $t > t_2$

Example Response



(ii) For including a conservation of momentum equation **Point A3**

Scoring Note: Part A (ii) and part A (iii) may be scored together, if necessary.

For setting $m_c v_c$ equal to the momentum of the block-cart system after the collision **Point A4**

Scoring Note: A correct, isolated, final expression of $v_f = \frac{5}{6} v_c$ earns points A3 and A4.

Example Response

$p_i = p_f$

$m_c v_c = (m_c + m_b) v_f = \left(m_c + \frac{1}{5} m_c\right) v_f$

$v_f = \frac{5}{6} v_c$

(iii)	For a multistep derivation that includes the correct relationship between kinetic energy, mass, and speed (i.e., $K = \frac{1}{2}mv^2$) **Scoring Note:** The minimum requirement to earn this point is to show an expression of kinetic energy or change in energy that goes beyond the given equation on the reference information. For example, substituting quantities from the problem into $K = \frac{1}{2}mv^2$ or indicating the change in kinetic energy is $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ earns the point.	**Point A5**
	For an indication that m_c and $\frac{6}{5}m_c$ are the initial and final masses, respectively, of the objects moving horizontally	**Point A6**
	For including an expression for v_f consistent with part A (ii) **Scoring Note:** A correct, isolated, final expression of $\Delta K = -\frac{1}{12}m_c v_c^{\,2}$ earns points A3, A4, A6, and A7.	**Point A7**

Example Response

$$\Delta K = K_f - K_0$$

$$K_0 = \frac{1}{2}m_c v_c^{\,2}$$

$$K_f = \frac{1}{2}\left(\frac{6}{5}m_c\right)\left(\frac{5}{6}v_c\right)^2 = \frac{5}{12}m_c v_c^{\,2}$$

$$\Delta K = -\frac{1}{12}m_c v_c^{\,2}$$

B	For indicating “Remains constant” **Scoring Note:** This point is scored independently from the graph drawn in Figure 2 in part A (i).	**Point B1**
	For indicating **one** of the following: • The frictional force exerted on the new block by the cart is equal in magnitude and opposite in direction to the frictional force exerted on the cart by the new block. • The frictional force is internal to the new block-cart system. • The net external force is still zero. • The frictional force does not cause a net external force.	**Point B2**
	For indicating **one** of the following: • The change in momentum of the new block is equal in magnitude and opposite in direction to the change in momentum of the cart. • The impulse exerted on the cart and the impulse exerted on the new block are equal in magnitude and opposite in direction. • The correct relationship between the change in the momentum of the block-cart system and the net external force.	**Point B3**
	Example Response *The momentum of the new block-cart system will remain constant because the frictional force exerted on the new block by the cart is internal to the system and only a net external force will cause a change in momentum of the system.*	

Question 2: Experimental Design — 12 points

(a)	For measuring the mass of at least one cylinder with the digital scale	**1 point**
	For measuring the period of oscillation of the cylinder-spring system with the stopwatch	**1 point**
	For a procedure that indicates that the cylinder hung on the spring should be set into oscillatory motion	**1 point**
	For a procedure that indicates a method to reduce experimental uncertainty	**1 point**

Accept **one** of the following:

- For using multiple masses
- For doing multiple trials with a single mass
- For measuring multiple oscillations and dividing by the number of oscillations

Example Response

Place a cylinder on the digital scale and record the mass. Hang the cylinder from the spring and pull the cylinder down a small distance so that the spring is stretched. Release the cylinder. Use the stopwatch to measure the amount of time necessary for the cylinder to complete ten full cycles (from maximum stretch length back to maximum stretch length). Repeat the procedure for cylinders of different masses.

Total for part (a) 4 points

(b)(i) For listing quantities that can be measured with a stopwatch and a digital scale and could be plotted to produce a linear graph whose slope can be used to determine k **1 point**

Accept **one** of the following:

- m vs. T^2
- T^2 vs. m
- $4\pi^2 m$ vs. T^2
- T^2 vs. $4\pi^2 m$
- $\dfrac{T^2}{4\pi^2}$ vs. m
- m vs. $\dfrac{T^2}{4\pi^2}$
- T vs. $\sqrt{m}$
- $\sqrt{m}$ vs. T
- $\dfrac{T}{2\pi}$ vs. $\sqrt{m}$
- $\sqrt{m}$ vs. $\dfrac{T}{2\pi}$
- T vs. $2\pi\sqrt{m}$
- $2\pi\sqrt{m}$ vs. T

Scoring Note: This point may be earned for any of the bullets above substituting $\dfrac{1}{f}$ for T .

Example Response

Vertical axis: m Horizontal axis: T^2

(b)(ii)	For correctly relating the slope of the best-fit line to the value of k	**1 point**

Example Response

Plotting the mass as a function of the period-squared would result in a graph whose slope could be used to find k by using the equation for the period of an oscillating cylinder-spring system.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{slope} = \frac{k}{4\pi^2}$$

$$k = (\text{slope})4\pi^2$$

Graph	**Slope**	k
m vs. T^2	$\text{slope} = \frac{k}{4\pi^2}$	$k = (\text{slope})4\pi^2$
T^2 vs. m	$\text{slope} = \frac{4\pi^2}{k}$	$k = \frac{4\pi^2}{\text{slope}}$
$4\pi^2 m$ vs. T^2	$\text{slope} = k$	$k = \text{slope}$
T^2 vs. $4\pi^2 m$	$\text{slope} = \frac{1}{k}$	$k = \frac{1}{\text{slope}}$
$\frac{T^2}{4\pi^2}$ vs. m	$\text{slope} = \frac{1}{k}$	$k = \frac{1}{\text{slope}}$
m vs. $\frac{T^2}{4\pi^2}$	$\text{slope} = k$	$k = \text{slope}$
T vs. $\sqrt{m}$	$\text{slope} = \sqrt{\frac{4\pi^2}{k}}$	$k = \frac{4\pi^2}{\text{slope}^2}$
$\sqrt{m}$ vs. T	$\text{slope} = \sqrt{\frac{k}{4\pi^2}}$	$k = \text{slope}^2 \times 4\pi^2$
$\frac{T}{2\pi}$ vs. $\sqrt{m}$	$\text{slope} = \sqrt{\frac{1}{k}}$	$k = \frac{1}{\text{slope}^2}$
$\sqrt{m}$ vs. $\frac{T}{2\pi}$	$\text{slope} = \sqrt{k}$	$k = \text{slope}^2$
T vs. $2\pi\sqrt{m}$	$\text{slope} = \sqrt{\frac{1}{k}}$	$k = \frac{1}{\text{slope}^2}$
$2\pi\sqrt{m}$ vs. T	$\text{slope} = \sqrt{k}$	$k = \text{slope}^2$

Total for part (b) **2 points**

(c)(i) For using the kinetic energy equation $K = \frac{1}{2}mv^2$ **1 point**

For substituting **one** of the following: **1 point**

- 0.25 kg as the mass
- 0.3 m/s as the initial velocity

For an answer that approximates the change in kinetic energy to be $\Delta K \approx -0.0113 \text{ J}$ **1 point**

Scoring Note: A correct response with no supporting work earns this point only.

Scoring Note: The unit and the negative sign are not required to earn this point.

Example Response

$$\Delta K = K_f - K_i$$
$$\Delta K = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}$$
$$\Delta K = \frac{1}{2}0.25(0)^2 - \frac{1}{2}0.25(0.3)^2$$
$$\Delta K = -0.0113 \text{ J}$$

(c)(ii) For indicating the magnitude of the change in momentum is zero **1 point**

For indicating the area under the force-time graph represents the value of the change in momentum **1 point**

Example Response

The area under the curve for a force vs time graph represents the impulse or change in momentum. The area under the curve for 0.5 s *to* 2.5 s *is zero.*

(c)(iii) For an explanation that compares the estimated value of the change in momentum from (c)(ii) to the data from the velocity-time graph **1 point**

Example Response

The velocity-time graph shows that velocity is 0.3 m/s *at both* 0.5 s *and* 2.5 s*, and momentum is mass times velocity, so the momentum is the same at both times. This agrees with my estimation from part (c)(ii) that the change in momentum is zero.*

Total for part (c) 6 points

Total for question 2 12 points

Question 3: Qualitative/Quantitative Translation — 12 points

(a) i. For a correct answer $v_D = \dfrac{F_H t_f}{M_D}$ — **1 point**

ii. For indicating the total momentum of the system is the same before and after the collision — **1 point**

Scoring Note: If the response only includes a correct final answer of $\dfrac{M_S}{M_D}$, the response earns this point but not the next point.

For correctly substituting the appropriate variables into a conservation of momentum equation — **1 point**

AND

an answer in the form $\dfrac{v_D}{v_S} = \ \ldots$

Scoring Notes:

This point can be earned only if the first point is earned.

The answer need not be correct to earn this point.

Example response for part (a)(ii)

$p_i = p_f$

$0 = M_S v_S - M_D v_D$

$\dfrac{v_D}{v_S} = \dfrac{M_S}{M_D}$

Total for part (a) — 3 points

(b) For two functions that are straight segments for $t < t_f$, — **1 point**

AND

begin at the origin,

AND

have two different positive slopes

For two functions that are horizontal functions for $t > t_f$ — **1 point**

AND

are continuous over the entire time range $0 < t < 2t_f$

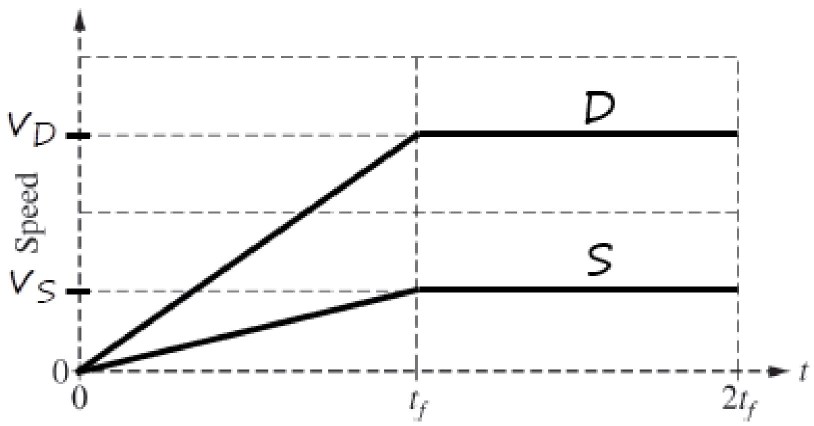
For labeling values on the vertical axis with $v_D > v_S$ — **1 point**

OR

The curve labeled D is greater than the curve labeled S for all t > 0

Scoring note: This point can still be earned if the labels are not on the vertical axis but clearly indicate that $v_D > v_S$.

Example response for part (b)



		Total for part (b)	**3 points**
(c)	**i.**	For stating or mathematically representing that if the disk is much more massive, then the block will have little effect on the motion of disk 1 **OR** For stating or mathematically representing that when $M_D >> M_B$, $v_{cm} = v_1$	**1 point**
	ii.	For correct reasoning. Correct answer: When $M_D << M_B$, $v_{cm} = 0$	**1 point**
		Example response for part (c) (ii) *If the block is much more massive, then it will barely move when the disk collides and sticks to it.*	
	iii.	For using conservation of momentum	**1 point**
		For a correct answer $v_{cm} = \dfrac{m_D v_1}{m_D + m_B}$	**1 point**
	iv.	For an attempt to use limiting-case reasoning or functional dependence with the equation in part (c)(iii)	**1 point**
		For recognizing the equation from (c)(iii) reduces to a simpler form and the simplified form is correctly compared to their answer in (c)(i)	**1 point**
		Example 1 response for part (c) (iv) *Yes. If M_B is very small, then the denominator of the equation simplifies to M_D, which then can cancel out of the equation leaving $v_{cm} = v_1$.*	
		Total for part (c)	**6 points**
		Total for question 3	**12 points**