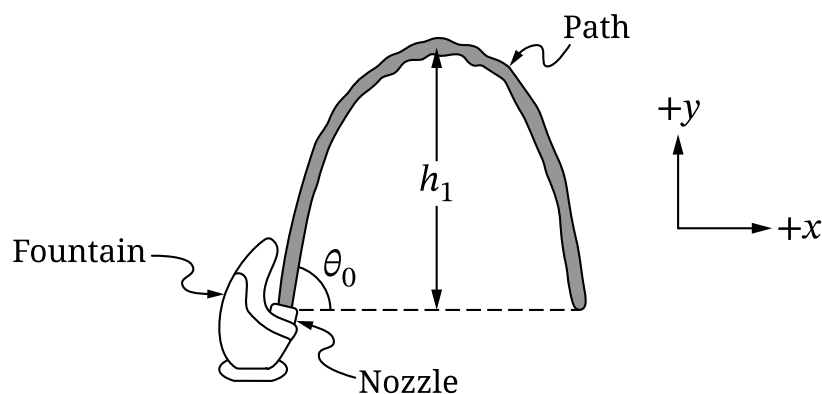


Question 1: Version J

1. Water exits the nozzle of a fountain at an angle θ_0 above the horizontal.

At time $t = 0$, a droplet of water exits the nozzle and follows the path shown in Figure 1. The droplet reaches a maximum height h_1 above the nozzle.

Figure 1



Note: Figure not drawn to scale.

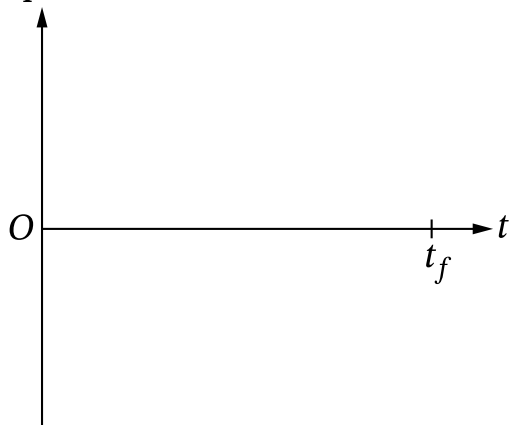
At $t = t_f$, the droplet returns to the same height at which the droplet exited the nozzle.

Part A

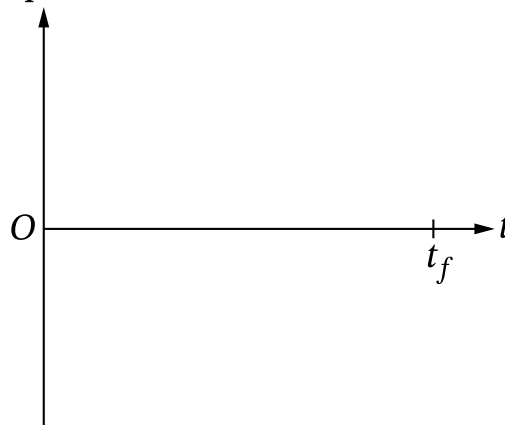
- i. On the axes shown in Figure 2, **sketch** graphs of the horizontal and vertical components of the velocity of the water droplet as functions of t from $t = 0$ to $t = t_f$.

Figure 2

Horizontal
Velocity
Component



Vertical
Velocity
Component



- ii. **Derive** an expression for the speed of the water exiting the nozzle. Express your final answer in terms of θ_0 , h_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
- iii. The nozzle has a circular cross-section with radius r_0 . **Derive** an expression for the volume flow rate of the water exiting the nozzle. Express your final answer in terms of θ_0 , h_1 , r_0 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part B

The fountain nozzle is replaced with a new nozzle with a radius smaller than r_0 . The water exits the new nozzle at the same angle θ_0 above the horizontal. The volume flow rate of the water exiting the new nozzle is equal to the volume flow rate of the water exiting the original nozzle. A water droplet exiting the new nozzle reaches a maximum height h_2 above the nozzle.

Indicate whether h_2 is greater than, less than, or equal to h_1 by writing one of the following.

- $h_2 > h_1$
- $h_2 < h_1$
- $h_2 = h_1$

Justify your answer. In your justification, include qualitative reasoning beyond mathematical derivations or expressions.

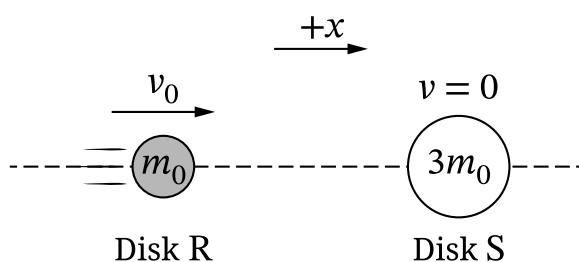
Question 2: Version J

2. Two small disks, R and S, are on a straight, horizontal track.

At time $t = 0$, Disk R of mass m_0 is located at position $x = 0$. Disk R is initially moving with speed v_0 in the $+x$ -direction. Disk S of mass $3m_0$ is initially at rest, as shown in the top view in Figure 1. Frictional forces are negligible.

At $t = t_1$, the disks collide. Immediately after the collision, Disk R moves with speed $\frac{1}{2}v_0$ in the $-x$ -direction.

Figure 1



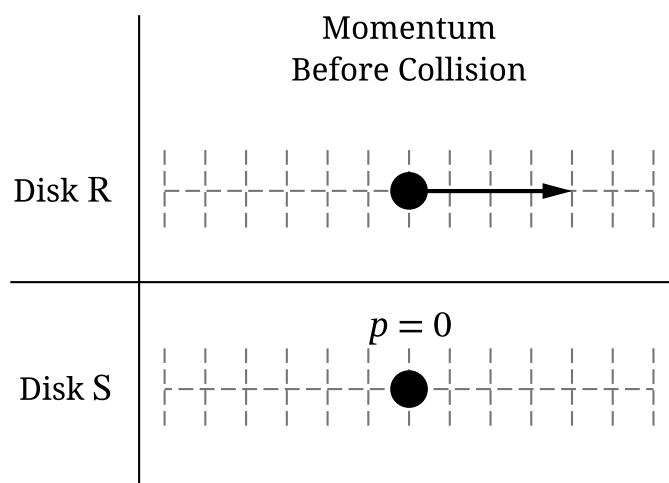
Top View

Note: Figure not drawn to scale.

Part A

The momentum-vector diagram in Figure 2 represents the momentums of Disk R and Disk S before the collision.

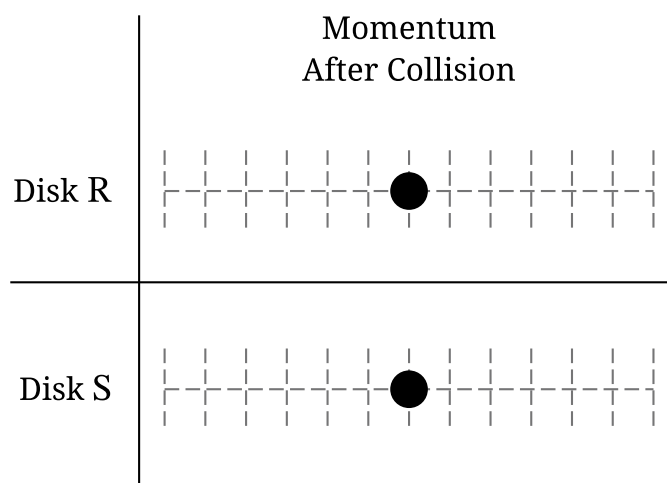
Figure 2



Draw arrows on Figure 3 to represent the momentum vectors of Disk R and Disk S after the collision.

- Each arrow must start on, and point away from, each dot.
- If the momentum of either disk is zero, write “ $p = 0$ ” next to the dot.
- Draw the length of each arrow to represent the magnitude of each momentum, consistent with the scale used in Figure 2.

Figure 3



Part B

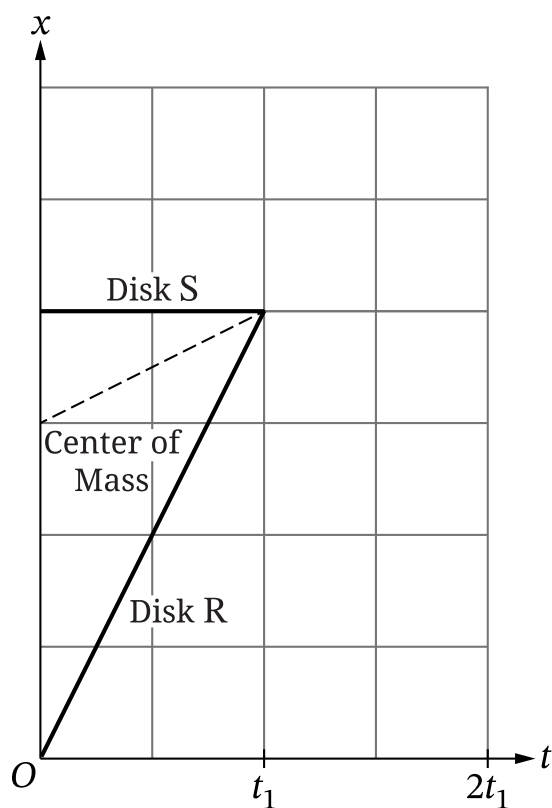
Starting with conservation of linear momentum, **derive** an expression for the kinetic energy of Disk S immediately after the collision. Express your final answer in terms of m_0 , v_0 , and physical constants, as appropriate. Begin your derivation by writing the fundamental physics principle or an equation from the reference information.

Part C

The graph shown in Figure 4 represents the positions x as functions of time t from $t = 0$ until $t = t_1$ for each of the following:

- Disk R
- Disk S
- The center of mass of the two-disk system

On the graph shown in Figure 4, **draw** three lines that represent the positions x of Disk R, Disk S, and the center of mass of the two-disk system as functions of t from $t = t_1$ to $t = 2t_1$. Distinctly **label** each line.

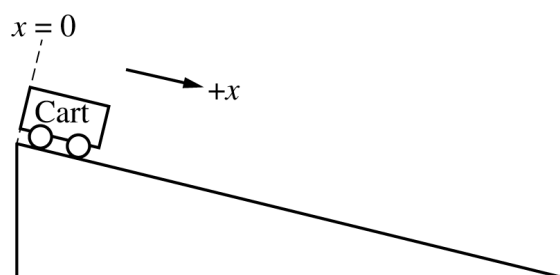
Figure 4**Part D**

During the collision, the magnitudes of the changes in momentum of Disk R and Disk S are $|\Delta p_R|$ and $|\Delta p_S|$, respectively.

Indicate whether $|\Delta p_R|$ is greater than, less than, or equal to $|\Delta p_S|$ by writing one of the following.

- $|\Delta p_R| > |\Delta p_S|$
- $|\Delta p_R| < |\Delta p_S|$
- $|\Delta p_R| = |\Delta p_S|$

Briefly **justify** your response by referencing a fundamental physics principle.



(12 points, suggested time 25 minutes)

- (a) Students conduct an experiment to determine the acceleration a of a cart. The cart is released from rest at the top of the ramp at time $t = 0$ and moves down the ramp. The x -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position x (m)	Time t (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

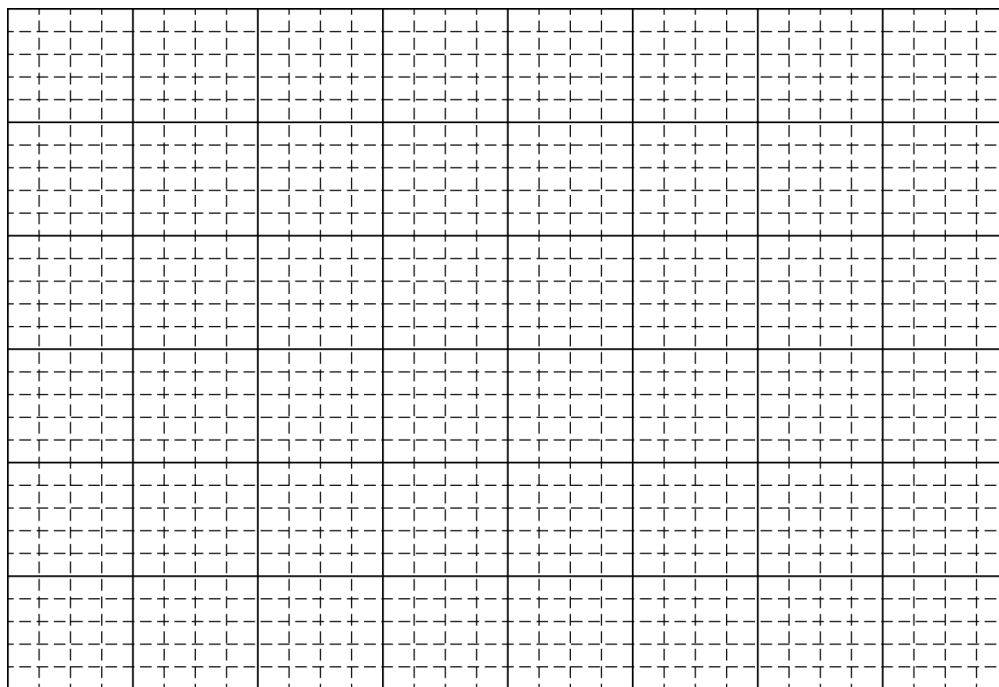
- i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration a of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis: _____ Horizontal axis: _____

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration a of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



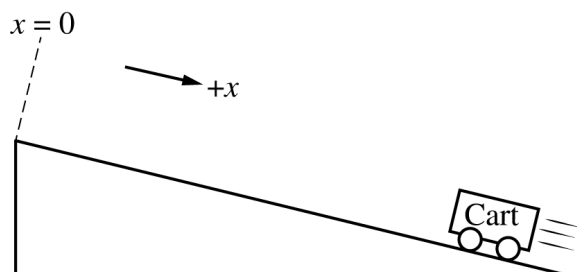
- iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration a of the cart as it rolls down the ramp.
- (b) The students are asked to determine an experimental value for the acceleration due to gravity g_{exp} using their data.
- i. What additional quantities do the students need to measure in order to calculate g_{exp} from a ?
- ii. Write an expression for the value of g_{exp} in terms of a .

GO ON TO THE NEXT PAGE.

(c) The students calculate the value of g_{exp} to be significantly lower than the accepted value of 9.8 m/s^2 .

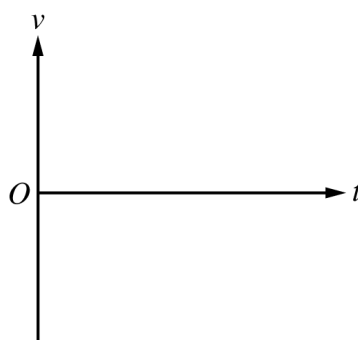
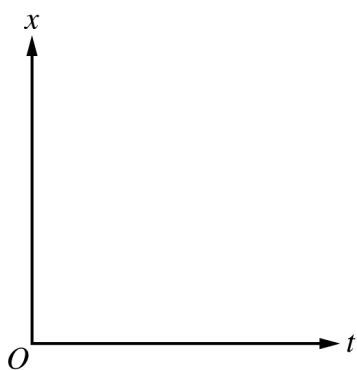
i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of g_{exp} ?

ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of g_{exp} .

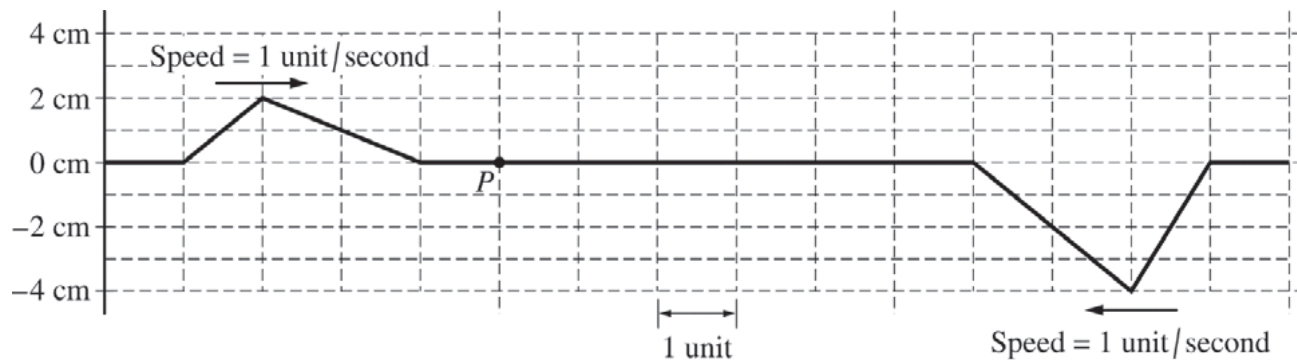


The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The x -axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position x and velocity v as functions of time t that correspond to the scenario shown while the cart moves up the ramp.



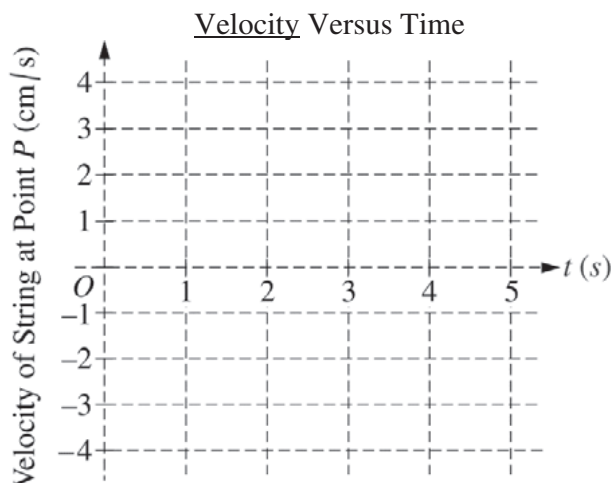
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5. (7 points, suggested time 13 minutes)

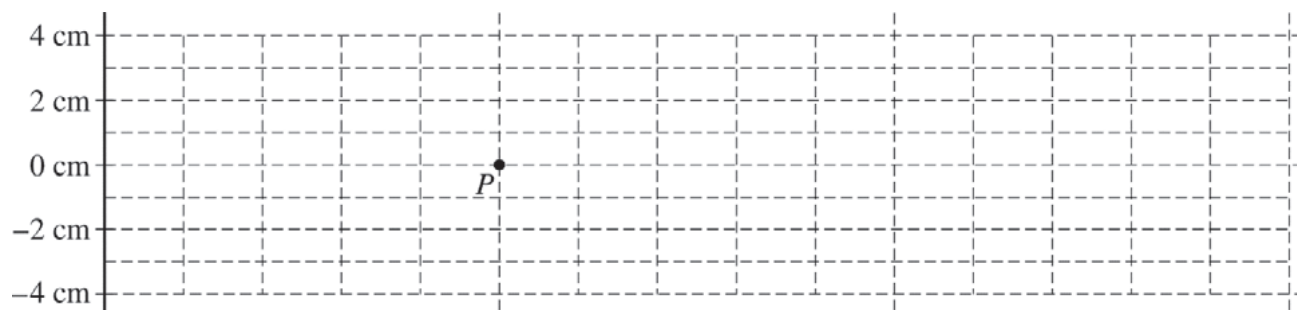
Two wave pulses are traveling in opposite directions on a string. The shape of the string at $t = 0$ is shown above. Each pulse is moving with a speed of one unit per second in the direction indicated.

- (a) Between time $t = 0$ and $t = 5$ seconds, the entire left-hand pulse approaches and moves beyond point P on the string. On the coordinate axes below, plot the velocity of the piece of string located at point P as a function of time between $t = 0$ and $t = 5$ seconds.

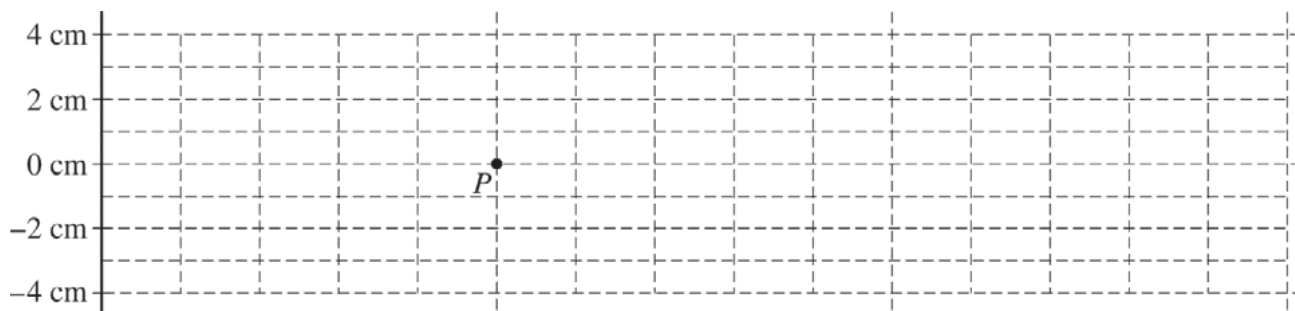
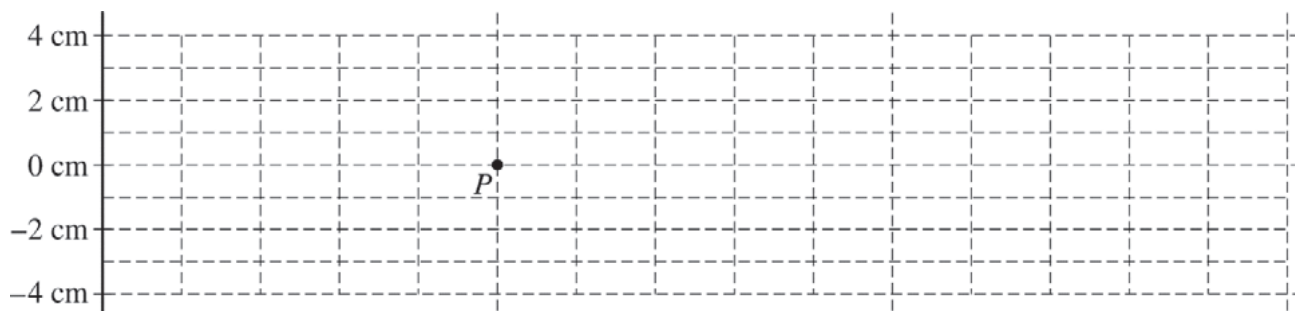


- (b) At $t = 5$ s, the pulses completely overlap. On the grid provided below, sketch the shape of the entire string at $t = 5$ s.

Note: Do any scratch (practice) work on the grids on the following page. You will only be graded for the sketch made on the grid on this page.

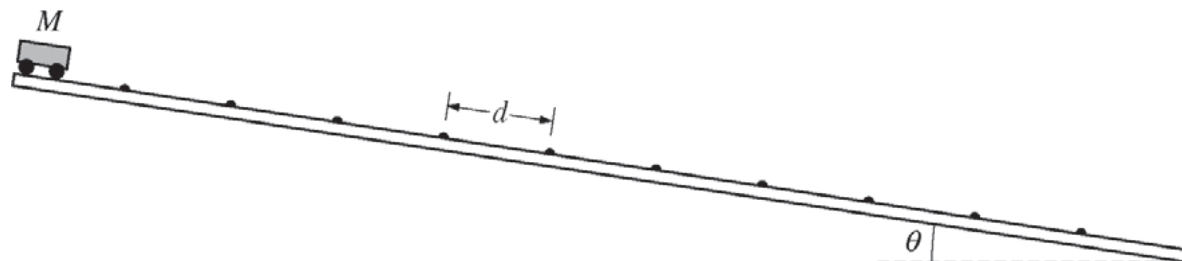


The grids below are provided for scratch work only. Sketches made below will NOT be graded.



STOP

END OF EXAM



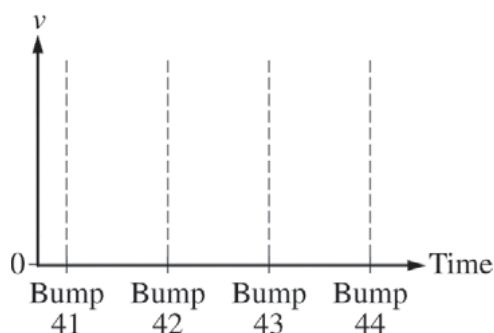
Note: Figure not drawn to scale.

3. (12 points, suggested time 25 minutes)

A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg} . This means the time interval taken to move from one bump to the next bump becomes constant.

(a) Consider the cart's motion between bump 41 and bump 44.

- In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.
- Over the same time interval, draw a dashed horizontal line at $v = v_{\text{avg}}$. Label this line " v_{avg} ".



(b) Suppose the distance between the bumps is increased but everything else stays the same.

Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together?

____ Greater than ____ Less than ____ The same as

Briefly explain your reasoning.

(c) With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ . Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller?

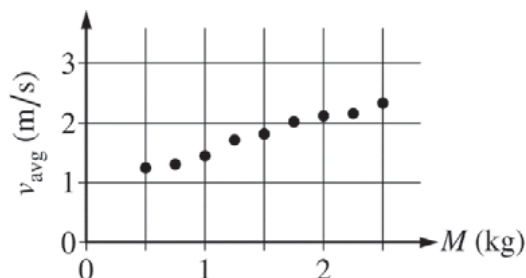
____ Greater than ____ Less than ____ The same as

Briefly explain your reasoning.

- (d) Before deriving an equation for a quantity such as v_{avg} , it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed:

$$v_{\text{avg}} = C \frac{Mg \sin \theta}{d}, \text{ where } C \text{ is a positive constant.}$$

- i. To test the equation, the student rolls a cart down the long track with speed bumps many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M .



Are these data consistent with the student's equation?

☐ Yes ☐ No

Briefly explain your reasoning.

- ii. Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense?

☐ Yes ☐ No

Briefly explain your reasoning.