

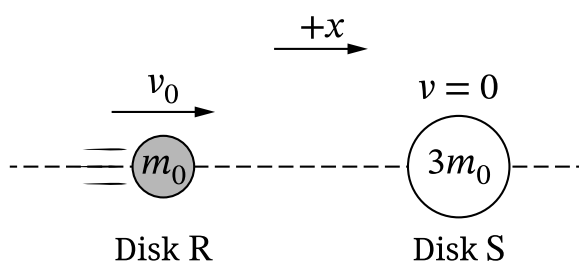
Question 2: Version J

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the $+x$ -direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the $-x$ -direction.

Figure 1



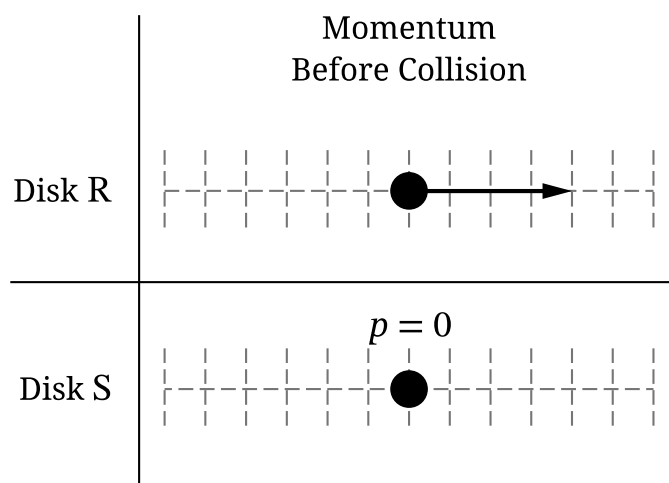
Top View

Note: Figure not drawn to scale.

Part A

The momentum-vector diagram in Figure 2 represents the momentums of Disk R and Disk S before the collision.

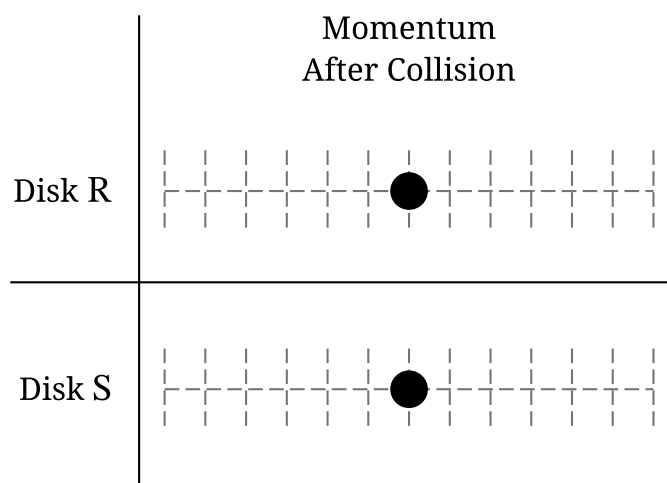
Figure 2



Draw arrows on Figure 3 to represent the momentum vectors of Disk R and Disk S after the collision.

- Each arrow must start on, and point away from, each dot.
- If the momentum of either disk is zero, write “ $p = 0$ ” next to the dot.
- Draw the length of each arrow to represent the magnitude of each momentum, consistent with the scale used in Figure 2.

Figure 3



Part B

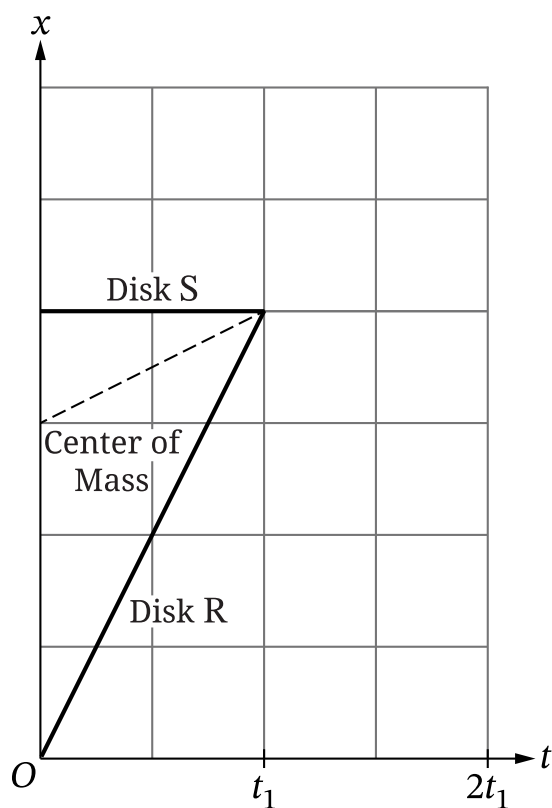
Starting with conservation of linear momentum, **derive** an expression for the kinetic energy of Disk S immediately after the collision. Express your final answer in terms of m_0 , v_0 , and physical constants, as appropriate. Begin your derivation by writing the fundamental physics principle or an equation from the reference information.

Part C

The graph shown in Figure 4 represents the positions x as functions of time t from $t = 0$ until $t = t_1$ for each of the following:

- Disk R
- Disk S
- The center of mass of the two-disk system

On the graph shown in Figure 4, **draw** three lines that represent the positions x of Disk R, Disk S, and the center of mass of the two-disk system as functions of t from $t = t_1$ to $t = 2t_1$. Distinctly **label** each line.

Figure 4**Part D**

During the collision, the magnitudes of the changes in momentum of Disk R and Disk S are $|\Delta p_R|$ and $|\Delta p_S|$, respectively.

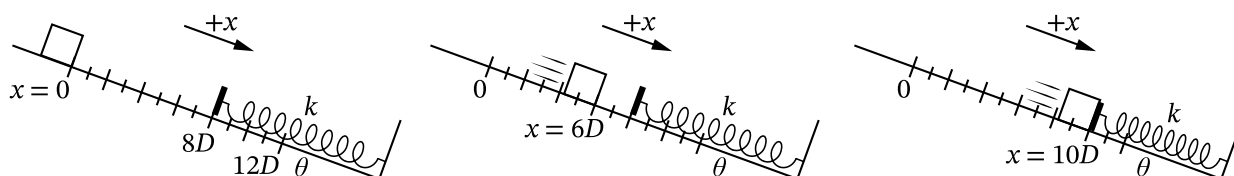
Indicate whether $|\Delta p_R|$ is greater than, less than, or equal to $|\Delta p_S|$ by writing one of the following.

- $|\Delta p_R| > |\Delta p_S|$
- $|\Delta p_R| < |\Delta p_S|$
- $|\Delta p_R| = |\Delta p_S|$

Briefly **justify** your response by referencing a fundamental physics principle.

Question 2: Version J

2. A block of mass M is released from rest at position $x = 0$ near the top of a ramp. The ramp makes an angle of θ with the horizontal. The block slides down the ramp with negligible friction. At $x = 8D$ the block makes contact with an uncompressed spring with spring constant k . The spring is then compressed and the block momentarily comes to rest at $x = 12D$. Figure 1 shows the instants when the block is at $x = 0$, $x = 6D$, and $x = 10D$, respectively.



- A. Figure 4 shows an energy bar chart that represents the kinetic energy K of the block, the gravitational potential energy U_g of the block-spring-Earth system, and the spring potential energy U_s of the block-spring-Earth system at the instant that the block is at $x = 10D$. The gravitational potential energy U_g of the block-spring-Earth system is defined to be zero when the block momentarily comes to rest at $x = 12D$.

Draw shaded bars that represent K , U_g , and U_s to complete the energy bar charts in Figure 2 and Figure 3 for when the block is released from rest at $x = 0$ and for when the block is at $x = 6D$, respectively.

- Shaded bars should start at the dashed line that represents zero energy.
- Represent any energy that is equal to zero with a distinct line on the zero-energy line.
- The relative heights of each shaded bar should reflect the magnitude of the respective energy consistent with the scale used in Figure 4.

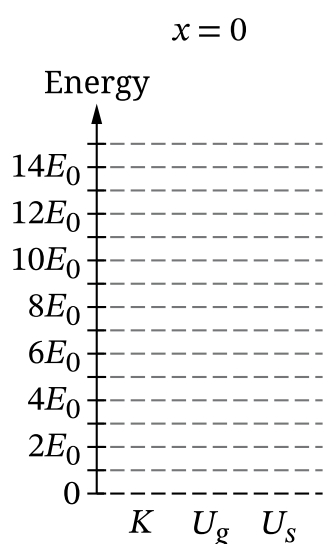


Figure 2

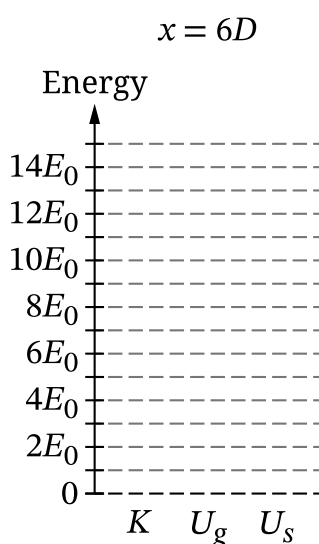


Figure 3

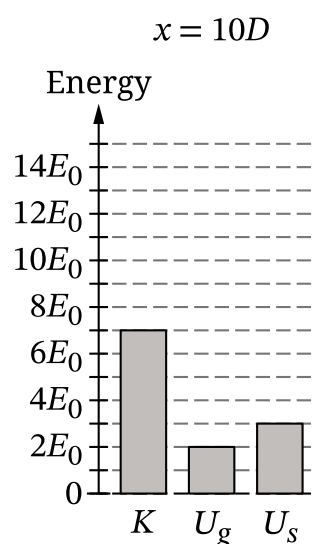


Figure 4

- B. Figure 5 shows the block at $x = 0$ when the block is released from rest and the block at $x = 12D$ when the block momentarily comes to rest against the compressed spring.

Figure 5



Starting with conservation of energy, **derive** an equation for the spring constant k . Express your answer in terms of M , θ , D , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. Figure 6 shows a graph of the energy of the system as a function of the position of the block from $x = 8D$ to $x = 12D$. The spring potential energy U_s of the block-spring-Earth system is shown on the graph.

On the axes shown in Figure 6, do the following.

- Sketch** and **label** a line or curve that represents the total mechanical energy E for the block-spring-Earth system as a function of the position of the block from $x = 8D$ to $x = 12D$.
- Sketch** and **label** a line or curve that represents the gravitational potential energy U_g for the block-spring-Earth system as a function of the position of the block from $x = 8D$ to $x = 12D$.

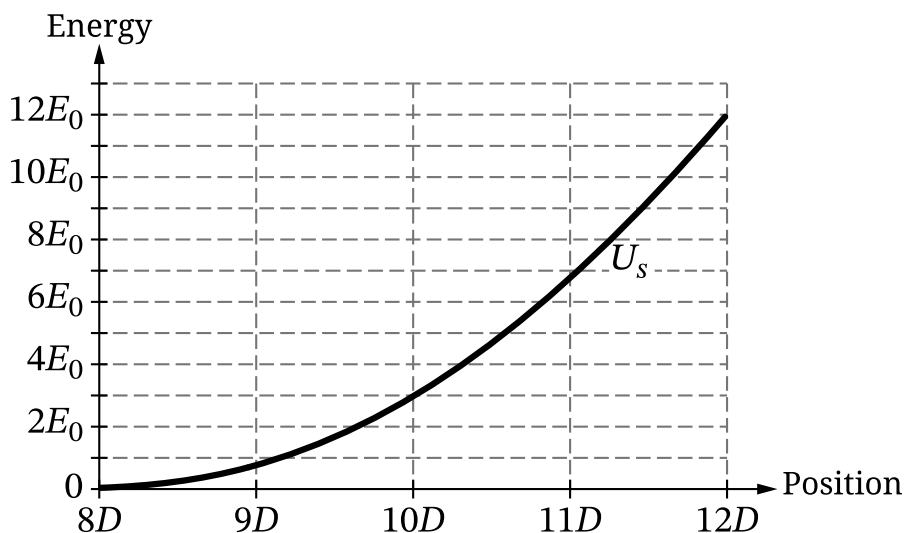


Figure 6

D. Indicate whether the speed v_{9D} of the block at $x = 9D$ is greater than, less than, or equal to the speed v_{8D} of the block at $x = 8D$.

_____ $v_{9D} > v_{8D}$

_____ $v_{9D} < v_{8D}$

_____ $v_{9D} = v_{8D}$

Justify how your response is consistent with the energy lines or curves you drew in Figure 6 in part C.

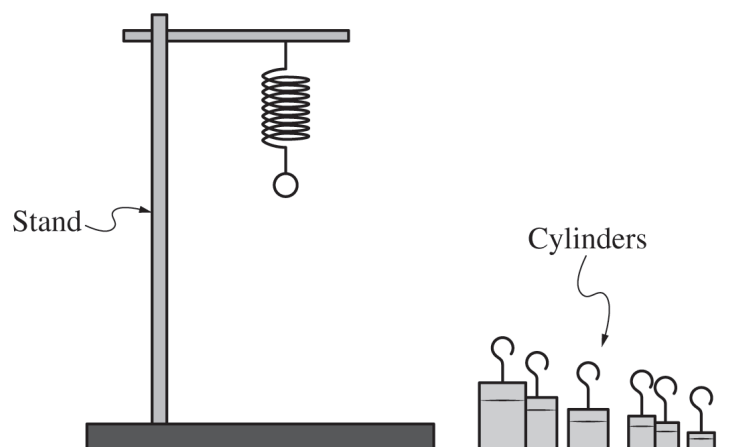
Begin your response to **QUESTION 2** on this page.

Figure 1

2. (12 points, suggested time 25 minutes)

A student hangs a spring of unknown spring constant k vertically by attaching one end to a stand, as shown in Figure 1. The other end of the spring has a small loop from which small cylinders can be hung. In addition to the spring, the student has access only to a variety of cylinders of unknown masses, a stopwatch, and a digital scale.

(a) Design an experimental procedure the student could use to determine the spring constant k of the spring.

In the following table, list the quantities that would be measured using only the provided equipment in your experiment. Define a symbol to represent each quantity.

In the space below the table, **describe** the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. If needed, you may include a simple diagram of the setup with your procedure.

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Continue your response to **QUESTION 2** on this page.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement
		Stopwatch
		Digital scale
Procedure (and diagram, if needed)		

- (b)
- i. **Indicate** the quantities that could be plotted to produce a linear graph whose slope can be used to determine the spring constant k of the spring.
- Vertical axis: _____ Horizontal axis: _____
- ii. Briefly **describe** how the slope of the graph would be analyzed to determine the spring constant k of the spring.

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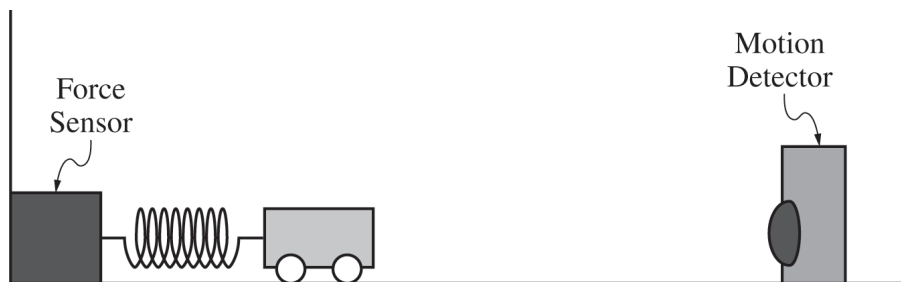
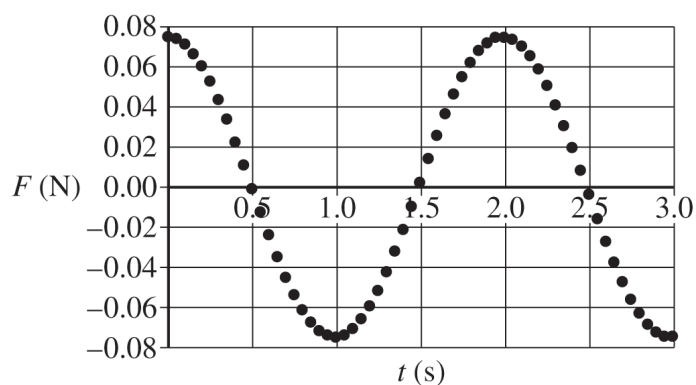
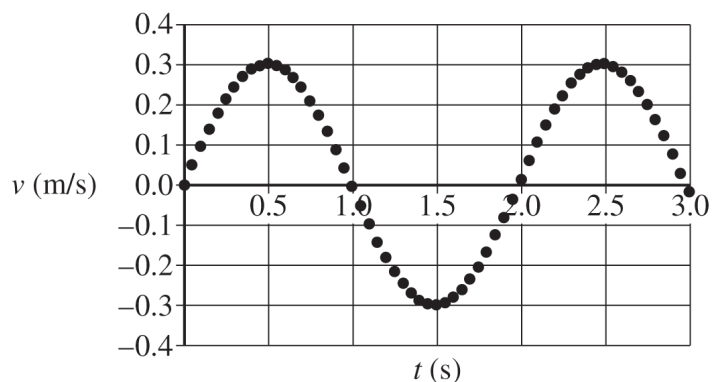
Continue your response to **QUESTION 2** on this page.

Figure 2

In a different experiment, the student attaches one end of a spring to a force sensor that is attached to a wall. The other end of the spring is attached to a cart with mass $m = 0.25$ kg. The student places a motion detector to the right of the cart, as shown in Figure 2, and pulls the cart to the right a small distance so that the spring is stretched. The student releases the cart from rest, and the cart-spring system oscillates.

The following graphs show the velocity v of the cart and the force F exerted on the cart by the spring as functions of time t .

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Continue your response to **QUESTION 2** on this page.

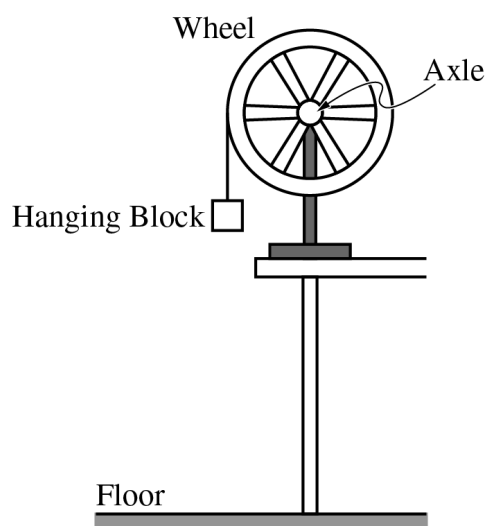
(c)

i. Using the data in the velocity-time graph, **calculate** the change in kinetic energy of the cart from $t = 0.5$ s to $t = 2.0$ s. Show your steps and substitutions.

ii. Using the data in the force-time graph, **estimate** the change in momentum of the cart from $t = 0.5$ s to $t = 2.5$ s. Briefly **explain** how you arrived at your estimation.

iii. Do the data from the velocity-time graph confirm your estimation from part (c)(ii) ? Briefly **explain**.

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. (12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

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Continue your response to **QUESTION 3** on this page.

- (a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

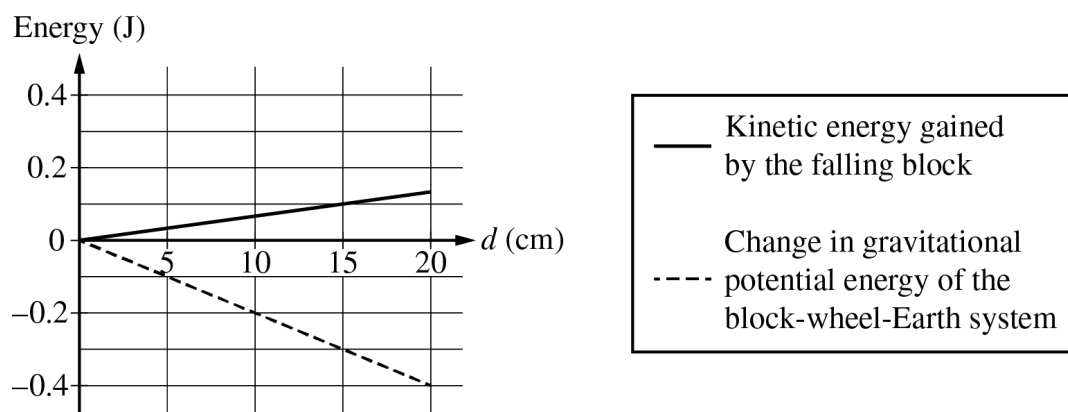
In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

If needed, you may include a simple diagram of the setup with your procedure.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement	Procedure (and diagram, if needed)

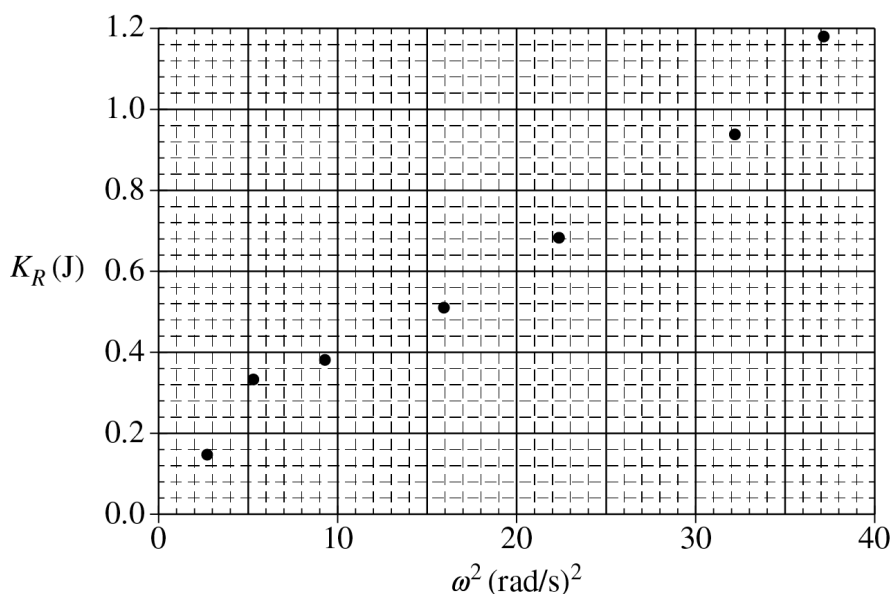
- (b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).

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Continue your response to **QUESTION 3** on this page.

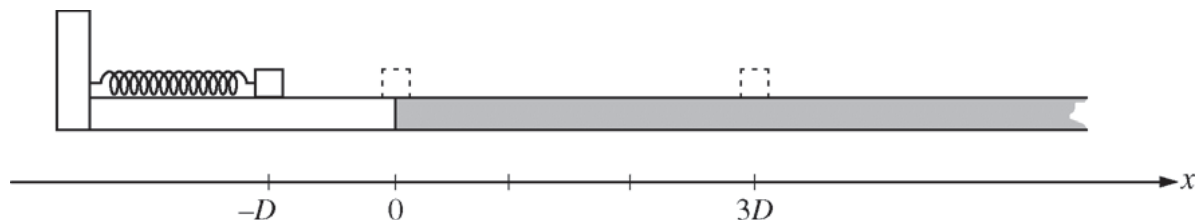
(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance d . On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance d .

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2 , as shown.



- On the above graph, draw a straight line that best represents the data.
- Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

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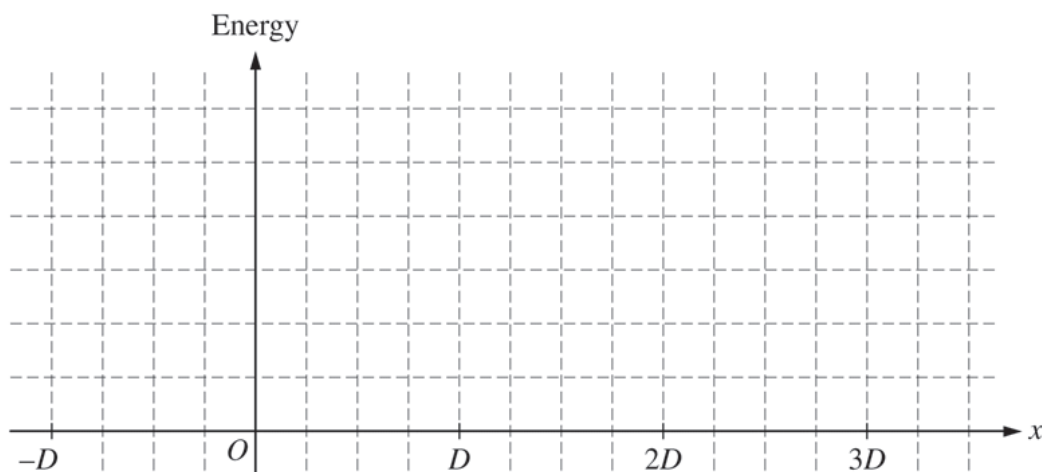


3. (12 points, suggested time 25 minutes)

A block is initially at position $x = 0$ and in contact with an uncompressed spring of negligible mass. The block is pushed back along a frictionless surface from position $x = 0$ to $x = -D$, as shown above, compressing the spring by an amount $\Delta x = D$. The block is then released. At $x = 0$ the block enters a rough part of the track and eventually comes to rest at position $x = 3D$. The coefficient of kinetic friction between the block and the rough track is μ .

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between $x = -D$ and $x = 3D$. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

- The kinetic energy K of the block
- The potential energy U of the block-spring system



The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at $x = 6D$. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position $x = 6D$.

- (b)
- Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.
 - Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.
- (c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D .
- (d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.