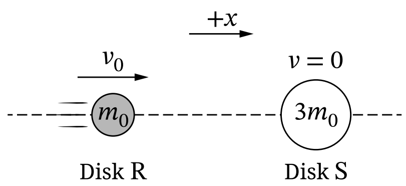
Question 2: Version J

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the $+x$-direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the $-x$-direction.

Figure 1



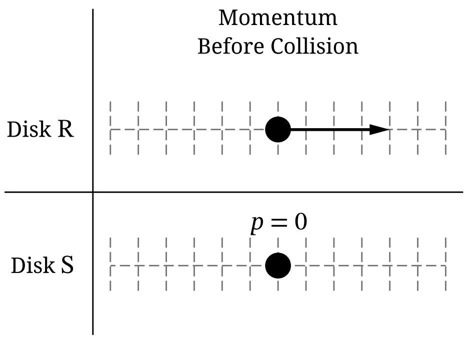
Top View

<u>Note</u>: Figure not drawn to scale.

Part A

The momentum-vector diagram in Figure 2 represents the momentums of Disk R and Disk S before the collision.

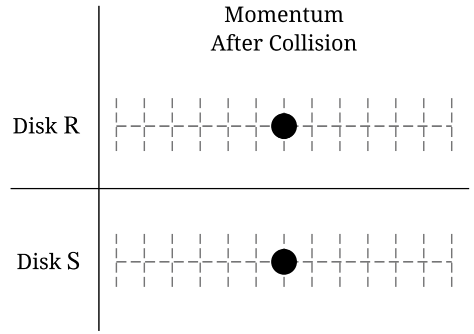
Figure 2



Draw arrows on Figure 3 to represent the momentum vectors of Disk R and Disk S after the collision.

- Each arrow must start on, and point away from, each dot.
- If the momentum of either disk is zero, write "$p = 0$" next to the dot.
- Draw the length of each arrow to represent the magnitude of each momentum, consistent with the scale used in Figure 2.

Figure 3



Part B

Starting with conservation of linear momentum, **derive** an expression for the kinetic energy of Disk S immediately after the collision. Express your final answer in terms of m_0, v_0, and physical constants, as appropriate. Begin your derivation by writing the fundamental physics principle or an equation from the reference information.

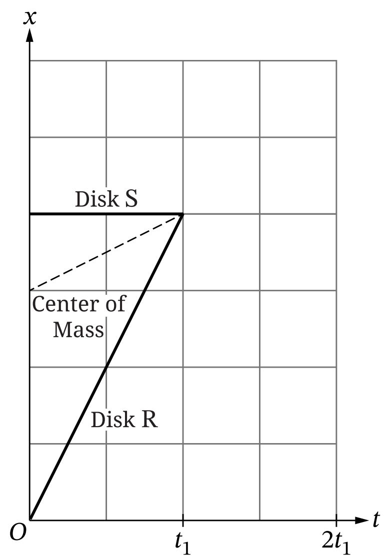
Part C

The graph shown in Figure 4 represents the positions x as functions of time t from $t = 0$ until $t = t_1$ for each of the following:

- Disk R
- Disk S
- The center of mass of the two-disk system

On the graph shown in Figure 4, **draw** three lines that represent the positions x of Disk R, Disk S, and the center of mass of the two-disk system as functions of t from $t = t_1$ to $t = 2t_1$. Distinctly **label** each line.

Figure 4



Part D

During the collision, the magnitudes of the changes in momentum of Disk R and Disk S are $|\Delta p_R|$ and $|\Delta p_S|$, respectively.

Indicate whether $|\Delta p_R|$ is greater than, less than, or equal to $|\Delta p_S|$ by writing one of the following.

- $|\Delta p_R| > |\Delta p_S|$
- $|\Delta p_R| < |\Delta p_S|$
- $|\Delta p_R| = |\Delta p_S|$

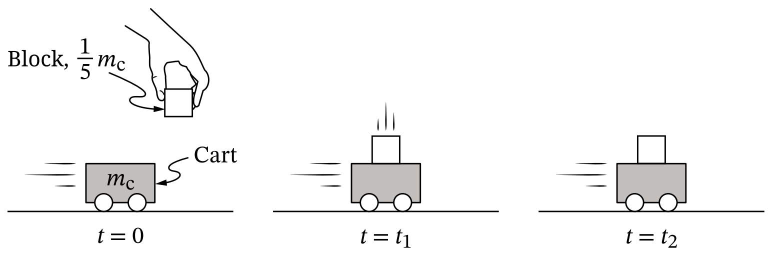
Briefly **justify** your response by referencing a fundamental physics principle.

Question 1: Version J

1. A student has a cart of mass m_c and a block of mass $\frac{1}{5}m_c$, as shown in Figure 1.

- At time $t = 0$, the cart is moving to the right across a horizontal surface with constant speed v_c, and the student releases the block from rest.
- At $t = t_1$, the block collides with and sticks to the top of the cart. The block does not slide on the cart.
- At $t = t_2$, the block-cart system continues to move to the right with constant speed v_f.

Figure 1



A.

i. On the axes shown in Figure 2, **sketch** a graph of the magnitude p_x of the x-component of the momentum of the block-cart system as a function of time t from $t = 0$ until $t > t_2$.

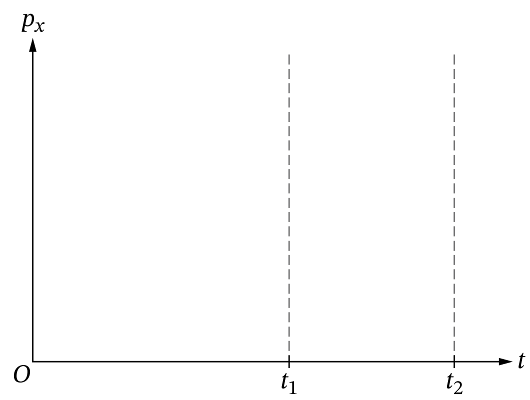


Figure 2

ii. **Derive** an expression for the speed v_f of the block-cart system after time $t = t_2$ in terms of m_c, v_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. **Derive** an expression for the change in the kinetic energy ΔK in the block-cart system from $t = 0$ to $t = t_2$ in terms of m_c, v_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider the case where a new block is dropped and collides with the top of the cart. The new block slides along the cart during the collision but does not slide off the cart. The time interval from when the new block collides with the cart and moves together with the cart is Δt. During Δt there is a frictional force between the new block and the cart.

Indicate whether the x-component of the momentum of the new block-cart system increases, decreases, or remains constant during Δt.

______ Increases
______ Decreases
______ Remains constant

Justify your response.

Begin your response to **QUESTION 2** on this page.

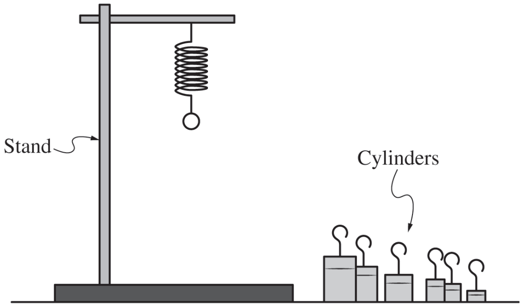


Figure 1

2. (12 points, suggested time 25 minutes)

A student hangs a spring of unknown spring constant k vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

(a) Design an experimental procedure the student could use to determine the spring constant k of the spring.

In the following table, list the quantities that would be measured using only the provided equipment in your experiment. Define a symbol to represent each quantity.

In the space below the table, **describe** the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. If needed, you may include a simple diagram of the setup with your procedure.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement
		Stopwatch
		Digital scale
Procedure (and diagram, if needed)		

(b)

i. **Indicate** the quantities that could be plotted to produce a linear graph whose slope can be used to determine the spring constant k of the spring.

Vertical axis:____________________ Horizontal axis:____________________

ii. Briefly **describe** how the slope of the graph would be analyzed to determine the spring constant k of the spring.

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Continue your response to **QUESTION 2** on this page.

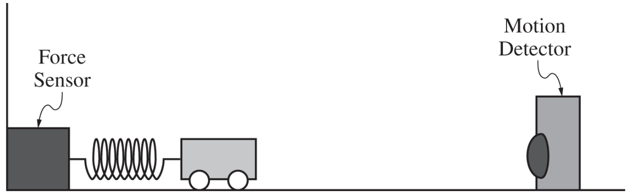
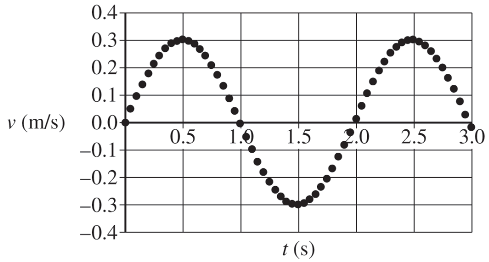
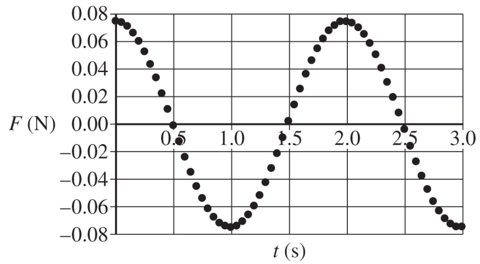


Figure 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity v of the cart and the force F exerted on the cart by the spring as functions of time t.





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Continue your response to **QUESTION 2** on this page.

(c)

i. Using the data in the velocity-time graph, **calculate** the change in kinetic energy of the cart from $t = 0.5$ s to $t = 2.0$ s. Show your steps and substitutions.

ii. Using the data in the force-time graph, **estimate** the change in momentum of the cart from $t = 0.5$ s to $t = 2.5$ s. Briefly **explain** how you arrived at your estimation.

iii. Do the data from the velocity-time graph confirm your estimation from part (c)(ii) ? Briefly **explain**.

GO ON TO THE NEXT PAGE.

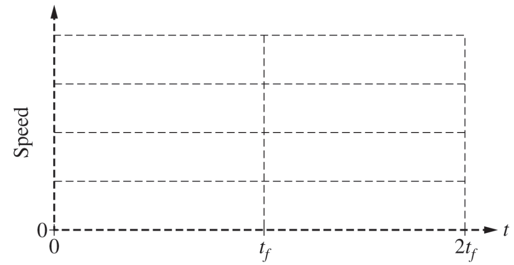
3. (12 points, suggested time 25 minutes)

(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

(b) Assume that the student's mass is greater than that of the disk ($M_S > M_D$). On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.



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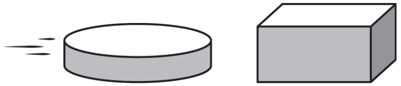

(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

____ Yes ____ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

GO ON TO THE NEXT PAGE.