Question 1: Mathematical Routines (MR) — 10 points

A **(i)** For sketching **one** of the following: **Point A1**

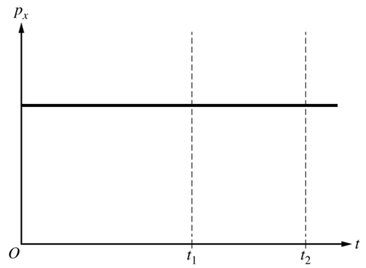
- A constant p_x for $t < t_1$
- A constant p_x for $t > t_1$

For sketching a line that demonstrates momentum is constant **Point A2**

Accept **one** of the following:

- A continuous, nonzero, horizontal line from $t = 0$ to $t = t_2$
- A continuous, nonzero, horizontal line from $t = 0$ to $t > t_2$

Example Response



(ii) For including a conservation of momentum equation **Point A3**

Scoring Note: Part A (ii) and part A (iii) may be scored together, if necessary.

For setting $m_c v_c$ equal to the momentum of the block-cart system after the collision **Point A4**

Scoring Note: A correct, isolated, final expression of $v_f = \frac{5}{6}v_c$ earns points A3 and A4.

Example Response

$p_i = p_f$

$m_c v_c = (m_c + m_b)v_f = \left(m_c + \frac{1}{5}m_c\right)v_f$

$v_f = \frac{5}{6}v_c$

(iii) For a multistep derivation that includes the correct relationship between kinetic energy, mass, and speed (i.e., $K = \frac{1}{2}mv^2$) **Point A5**

Scoring Note: The minimum requirement to earn this point is to show an expression of kinetic energy or change in energy that goes beyond the given equation on the reference information. For example, substituting quantities from the problem into $K = \frac{1}{2}mv^2$ or indicating the change in kinetic energy is $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ earns the point.

For an indication that m_c and $\frac{6}{5}m_c$ are the initial and final masses, respectively, of the objects moving horizontally **Point A6**

For including an expression for v_f consistent with part A (ii) **Point A7**

Scoring Note: A correct, isolated, final expression of $\Delta K = -\frac{1}{12}m_c v_c{}^2$ earns points A3, A4, A6, and A7.

Example Response

$\Delta K = K_f - K_0$

$K_0 = \frac{1}{2}m_c v_c{}^2$

$K_f = \frac{1}{2}\left(\frac{6}{5}m_c\right)\left(\frac{5}{6}v_c\right)^2 = \frac{5}{12}m_c v_c{}^2$

$\Delta K = -\frac{1}{12}m_c v_c{}^2$

B For indicating "Remains constant" **Point B1**

Scoring Note: This point is scored independently from the graph drawn in Figure 2 in part A (i).

For indicating **one** of the following: **Point B2**

- The frictional force exerted on the new block by the cart is equal in magnitude and opposite in direction to the frictional force exerted on the cart by the new block.
- The frictional force is internal to the new block-cart system.
- The net external force is still zero.
- The frictional force does not cause a net external force.

For indicating **one** of the following: **Point B3**

- The change in momentum of the new block is equal in magnitude and opposite in direction to the change in momentum of the cart.
- The impulse exerted on the cart and the impulse exerted on the new block are equal in magnitude and opposite in direction.
- The correct relationship between the change in the momentum of the block-cart system and the net external force.

Example Response

The momentum of the new block-cart system will remain constant because the frictional force exerted on the new block by the cart is internal to the system and only a net external force will cause a change in momentum of the system.

Question 2: Translation Between Representations (TBR) — 12 points

A For drawing one bar in Figure 2 that shows only gravitational potential energy (U_g) **Point A1**

Scoring Note: The correct height of the bar is not required to earn this point.

For including only K and U_g in Figure 3 **Point A2**

Scoring Note: The correct heights of the bars are not required to earn this point.

For drawing bars in figures 2 and 3 whose total height, respectively, equals $12E_0$ **Point A3**

Example Response

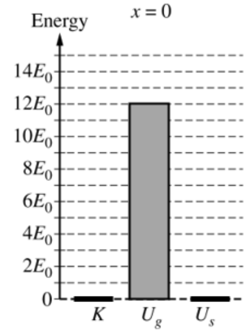


Figure 2

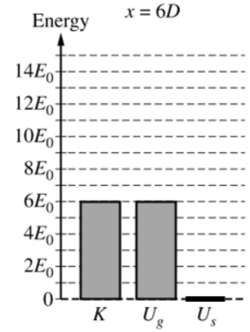
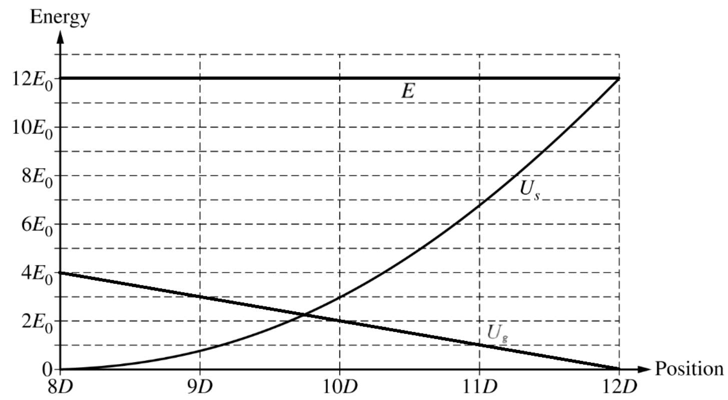


Figure 3

B	For a multistep derivation that includes conservation of energy	**Point B1**
	For equating the gravitational potential energy to the spring potential energy	**Point B2**
	For **one** of the following: • Substituting $12D\sin\theta$ for Δy in a gravitational potential energy expression • Substituting $4D$ in for Δx in a spring potential energy expression	**Point B3**
	For a correct expression for k Accept any algebraic equivalent of **one** of the following: • $k = \frac{3}{2}\frac{Mg\sin\theta}{D}$ • $\frac{3}{2}\frac{Mg\sin\theta}{D}$ **Scoring Note:** A correct, isolated, final expression of $k = \frac{3}{2}\frac{Mg\sin\theta}{D}$ earns points B2, B3, and B4.	**Point B4**
	Example Response $E_0 = E_f$ $U_g = U_s$ $mg\Delta y = \frac{1}{2}k(\Delta x)^2$ $Mg(12D\sin\theta) = \frac{1}{2}k(4D)^2$ $k = \frac{3}{2}\frac{Mg\sin\theta}{D}$	

C	**(i)**	For sketching a horizontal line at $12E_0$, continuous from $8D$ to $12D$, with a label that indicates total mechanical energy	**Point C1**
	(ii)	For sketching a straight line that is decreasing with a label that indicates gravitational potential energy	**Point C2**
		For starting the line that represents gravitational potential energy at the point $(8D, 4E_0)$	**Point C3**

Example Response



D	For **one** of the following: • Indicating $v_{9D} > v_{8D}$ if the graph in part C shows $U_g + U_s$ at $x = 8D$ is **greater than** $U_g + U_s$ at $x = 9D$ • Indicating $v_{9D} < v_{8D}$ if the graph in part C shows $U_g + U_s$ at $x = 8D$ is **less than** $U_g + U_s$ at $x = 9D$ • Indicating $v_{9D} = v_{8D}$ if the graph in part C shows $U_g + U_s$ at $x = 8D$ is **equal to** $U_g + U_s$ at $x = 9D$	**Point D1**
	For a justification consistent with the graph that correctly relates the speed to the gravitational potential energy, spring potential energy, and either kinetic energy or total energy	**Point D2**
	Example Response *The speed at $x = 9D$ is greater than the speed at $x = 8D$. From the graph, the kinetic energy at $x = 8D$ must be $8E_0$. At $x = 9D$, the spring potential energy is $< E_0$ and the gravitational potential energy is $3E_0$, leaving $> 8E_0$ of the energy as kinetic. Therefore, the speed must be greater at $x = 9D$ than at $x = 8D$.*	

Question 3: Experimental Design and Analysis (LAB) — 10 points

A For describing a procedure that includes **both** of the following: **Point A1**

- Attaching the block to the meterstick
- Measuring the force exerted on the left end of the meterstick while the block is attached

For a procedure that indicates a reasonable method of reducing experimental uncertainty **Point A2**

Examples of acceptable responses may include the following:

- Repeating the force measurement for one location of the block
- Collecting force measurements for multiple locations of the block

Example Response

Attach the block of unknown mass m_0 *through one of the small holes in the meterstick. Measure the distance from the stand to the block. Record the force reading from the spring scale when the meterstick is horizontal. Repeat data collection by attaching the block to the meterstick through a different small hole in the meterstick. Repeat the measurements by attaching the block at each of the small hole locations numerous times.*

B For indicating appropriate quantities that can be plotted so that m_0 can be determined **Point B1**

Scoring Notes:

- Any response that correctly identifies the functional dependence between force and distance earns this point, regardless of any coefficients that contain numbers or physical/fundamental constants, or which axis is chosen to graph each of those quantities.
- This point may be earned independently of the response in part A.

For describing how the slope is related to the unknown mass m_0 **Point B2**

Examples of acceptable responses may include the following:

Graph	Analysis
Force as a function of the distance from the stand to the block	The slope is equal to m_0 times g divided by the distance from the stand to the spring scale. OR m_0 is equal to the slope times the distance from the stand to the spring scale divided by g.
Force as a function of the ratio of the distance from the stand to the block to the distance from the stand to the spring scale	The slope is equal to m_0 times g. OR m_0 is equal to the slope divided by g.
Force as a function of g times the distance from the stand to the block	The slope is equal to m_0 divided by the distance from the stand to the spring scale. OR m_0 is equal to the slope times the distance from the stand to the spring scale.
Force as a function of g times the ratio of the distance from the stand to the block to the distance from the stand to the spring scale	The slope is equal to m_0. OR m_0 is equal to the slope.
Force times the distance from the stand to the spring scale as a function of the distance from the stand to the block	The slope is equal to m_0 times g. OR m_0 is equal to the slope divided by g.
Force times the distance from the stand to the spring scale as a function of g times the distance from the stand to the block	The slope is equal to m_0. OR m_0 is equal to the slope.

Example Response

A graph of the force as a function of the distance from the stand to the block could be used to determine the unknown mass. The slope of the graph times the distance from the stand to the spring scale divided by g *will result in the value of the unknown mass* m_0.

C	**(i)**	For listing a quantity that could be plotted on the vertical axis to produce a linear graph whose slope can be used to determine the mass of the meterstick[1]	**Point C1**

Examples of acceptable responses may include the following:

- F_T
- $\dfrac{F_T}{g}$
- $\dfrac{6}{5}\dfrac{F_T}{g}$
- $\dfrac{6}{5}F_T$

Scoring Note: Any response that correctly identifies the functional dependence between tension and angle earns this point, regardless of any coefficients that contain numbers or physical/fundamental constants.

Example Response

Vertical axis: F_T

(ii)	For labeling the vertical axis (including units) with a linear scale	**Point C2**
	For correctly plotting data points consistent with **one** of the following:	**Point C3**

- The quantities indicated in part C (i)
- The quantities indicated on the axes
- The quantities indicated in Table 2

(iii)	For drawing an appropriate best-fit line that approximates the trend of the plotted data	**Point C4**

Scoring Note: If the graph produced is nonlinear, then a line or curve that approximates the trend of that data can earn this point.

Example Response

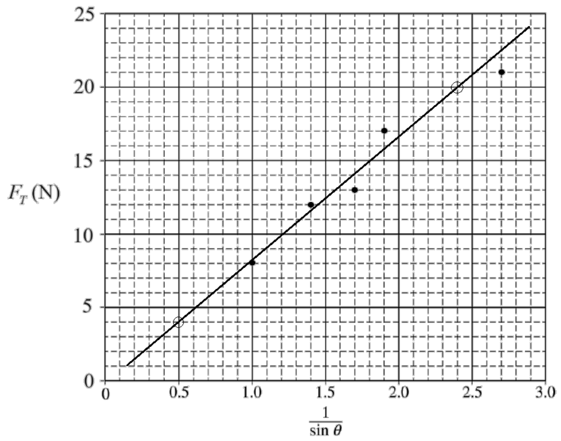


Figure 3

D	For correctly relating the slope of the best-fit line to the value of M	Point D1

Examples of acceptable responses may include the following:

Graph	Analysis
F_T vs. $\dfrac{1}{\sin\theta}$	$\text{slope} = \dfrac{5}{6}Mg$ OR $M = \dfrac{6}{5}\dfrac{\text{slope}}{g}$
$\dfrac{F_T}{g}$ vs. $\dfrac{1}{\sin\theta}$	$\text{slope} = \dfrac{5}{6}M$ OR $M = \dfrac{6}{5}\text{slope}$
$\dfrac{6}{5}\dfrac{F_T}{g}$ vs. $\dfrac{1}{\sin\theta}$	$\text{slope} = M$ OR $M = \text{slope}$
$\dfrac{6}{5}F_T$ vs. $\dfrac{1}{\sin\theta}$	$\text{slope} = Mg$ OR $M = \dfrac{\text{slope}}{g}$

For a value of M within a range of 0.90 kg to 1.15 kg **Point D2**

Example Response

$$F_T = \frac{5Mg}{6\sin\theta}$$
$$F_T = \frac{5}{6}Mg\frac{1}{\sin\theta}$$
$$\text{slope} = \frac{5}{6}Mg$$
$$M = \frac{6(\text{slope})}{5g}$$

$$\text{slope} = \frac{20\text{ N} - 4\text{ N}}{2.4 - 0.5}$$
$$\text{slope} = 8.4\text{ N}$$
$$M = \frac{(6)(8.4\text{ N})}{(5)\left(9.8\ \frac{\text{N}}{\text{kg}}\right)}$$
$$M = 1.0\text{ kg}$$

Question 4: Qualitative Quantitative Translation (QQT) — 8 points

A	For indicating $a_1 < a_2$	**Point A1**
	For a justification that indicates that the blocks have the same weight or mass	**Point A2**
	For a justification that indicates that the salt water exerts a larger buoyant force than the freshwater exerts (or the freshwater exerts a smaller buoyant force than the saltwater exerts)	**Point A3**

Example Response

There are identical downward forces (mg) on the block in both liquids. Each submerged block also has an upward buoyant force. Because the buoyant force is proportional to the density of the fluid, the block in salt water has a larger buoyant force exerted on it. Therefore, the block in salt water has a larger net force and so has a larger acceleration.

B	For a multistep derivation that includes Newton's second law	**Point B1**
	For indicating ρVg is the buoyant force	**Point B2**
	For a correct expression for the initial upward acceleration a, consistent with the indicated expression for buoyant force **Scoring Note:** A correct, isolated, final expression for a earns points B2 and B3.	**Point B3**

Example Response

$$\Sigma F = m_{sys}a_{sys}$$
$$F_B - mg = ma$$
$$\rho Vg - mg = ma$$
$$a = \frac{\rho Vg}{m} - g$$

C	For attempting to address the functional dependence between a and ρ in the equation derived in part B **Scoring Note:** It is not necessary to use the functional dependence correctly to earn this point. The response only needs to use functional dependence language such as proportional, inversely proportional, related, numerator, and denominator, to relate a and ρ.	**Point C1**
	For correctly using functional dependence to evaluate the relationship between a and ρ consistent with the expression derived in part B and the claim made in part A	**Point C2**

Example Response

The expression for the acceleration a is consistent with part A. In part A, I said the acceleration would be larger in salt water because the density is greater. In my expression, the acceleration increases with increasing density because of the $\frac{\rho Vg}{m}$ *term.*

Question 2: Experimental Design — 12 points

(a)(i)	For indicating two quantities that, when graphed together, produce a straight line whose slope can be used to determine the acceleration a	**1 point**

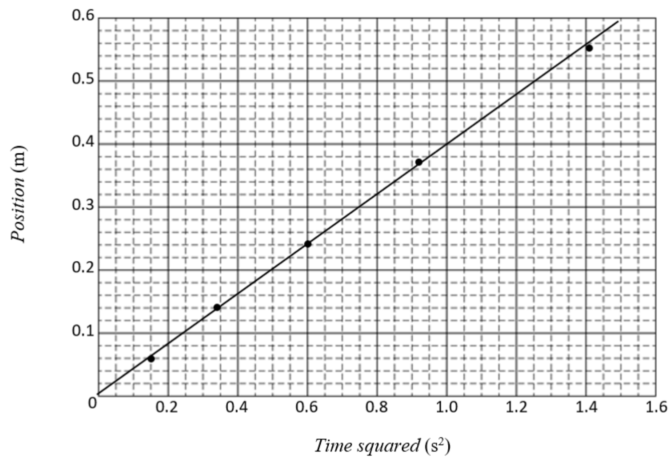
Example Response

Vertical Axis : *Position*   Horizontal Axis : *Time squared*

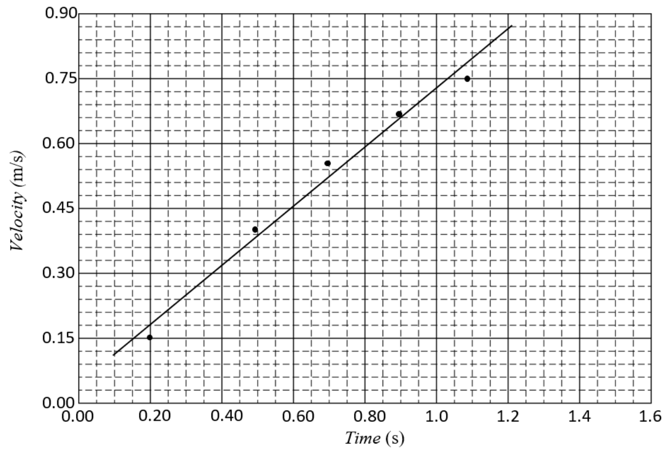
Position x (m)	Time t (s)	Time squared t^2 $\left(s^2\right)$
0.06	0.39	0.15
0.14	0.59	0.35
0.24	0.77	0.59
0.37	0.96	0.92
0.55	1.20	1.44

(a)(ii)	The axes have a linear scale and are identified (labels **OR** units) so that when graphed correctly, the data will span more than half of the horizontal and vertical axes	**1 point**
	For plotting at least 4 of the data points correctly	**1 point**
	For drawing a best-fit line that approximates the trend of the data	**1 point**

Example Response



Alternate Example Response



Scoring Note: The following tables represent the most common linearized graphs with the data that were used to determine the acceleration.

Graph: v vs. t	
$v\left(\frac{m}{s}\right)$	t (s)
0.15	0.20
0.40	0.49
0.56	0.68
0.68	0.87
0.75	1.08

Graph: $2x$ vs. t^2	
$2x$ (m)	t^2 $\left(s^2\right)$
0.12	0.15
0.28	0.35
0.48	0.59
0.74	0.92
1.10	1.44

Graph: $2v_{avg}$ vs. t	
$2v_{avg}\left(\frac{m}{s}\right)$	t (s)
0.31	0.39
0.47	0.59
0.62	0.77
0.77	0.96
0.92	1.20

Graph: x vs. $\frac{1}{2}t^2$	
x (m)	$\frac{1}{2}t^2$ $\left(s^2\right)$
0.06	0.08
0.14	0.17
0.24	0.30
0.37	0.46
0.55	0.72

Graph: $v_{avg}{}^2$ vs. x	
$v_{avg}{}^2\left(\frac{m^2}{s^2}\right)$	x (m)
0.02	0.06
0.06	0.14
0.10	0.24
0.15	0.37
0.21	0.55

Graph: $\sqrt{x}$ vs. t	
$\sqrt{x}$ $\left(\sqrt{m}\right)$	t (s)
0.24	0.39
0.37	0.59
0.49	0.77
0.61	0.96
0.74	1.20

(a)(iii) For attempting to find the slope, $\left(\frac{\text{rise}}{\text{run}}\right)$ or $\left(\frac{\Delta y}{\Delta x}\right)$, of the best-fit line drawn in part (a)(ii) **1 point**

Scoring Note: An indication that a calculator was used for linear regression to determine the value of the slope may earn this point.

For using the slope in a valid kinematic equation to calculate the acceleration **1 point**

Scoring Note: This point can be earned if evidence of a kinematic equation exists in graphed quantities (e.g., a graph of position as a function of $\frac{1}{2}t^2$).

Example Response

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta\ \text{position}}{\Delta\ \text{time}^2} = \frac{0.48\ \text{m} - 0.18\ \text{m}}{1.2\ \text{s}^2 - 0.4\ \text{s}^2} = 0.375\ \frac{\text{m}}{\text{s}^2}$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$\frac{\Delta x}{t^2} = \frac{1}{2}a$$

$$\text{slope} \times 2 = a$$

$$a = 0.75\ \frac{\text{m}}{\text{s}^2}$$

Total for part (a) 6 points

(b)(i) For indicating a quantity to be measured **1 point**

Accept **one** of the following:

- The angle θ with the horizontal
- The height h and length L of the ramp

Scoring Note: Stating only the height needs to be measured can earn this point if an energy approach is used.

(b)(ii) For providing a correct expression relating the acceleration of gravity to the acceleration measured **1 point**

Scoring Note: If $\cos\theta$ is used, the response must specify that θ was measured from the vertical.

Example Response

$$mg_{\text{exp}} \sin\theta = ma$$

$$g_{\text{exp}} = \frac{a}{\sin\theta}$$

OR

$$\sin\theta = \frac{h}{L}$$

$$g_{\text{exp}} = \left(\frac{L}{h}\right)a$$

OR

$$mg_{\text{exp}}h = \frac{1}{2}mv^2$$

$$g_{\text{exp}}h = \frac{1}{2}v^2$$

$$v = \sqrt{2g_{\text{exp}}h}$$

$$v = at$$

$$at = \sqrt{2g_{\text{exp}}h}$$

$$g_{\text{exp}} = \frac{a^2t^2}{2h}$$

Total for part (b) 2 points

(c)(i) For identifying a physical factor that could have affected the result **1 point**

Accept **one** of the following:

- A physical factor in the materials used (e.g., the wheels have nonnegligible rotational inertia, the ramp was bumpy, the wheels were wobbly or not perfectly round, the base of the ramp was not level, the floor was not level.)
- A physical factor in the environment (e.g., the room was being accelerated, elevator, the experiment was performed at high elevation or on a different planet.)
- A physical error in measurement collection (e.g., time, position, or angle was measured incorrectly.)

Scoring Note: A statement of "Human error" does not earn this point.

(c)(ii) For correctly indicating the functional dependence between the reason listed in part (c)(i) and g_{exp} **1 point**

Accept **one** of the following:

- Correctly indicating the functional dependence between the physical factor in the materials used and g_{exp} (e.g., if the rotational inertia of the rotating wheels is nonnegligible, the cart will have a smaller acceleration and g_{exp} will be smaller.)
- Correctly indicating the functional dependence between the physical factor in the environment and g_{exp} (e.g., if the experiment was performed at a high elevation, the acceleration will be smaller and g_{exp} will be smaller.)
- Correctly indicating the functional dependence between the physical error in the measurement collection and g_{exp} (e.g., if the angle of the ramp is smaller than the measured value, the cart will have a smaller acceleration and g_{exp} will be smaller.)

Example Response

The expression I derived for the value for g_{exp} *did not take into consideration that the wheels had any rotational inertia. If the wheels have rotational inertia and are rotating, the acceleration of the cart would be less than* $g\sin\theta$*, so the value of* g_{exp} *would be less than* $9.8\ \frac{\text{m}}{\text{s}^2}$.

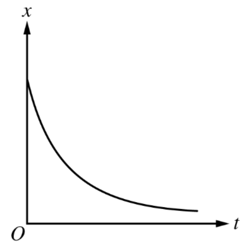
Total for part (c) 2 points

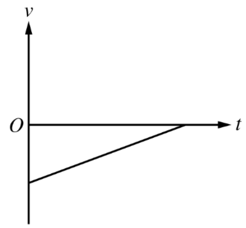
(d) For sketching a concave up curve with an initially negative slope for the graph of position as a function of time **1 point**

For **one** of the following: **1 point**

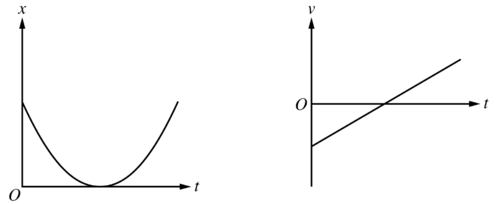
- Drawing a line with a positive slope and a negative vertical intercept for the v vs. t graph
- Drawing a v vs. t graph that is consistent with the x vs. t graph that shows acceleration

Example Response

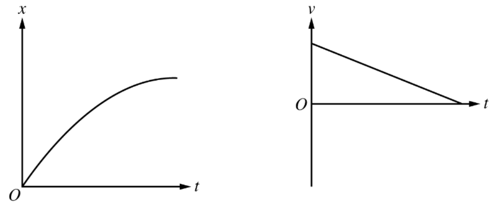




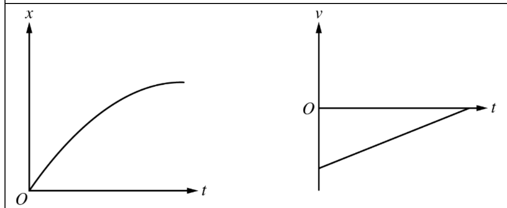
Scoring Note: The following are alternate example graphs with the points the response would earn.



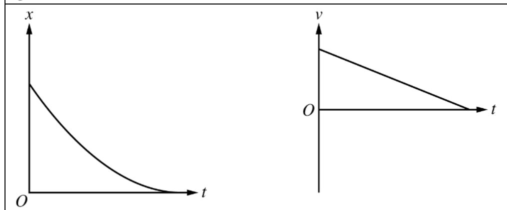
Earns 2 points



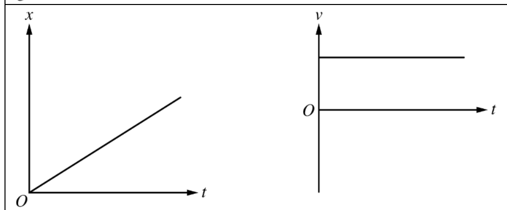
Earns 1 point



Earns 1 point



Earns 1 point



Earns 0 points

Total for part (d) 2 points

Total for question 2 12 points

Question 4: Short Answer Paragraph Argument — 7 points

(a) For a correct expression for the angular acceleration of the pulley in terms of the appropriate quantities: $\alpha_D = \dfrac{2F_T}{MR}$ — **1 point**

Example Response

$$\alpha_D = \frac{RF_T}{\frac{1}{2}MR^2} \quad \textbf{\textit{OR}} \quad \alpha_D = \frac{2F_T}{MR}$$

Total for part (a) 1 point

(b) For indicating that the torque, τ, is the same for both pulleys — **1 point**

For indicating that the impulse, $\tau\Delta t$, (or change in momentum ΔL) is the same for both pulleys because τ and Δt are the same — **1 point**

For indicating that the rotational inertia, I, of the disk and hoop are different — **1 point**

For providing reasoning that because the rotational inertia, I, are different for the disk and hoop, the kinematic quantities ($\Delta\theta$, ω, α) are also different for the disk and hoop — **1 point**

For **one** of the following: — **1 point**

- Relating I and ω to reason that ΔK is greater for the disk
- Indicating that because $\Delta\theta$ is greater for the disk the work done on the disk is greater

For a logical, relevant, and internally consistent argument that follows the guidelines described in the published requirements for the paragraph-length response — **1 point**

Example Response

The rotational inertia, I, of the hoop is larger than the rotational inertia of the disk because the hoop's mass is all on the outside instead of distributed throughout like the disk. Equal forces are applied to both pulleys at the same distance, which means that the torques exerted on the pulleys will also be equal. Since the same torque is applied to both pulleys for the same time period, the change in angular momentum will be the same for the disk and hoop. The magnitude of the angular velocity for the hoop will be smaller than that of the disk since angular velocity is inversely proportional to the rotational inertia $\left(\omega = \dfrac{L}{I}\right)$.

Since kinetic energy is proportional to rotational inertia and the square of angular velocity $\left(K_R = \dfrac{1}{2}I\omega^2\right)$, the difference in angular velocity more greatly affects the rotational kinetic energy. That means the disk will have a greater rotational kinetic energy than the hoop.

Total for part (b) 6 points

Total for question 4 7 points